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PROPAGATION OF RAYLEIGH WAVES IN A MICROPOLAR THERMOELASTIC HALF-SPACE WITH IMPEDANCE BOUNDARY CONDITIONS

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Abstract. This paper deals with the propagation of Rayleigh waves in a micropolar thermoelastic half space with impedance boundary conditions. The boundary of the half space is thermally insulated / isothermal and it is assumed that normal traction, shear traction and shear couple traction at the surface, varies linearly with normal, tangential components of displacement and microrotation respectively. The secular equation for Rayleigh wave with impedance boundary conditions is obtained and this equation is in agreement with the classical secular equation for elastic solid with traction free boundary conditions when micropolar, thermal and impedance parameters are removed. The non-dimensional speed of Rayleigh wave is computed as a function of impedance parameters and presented graphically for a particular micropolar thermoelastic material.

Keywords: micropolar thermoelasticity; Rayleigh waves; impedance boundary conditions; secular equation.

1. Introduction

Eringen's [1] micropolar theory of elasticity is now well known due to its possible utility in examining the deformation properties of materials such as cellular solids, polymers, composite fibrous, granular material, masonry, bones and many more with microstructures. This theory takes into account the intrinsic rotation along with linear displacement in the materials possess microstructure and the motion is governed by six degrees of freedom, three of microrotation and three of classical translation. The classical theory of elasticity, which ignores the microrotation degrees of freedoms, can explain the behavior of common solid materials like coal, concrete etc. This theory is inadequate to explain the behavior of materials with inner microstructure such as polycrystalline and materials with fibrous or coarse grain. Therefore, micropolar theory was developed to explain the microscopic motion and long-range interactions in solids. This theory is also identical with Cosserat elasticity which was further developed by Aero and Kuvshinskii [2].

Mechanical and thermal fields are associated in almost all practical engineering problems as the application of mechanical forces can change the temperature of the system. Keeping this important interaction in view, Nowacki [3] and Eringen [4] extended the micropolar theory by including thermal effects and presented linear theory of micropolar thermoelasticity. Tauchert, Claus Jr and Ariman [5] developed the linear theory of micropolar thermoelasticity and

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formulate the constitutive equations. Various problems on micropolar thermoelasticity has been investigated extensively by researcher due its applications in various fields like earthquake, nuclear reactors, aeronautics, astronautics and modern sensor devices.

The generalized theory of thermoelasticity is a modified version of classical uncoupled and coupled theory of thermoelasticity and has been developed in order to remove the paradox of impossible phenomena of infinite velocity of thermal signals in the classical coupled theory of thermoelasticity. Lord and Shulman [6] and Green and Lindsay [7] include the concept of thermal relaxation time and eliminate this paradox of infinite velocity of thermal signals. Based upon generalized theory of thermoelasticity given by Green and Lindsay, Boschi and Ieşan [8] proposed a generalized theory of linear micropolar thermoelasticity that admits the possibility of second sound effect. Ciarletta [9] established the finite speed of thermal waves by using theory of micropolar thermoelasticity without energy dissipation. Based upon Lord and Shulman theory [10] Sherief, Hamza and EI-Sayed [9] derived the generalized equation for the linear theory of micropolar thermoelasticity.

A comprehensive study is available on the phenomenon of wave propagation in micropolar-generalized thermoelastic solid because of their practical applicability in the various fields of science and technology such as, seismology, acoustics, aerospace and submarine structures. Surface waves due to their destructive nature during earthquake are of particular importance in the study of seismology. Lord Rayleigh [11] was the first to study the wave propagating along the isotropic elastic solid and such waves after his name are known as Rayleigh waves. Several researchers have explored the concept of Rayleigh waves in different type of elastic materials. For example, Lockett [12] discussed the effects of thermal properties of an isotropic thermoelastic material on velocity of Rayleigh waves. Maugin [13] studied conditions of propagation of accelerations waves in different type of micropolar media. Kumar and Singh [14] discussed about the existence Rayleigh wave in micropolar generalized thermoelastic half space with stretch. Rao and Reddy [15] studied the Rayleigh type wave propagation in a micropolar cylindrical surface. Kumar, Kaur and Rajvanshi [16] investigated the propagation of Lamb waves in micropolar-generalized thermoelastic solid with two temperatures bordered with layer of inviscid liquid. Kumar and Partap [17] studied propagation of Rayleigh Lamb waves in a micropolar elastic cylindrical plate.

The boundary conditions in almost all the problems related to Rayleigh waves are considered as a traction free surface that is stresses vanishes on the surface. The possibilities of other type of boundary conditions are rarely consider in seismology or geophysics but there are other fields of physics like electromagnetism and acoustics, where it is common to use impedance boundary conditions. The impedance boundary conditions prescribed on the boundary is the linear combination of the unknown function and their derivatives. Tiersten [18] encountered these types of boundary conditions while studying the wave propagation in an isotropic elastic solid coated with thin film of different material. Malischewsky [19] modified the Tiersten’s conditions in terms of stresses and displacement and obtained the secular equation for Rayleigh waves. Godoy, Duran and Nedelec [20] proved the existence of surface waves in an elastic half space with impedance boundary conditions and derived the secular equation with these conditions. Vinh and Hue [21] used impedance boundary conditions to investigate Rayleigh waves in an orthotropic and monoclinic half space. Recently Singh [22] studied about the Rayleigh wave in a thermoelastic solid half space subjected to impedance boundary conditions.

Rayleigh waves are extremely useful for material characterization and to remove defects in the objects, as these are very sensitive to surface defects. Very few papers on Rayleigh waves with impedance boundary conditions are available but this concept has not been used in micropolar thermoelastic material. In this paper the propagation Rayleigh waves in a micropolar thermoelastic half space with impedance boundary condition has been investigated. Secular equation for thermally insulated and isothermal surface is obtained and this equation coincides with the secular equation of Rayleigh waves in thermoelastic solid when the micropolar effect is removed. On removing the micropolar effects, impedance parameters and thermal effects this equation reduces to famous secular equation of Rayleigh wave in isotropic elastic solid with traction free boundary conditions. Effect of micropolarity present in the medium on the phase velocity is highlighted through comparative study with respect impedance parameter.

2. Basic equations

Following Eringen [1] the governing equations of motion for homogeneous, isotropic micropolar thermoelastic solid are:

$$(\mu + K)\nabla^2\vec{u} + (\lambda + \mu)\nabla(\nabla.\vec{u}) + K\nabla \times \vec{\phi} - \nu\nabla T = \rho\frac{\partial^2\vec{u}}{\partial t^2} \tag{1}$$

$$(\alpha + \beta + \gamma)\nabla(\nabla.\vec{\phi}) - \gamma\nabla \times (\nabla \times \vec{\phi}) + K\nabla \times \vec{u} - 2K\vec{\phi} = \rho j\frac{\partial^2\vec{\phi}}{\partial t^2}, \tag{2}$$

where $\vec{u}$ is the displacement vector, ρ is the density of the material, j is the microinertia, $\vec{\phi}$ is the microrotation vector, λ, μ are Lame's constants K, α, β, γ are micropolar isotropic material constants.

The constitutive relations are given by:

$$\sigma_{ij} = \lambda u_{r,r}\,\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + K(u_{j,i} - \epsilon_{ijr}\phi_r) - \nu T\delta_{ij} \tag{3}$$

$$m_{ij} = \alpha\,\phi_{r,r}\,\delta_{ij} + \beta\,\phi_{i,j} + \gamma\,\phi_{j,i}, \tag{4}$$

where $(i, j, r = 1,2,3)$, σ_{ij} is the stress tensor, m_{ij} is the couple stress tensor and δ_{ij} is the kronecker delta.

Following Lord and Shulman [6], the heat conduction equation is

$$K^*\nabla^2 T = \rho C^*\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)T + \nu T_0\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\nabla.\vec{u}, \tag{5}$$

where K^* is the coefficient of thermal conductivity, $\nu = (3\lambda + 2\mu + K)\alpha_t$, C^* is the specific heat at constant strain, α_t is the coefficient of thermal linear expansion, T is the change in temperature of the medium at any time, T_0 is the reference temperature of the body and τ_0 is the thermal relaxation time.

3. Formulation of the problem

We consider a homogeneous and isotropic micropolar thermoelastic half space at uniform temperature T_0 in the undeformed state. Origin is placed at the plane surface and y-axis pointing vertically downward into the half space. The direction of propagation of the waves is considered along x-axis so that all particles vibrating on a line parallel to z-axis are equally displaced. Therefore, all the field quantities will be independent of z-coordinates. For the two-dimensional problem, we assume the components of the displacement $\vec{u}$ and microrotation vector $\vec{\phi}$ of the form

$$\vec{u} = (u, v, 0),\ \vec{\phi} = (0, 0, \phi) \tag{6}$$

Using (6), equation (1) and (2) can be written as:

$$(\lambda + 2\mu + K)\frac{\partial^2 u}{\partial x^2} + (\mu + K)\frac{\partial^2 u}{\partial y^2} + (\lambda + \mu)\frac{\partial^2 v}{\partial x\partial y} + K\frac{\partial\phi}{\partial y} - \nu\frac{\partial T}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2} \tag{7}$$

$$(\lambda + 2\mu + K)\frac{\partial^2 v}{\partial y^2} + (\mu + K)\frac{\partial^2 v}{\partial x^2} + (\lambda + \mu)\frac{\partial^2 u}{\partial x\partial y} - K\frac{\partial\phi}{\partial x} - \nu\frac{\partial T}{\partial y} = \rho\frac{\partial^2 v}{\partial t^2} \tag{8}$$

$$\gamma\left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}\right) + K\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - 2K\phi = \rho j\frac{\partial^2\phi}{\partial t^2} \tag{9}$$

Using Helmholtz's representation the displacement components u and v can be written in terms of potential functions as:

$$u = \frac{\partial\phi_1}{\partial x} + \frac{\partial\psi_1}{\partial y},\ v = \frac{\partial\phi_1}{\partial y} - \frac{\partial\psi_1}{\partial x} \tag{10}$$

Substituting (10) in equations (5) and (7)-(9), we obtained:

$$(\lambda + 2\mu + K)\nabla^2\phi_1 - \nu T = \rho \frac{\partial^2 \phi_1}{\partial t^2} \quad (11)$$

$$(\mu + K)\nabla^2\psi_1 + K\phi = \rho \frac{\partial^2 \psi_1}{\partial t^2} \quad (12)$$

$$\gamma\nabla^2\phi - 2K\phi - K\nabla^2\psi_1 = \rho j \frac{\partial^2 \phi}{\partial t^2} \quad (13)$$

$$K^*\nabla^2 T = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)(\rho C^* T + \nu T_0 \nabla^2 \phi_1) \quad (14)$$

4. Solution of the Problem

The surface wave solutions of the equations (11)-(14) may be consider as:

$$\{\phi_1, \psi_1, T, \phi\} = \{\overline{\phi_1}(y), \overline{\psi_1}(y), \bar{T}(y), \bar{\phi}(y)\}\, e^{ik(x-ct)}, \quad (15)$$

where c is the phase velocity , k is the wave number, $\omega = kc$ is the circular frequency. It is assumed that the Rayleigh surface waves possibly damped in time, propagating along x-axis with wave speed $Re(c) = V > 0$ and $Im(c) \leq 0$.

Using (15) in the equations (11) – (14), we have:

$$[D^4 - AD^2 + B](\overline{\phi_1}(y), \bar{T}(y)) = 0 \quad (16)$$

$$[D^4 - A'D^2 + B'](\overline{\psi_1}(y), \bar{\phi}(y)) = 0 \quad (17)$$

Here

$$D = \frac{d}{dy},\ A = k^2\left[2 - \frac{c^2\left(1 + A_2 + \frac{A_1}{c_1^2}\right)}{A_1}\right],\ B = k^4\left[\frac{A_1 - c^2\left(1 + A_2 + \frac{A_1}{c_1^2}\right) + \frac{c^4}{c_1^2}}{A_1}\right]$$

$$A' = k^2\left(1 - \frac{c^2}{c_2^2}\right) + k^2 - \frac{k^2c^2\rho j}{\gamma} + \frac{2K}{\gamma} - \frac{K^2}{\gamma(\mu + K)} \quad (18)$$

$$B' = k^2\left(k^2 - \frac{k^2c^2\rho j}{\gamma} + \frac{2K}{\gamma}\right)\left(1 - \frac{c^2}{c_2^2}\right) - \frac{k^2K^2}{\gamma(\mu + K)}$$

$$c_1^2 = \frac{\lambda + 2\mu + K}{\rho},\ c_2^2 = \frac{\mu + K}{\rho},\ \tau^* = \tau_0 + \frac{i}{\omega},\ A_1 = \frac{K^*}{\rho C^* \tau^*},\ A_2 = \frac{\nu^2 T_0}{\rho^2 c_1^2 C^*}$$

Using the radiation conditions $\overline{\phi_1}(y), \overline{\psi_1}(y), \bar{T}(y), \bar{\phi}(y) \to 0$ as $y \to \infty$ on the general solutions of the equations (16) and (17) and using (15), we obtained

$$\phi_1 = (B_1 e^{-kb_1 y} + B_2 e^{-kb_2 y})e^{ik(x-ct)} \quad (19)$$

$$\psi_1 = (B_3 e^{-kb_3 y} + B_4 e^{-kb_4 y})e^{ik(x-ct)} \quad (20)$$

$$T = (r_1 B_1 e^{-kb_1 y} + r_2 B_2 e^{-kb_2 y})e^{ik(x-ct)} \quad (21)$$

$$\phi = (r_3 B_3 e^{-kb_3 y} + r_4 B_4 e^{-kb_4 y})e^{ik(x-ct)}, \quad (22)$$

where

$$b_1^2 + b_2^2 = \frac{A}{k^2},\ b_1^2 b_2^2 = \frac{B}{k^4},\ b_3^2 + b_4^2 = \frac{A'}{k^2},\ b_3^2 b_4^2 = \frac{B'}{k^4} \quad (23)$$

$$\begin{cases} r_i = k^2\left[\frac{(b_i^2 - 1)(\lambda + 2\mu + K) + \rho c^2}{\nu}\right], & (i = 1,2) \\ r_j = \frac{k^2(\mu + K)}{K}\left[1 - \frac{c^2}{c_2^2} - b_j^2\right], & (j = 3,4) \end{cases} \quad (24)$$

and B_1 , B_2 , B_3 and B_4 are arbitrary constants.

5. Boundary conditions and secular equation

The general form of impedance boundary conditions in two dimensions in terms of displacements and stresses given by Malischewsky [19] can be written as $\sigma_{i2} + \epsilon_i u_i = 0$, for $y = 0$, where ϵ_i are the impedance parameters and have the dimensions of stress/length. For elastic half space, Godoy, Duran and Nedelec [20], expressed ϵ_i as $\epsilon_i = \omega Z_i$. Here Z_i are impedance real valued parameters, has dimensions of stress/velocity and $\omega = kc$ is the circular frequency. Here the impedance boundary conditions at the surface

$y = 0$ of a micropolar thermoelastic solid are consider as $\sigma_{2i} + \omega Z_i u_i = 0$, which can be written as:

$$\sigma_{21} + \omega Z_1 u = 0, \sigma_{22} + \omega Z_2 v = 0, m_{23} + \omega Z_3 \phi = 0, \frac{\partial T}{\partial y} + hT = 0, \quad (25)$$

where $h \to 0$ corresponds to thermally insulated surface and $h \to \infty$ corresponds to isothermal surface.

Imposing boundary conditions (25) on the surface $y = 0$, we get a system of four homogeneous equations. For a non-trivial solution the determinant of the coefficients B_1, B_2, B_3 and B_4 must vanishes which yields the following secular equation for the velocity of propagation of the Rayleigh waves:

$$m_1[T_1(l_2 n_4 - n_2 l_4) - T_2(l_1 n_4 - n_1 l_4)] = m_2[T_1(l_2 n_3 - n_2 l_3) - T_2(l_1 n_3 - n_1 l_3)], \quad (26)$$

where

$$l_i = V_1 Z_1^* - b_i - \left(1 + \frac{K}{\mu}\right) b_i, \qquad (i = 1,2)$$

$$l_j = V_1 Z_1^* b_j - 1 - \left(1 + \frac{K}{\mu}\right)\left(1 - \frac{c^2}{c_2^2}\right), \quad (j = 3,4)$$

$$n_i = 2 + \frac{K}{\mu} - V_1^2 - V_1 Z_2^* b_i, \qquad (i = 1,2)$$

$$n_j = \left(2 + \frac{K}{\mu}\right) b_j - V_1 Z_2^*, \qquad (j = 3,4)$$

$$m_1 = (\mu V_1 Z_3^* - \gamma b_3)\left(1 - \frac{c^2}{c_2^2} - b_3^2\right), \quad m_2 = (\mu V_1 Z_3^* - \gamma b_4)\left(1 - \frac{c^2}{c_2^2} - b_4^2\right)$$

$$V_1 = \sqrt{\frac{\rho c^2}{\mu}}, \qquad Z_i^* = \frac{Z_i}{\sqrt{\rho\mu}}, \qquad (i = 1,2,3)$$

For thermally insulated surface:

$$T_i = b_i \left[\left(2 + \frac{\lambda + K}{\mu}\right)(b_i^2 - 1) + V_1^2\right], \qquad (i = 1,2)$$

For isothermal surface:

$$T_i = \left[\left(2 + \frac{\lambda + K}{\mu}\right)(b_i^2 - 1) + V_1^2\right], \qquad (i = 1,2)$$

6. Particular cases

1. In the absence of micropolar effect, the equation (26) reduces to the secular equation for the phase velocity of Rayleigh waves in a thermoelastic half space with impedance boundary conditions. Neglecting micropolar constants ($K = j = 0$) in the condition (18), we obtained:

$$A' = k^2\left(1 - \frac{c^2}{c_2^2}\right) + k^2, \; B' = k^4\left(1 - \frac{c^2}{c_2^2}\right)$$

Using equation (23), we get:

$b_3^2 = 1 - \frac{c^2}{c_2^2}$, $b_4^2 = 1$, $m_1 = 0$ and m_2 be a non-zero value.

Consequently, the secular equation (26) reduces to:

$$l_3(n_1 T_2 - n_2 T_1) - n_3(l_1 T_2 - l_2 T_1) = 0. \quad (27)$$

The equation (27) coincides with the secular equation, obtained by author Singh [22] for Rayleigh waves in thermoelastic solid half space with impedance boundary conditions.

2. Further equation (27) reduces to secular equation for Rayleigh wave velocity with traction free boundary conditions when $Z_i^* = 0$, $(i = 1,2,3)$.

3. If we neglect the impedance parameter, micropolarity and thermal effects from the model i.e. $K = j = Z_1^* = Z_2^* = Z_3^* = \nu = 0$, the equation (26) reduces to:

$$\left(2 - \frac{c^2}{c_2^2}\right)^2 = 4\sqrt{1 - \frac{c^2}{c_1^2}}\sqrt{1 - \frac{c^2}{c_2^2}}, \quad (28)$$

where $c_1^2 = \frac{\lambda + 2\mu}{\rho}$, $c_2^2 = \frac{\mu}{\rho}$.

Equation (28) is the well-known dispersion equation for the phase velocity of Rayleigh waves in classical elastic half space.

7. Numerical results and discussions

To illustrate the theoretical results numerical computations have been carried out and non-dimensional Rayleigh wave speed has been calculated in a micropolar thermoelastic solid. The aluminum epoxy composite is taken as a micropolar thermoelastic solid and following Gauthier [23], the values of relevant physical constants of this material are
$\rho = 2.19 \times 10^3 kg/m^3$, $\lambda = 7.59 \times 10^{10} N/m^2$, $\mu = 1.89 \times 10^{10} N/m^2$,
$K = 0.0149 \times 10^{10} N/m^2$, $\alpha = 0.01 \times 10^6 N$, $\beta = 0.015 \times 10^6 N$, $\gamma = 0.268 \times 10^6 N$,
$j = 0.196 \times 10^4 m^2$, $K^* = 0.492 \times 10^2 W/m\,K$, $C^* = 1.89 \times 10^{10} J/kg.K.$, $\tau_0 = 0.5 \times 10^{-10} s$, $T_0 = 298K$, $\alpha_t = 2.36 \times 10^{-6} K^{-1}$.

Under the assumption that c is a complex constant parameter with $Re(c) = V \geq 0$, the non-dimensional Rayleigh wave speed $V_1 = \sqrt{\frac{\rho V^2}{\mu}}$ is calculated by solving secular equation (26) using functional iteration method. The effects of micropolar, impedance parameters and the dependence of Rayleigh wave speed on wave number have been shown graphically in Fig. 1 – Fig. 7. Fig. 1 – Fig. 3 depicts the influence of micropolarity on the non-dimensional Rayleigh wave speed V_1 with respect to impedance parameters Z_1^*, Z_2^* and Z_3^*. The variations of wave speed V_1 with respect to impedance parameter in a micropolar thermoelastic half space under thermally insulated and isothermal boundary conditions are presented in Fig. 4 – Fig. 6. Fig. 7 shows the variation of V_1 with wave number for different values of impedance parameters.

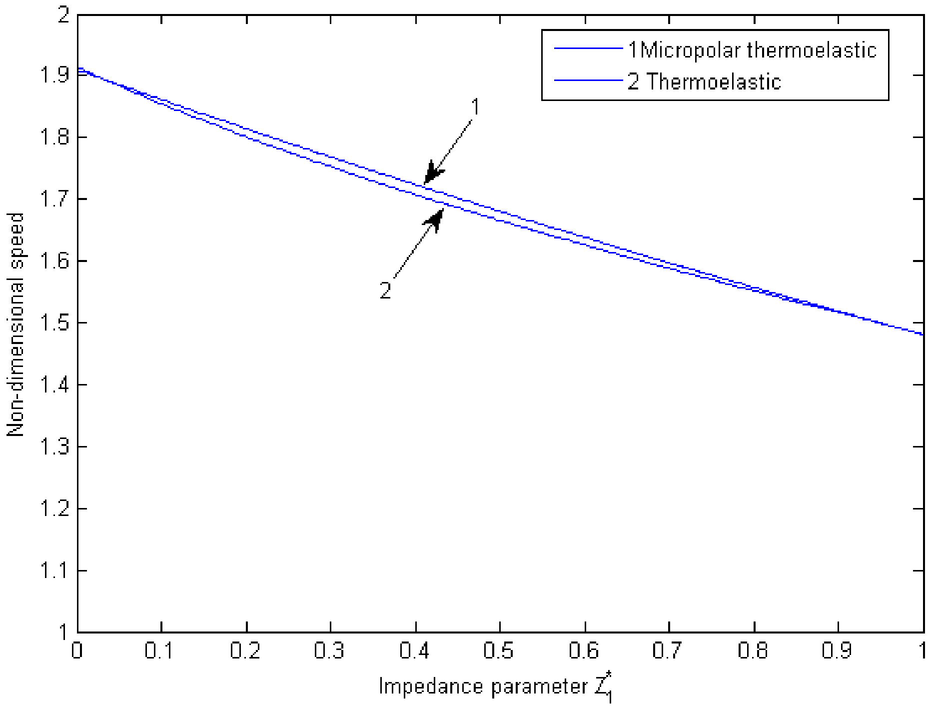

Fig. 1. Micropolar effects w.r.t. Impedance parameter $\boldsymbol{Z_1^*}$ on non-dimensional speed $\boldsymbol{V_1}$ of Rayleigh wave.

The effects of micropolar parameters on the non-dimensional wave speed V_1 are quite pertinent and can be easily noticed from the Fig. 1- Fig. 3. In Fig. 1 the non-dimensional wave speed V_1 has been plotted against the non-dimensional impedance parameter Z_1^* at constant frequency $\omega = 10\,rad/s$ and keeping the boundary free of normal and couple traction ($Z_2^* = 0, Z_3^* = 0$).

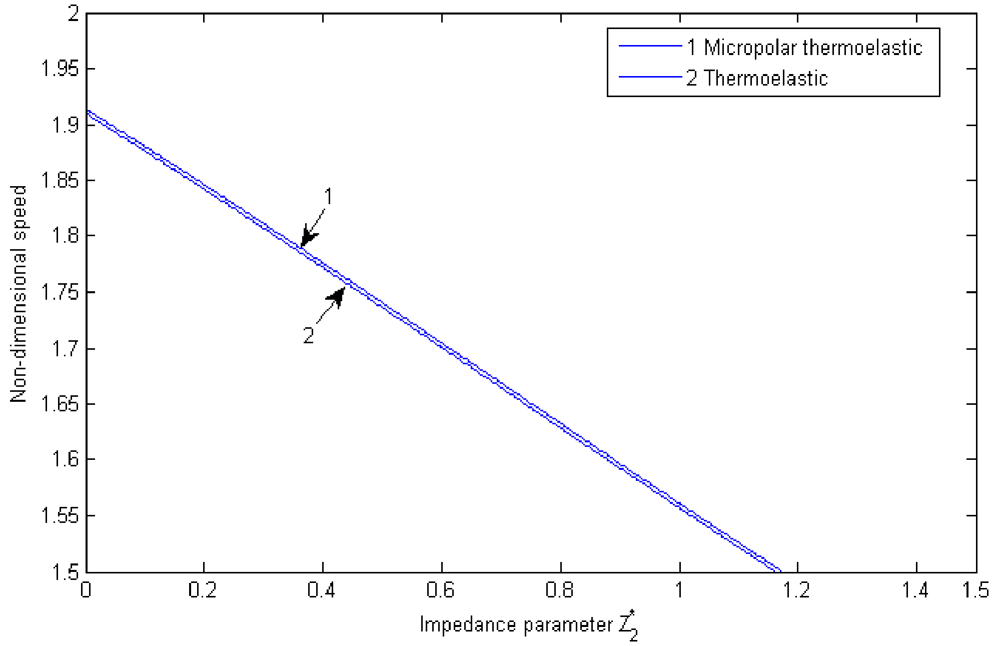

Fig. 2. Micropolar effects w.r.t. impedance parameter $\mathbf{Z_2^*}$ on non-dimensional speed $\boldsymbol{V_1}$of Rayleigh wave.

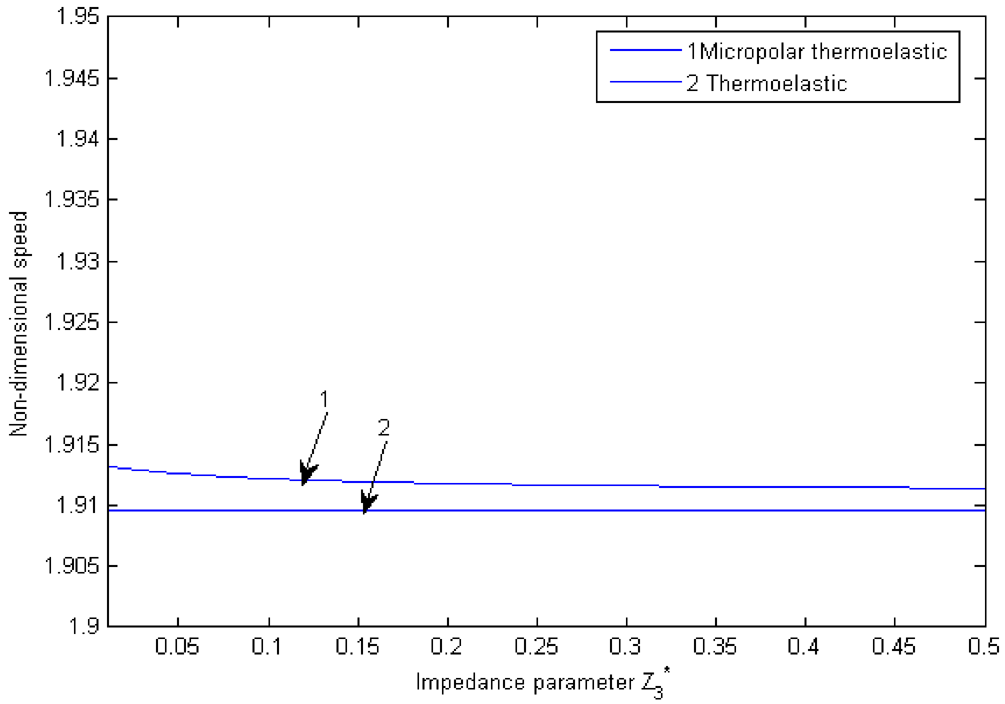

Fig. 3. Micropolar effects w.r.t. Impedance parameter $\mathbf{Z_3^*}$ on non-dimensional speed $\boldsymbol{V_1}$ Rayleigh wave.

It is observed that the inclusion of micropolar effect in a thermoelastic half space increases the non-dimensional wave speed for same values of Z_1^* in the interval $0 \leq Z_1^* \leq 1$. Fig. 2 and Fig. 3 shows the variations of wave speed V_1 with respect to non-dimensional impedance parameter Z_2^* and Z_3^* respectively keeping the other two impedance parameters fixed at zero value. From both the figures, it is clearly visible that wave speed increases in case of micropolar thermoelastic solid as compared to thermoelastic solid.

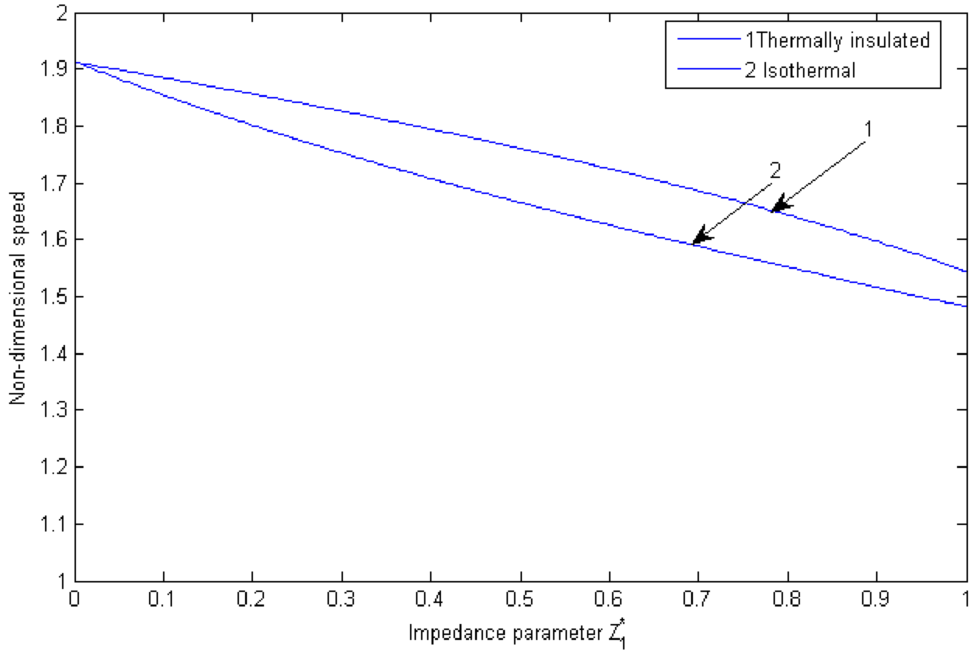

Fig. 4. Variation of non-dimensional wave speed $\boldsymbol{V_1}$ w.r.t. impedance parameter $\boldsymbol{Z_1^*}$ in a micropolar thermally insulated and isothermal half space.

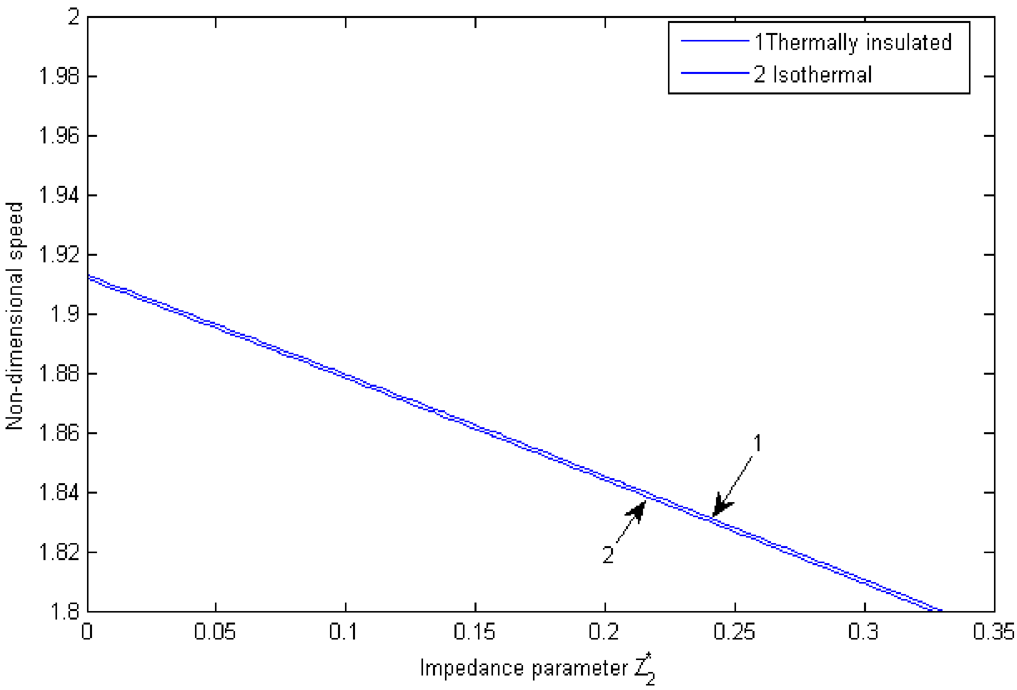

Fig. 5. Variation of non-dimensional wave speed $\boldsymbol{V_1}$ w.r.t. impedance parameter $\boldsymbol{Z_2^*}$ in a micropolar thermally insulated and isothermal half space.

Fig. 4 shows the non-dimensional wave speed of Rayleigh wave as a function of impedance parameter Z_1^* when the boundary is free of normal and couple traction $(Z_2^* = 0, Z_3^* = 0)$. Comparison of wave speed V_1 has been shown when the solid half space is subjected to thermally insulated and isothermal boundary conditions. It is noticed that the non-dimensional wave speed V_1 decreases in case of isothermal surface (curve-2) as compared to thermally insulated surface (curve-1) for same value of impedance parameter Z_1^* when (Z_2^*=Z_3^* =0, $\omega = 10\, rad/sec$). It can be seen from the graph that wave speed decreases gradually with increase in impedance parameter Z_1^* in the interval $0 \le Z_1^* \le 1$ for both thermally insulated and isothermal surface. The same pattern of variations of non-dimensional wave speed is observed with respect to the impedance parameter Z_2^* and Z_3^* as shown in Fig. 5 and Fig. 6. The wave speed is higher in case of thermally insulated condition as compared to isothermal condition.

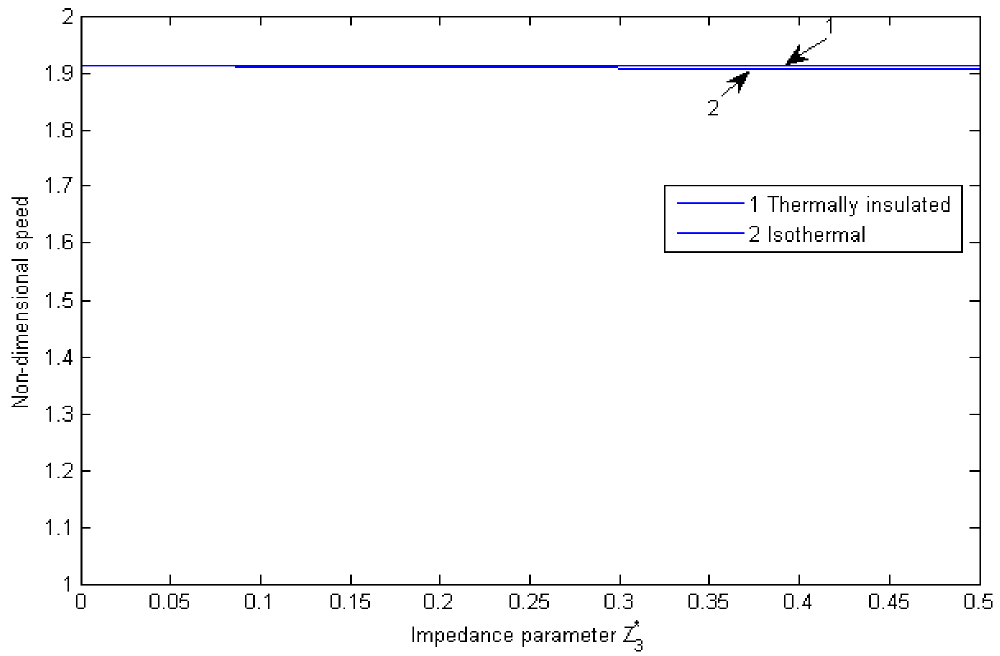

Fig. 6. Variation of non-dimensional wave speed $\boldsymbol{V_1}$ w.r.t. impedance parameter $\boldsymbol{Z_3^*}$ in a micropolar thermally insulated and isothermal half space.

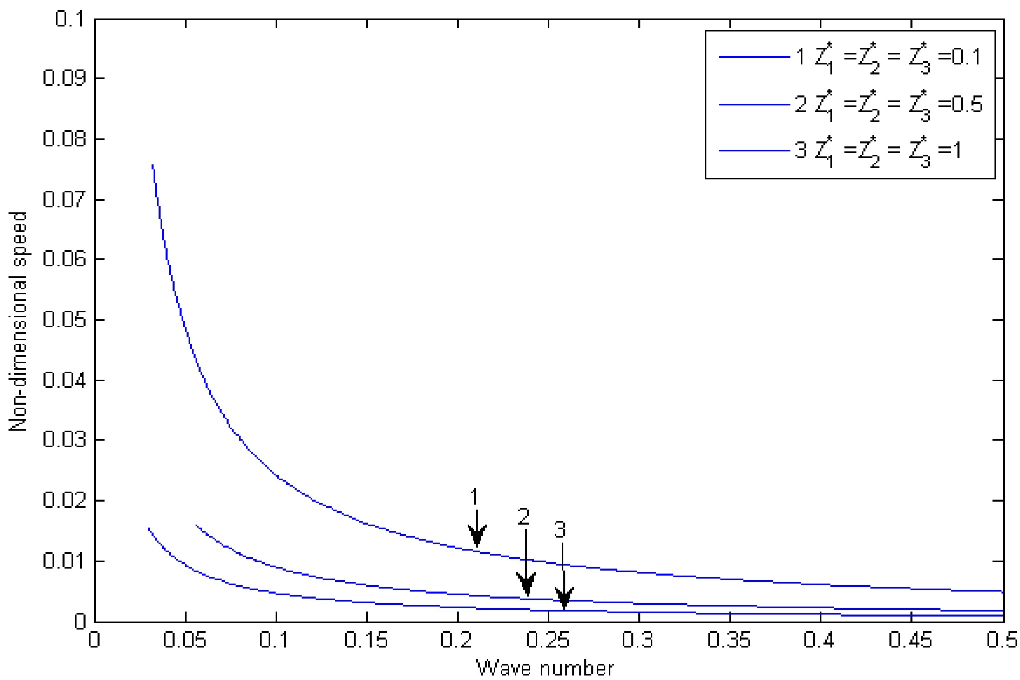

Fig. 7. Variation of non-dimensional wave speed $\boldsymbol{V_1}$ w.r.t. wave number for different values of impedance parameters.

Fig. 7 describes the dependence of non-dimensional wave speed on the wave number for different values of impedance parameters. It can be seen that non-dimensional wave speed decreases sharply for low wave number in the range $0 < k < 0.15$ and then decreases gradually with higher wave number. It reveals that the Rayleigh wave speed is dispersive in a micropolar thermoelastic solid with impedance boundary conditions.

7. Conclusion

In the present study, The Rayleigh waves in a micropolar thermoelastic half space with impedance boundary conditions for thermally insulated and isothermal surface is studied. The secular equation for Rayleigh waves satisfying impedance boundary conditions is obtained in the explicit form. The secular equation is in agreement with the secular equation of Rayleigh waves for thermoelastic half space with impedance boundary conditions when the micropolar parameters are removed. Further, on the removal of impedance and thermal effects, this equation reduces to classical equation for an elastic solid as obtained by Lord Rayleigh. From the numerical analysis, we may conclude that:

– The Rayleigh waves exist in a micropolar thermoelastic material with impedance boundary conditions.

– The inclusion of micropolarity effects in thermoelastic solid increases the non-dimensional speed of Rayleigh waves when it is calculated as a function of impedance parameters.

– The non-dimensional Rayleigh wave speed is highly or low dispersive depends upon wave number and the range of impedance parameters.

– The non-dimensional wave speed increases in case of thermally insulated boundary as compared to isothermal boundary conditions when calculated as a function of impedance parameters.

The study of waves in micropolar thermoelastic material is quite significant in the field earthquake where waves in certain rock behaving like micropolar solids and Rayleigh waves are particularly important due their destructive nature during earthquake. Therefore, the present study of Rayleigh waves in micropolar thermoelastic solid with impedance boundary conditions although, a theoretical modal but the results obtained in this modal may be useful for the researcher working in the field of seismology, geological material, earthquake engineering and geophysics.

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