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ON THE BEHAVIOR OF MATERIALS WITH DEFECTIVE COATING UNDER DIFFERENT CONTACT CONDITIONS

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Abstract. On a multi-layered base described by the Lamé equations there is a defective coating in the form of Kirchhoff plates. Defects of two types divide the covering into two half-planes with parallel ends, which, in the first case, are spaced apart from each other by a finite distance, and in the second case the distance is absent. All types of contact of coatings with a base are studied: in the absence of friction, in the presence of shearing stresses, with complete cohesion of the coatings and the base. The stress concentration in the end zone is investigated. The block-element method is used, which allows for studying the boundary problems mathematically. A complete analysis of the stress concentration features in the problem is performed.

Keywords: block element; factorization; topology; integral and differential factorization methods; exterior forms; block structures; boundary problems; bodies with coverings; hidden defects.

1. Introduction

The boundary problem of rigid adhesion of plates modeled by Kirchhoff plates to a three-dimensional deformable layered medium as a base is considered. Thus, two states of the block structure are considered in the static mode. In the first case, the semi-infinite plates in the form of half-planes are remote from each other, so that the distance between the ends is different from zero as the lithospheric plates in [1,2]. In the second case plates are brought together to a zero distance between them. It is proved that in this case the break can occur, capable of distroing the surface of the base, forming new crack. The theory of the cracks has been created and studied by leading scientists [3-12]. In the previous articles of the authors, scalar and vector cases of effect on plates have been studied. In the scalar case of vertical effect on plates it was assumed that tangent contact stresses are absent in the domain of contact of the plates with the base. In the vector case of horizontal effects on plates it was assumed that there are no vertical components in the contact domain in the presence of two components of contact tangent stresses. In the case of rigid adhesion of plates to the base, both vertical and horizontal components of contact stresses are present in the contact domain, and they are determined by solving the complete three-dimensional boundary value problem. The result is not equal to the sum of solutions to previous problems and has new properties.

2. Formulation of the problem

Assume that the coverings are semi-infinite Kirchhoff plates in the form of half-planes, the boundaries of which are parallel to each other and are spaced apart by 2θ, $\theta \geq 0$, each

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having individual mechanical properties. Assume the coordinate axes x_1ox_2 lying in the plane of the plates, and the axis x_3 directed along the outward normal to the base. Let us consider the case of static effects on the surface of the plates with rigid adhesion to the base. Then the equations of the boundary problem for the plates can be represented as

$$\mathbf{R}_b\left(\partial x_1,\partial x_2\right)\mathbf{u}_b-\mathbf{s}_b(x_1,x_2)=0,\quad b=\lambda,r \tag{1}$$

Each plate is considered as a manifold with boundary, where $\mathbf{u}_b=\{u_{1b},u_{2b},u_{3b}\}$ is a vector of the plates points displacement in horizontal u_{1b},u_{2b} and vertical u_{3b} directions of the middle surface, $b=\lambda$ for the left plate and $b=r$ for the right plate. The notations used:

$$\mathbf{s}_b(x_1,x_2)=\left\|\begin{matrix}-\varepsilon_{5b}s_{1b}(x_1,x_2) & 0 & 0\\ 0 & -\varepsilon_{5b}s_{2b}(x_1,x_2) & 0\\ 0 & 0 & \varepsilon_{53b}s_{3b}(x_1,x_2)\end{matrix}\right\|,\quad s_{nb}(x_1,x_2)=(t_{nb}+g_{nb})$$

$$\mathbf{R}_b\left(\partial x_1,\partial x_2\right)\mathbf{u}_b=$$

$$=\left\|\begin{matrix}(\frac{\partial^2}{\partial x_1^2}+\varepsilon_{1b}\frac{\partial^2}{\partial x_2^2})u_{1b} & (\varepsilon_{2b}\frac{\partial^2}{\partial x_1\partial x_2})u_{2b} & 0\\ (\varepsilon_{2b}\frac{\partial^2}{\partial x_1\partial x_2})u_{1b} & (\frac{\partial^2}{\partial x_2^2}+\varepsilon_{1b}\frac{\partial^2}{\partial x_1^2})u_{2b} & 0\\ 0 & 0 & (\frac{\partial^4}{\partial x_1^4}+2\frac{\partial^2}{\partial x_1^2}\frac{\partial^2}{\partial x_2^2}+\frac{\partial^4}{\partial x_2^4})u_{3b}\end{matrix}\right\| \tag{2}$$

The Fourier transform of the differential part of the system of equations (1) has the following form:

$$\mathbf{R}_b(-i\alpha_1,-i\alpha_2)\mathbf{U}_b=$$

$$=-\left\|\begin{matrix}(\alpha_1^2+\varepsilon_{1b}\alpha_2^2)U_{1b} & \varepsilon_{2b}\alpha_1\alpha_2U_{2b} & 0\\ \varepsilon_{2b}\alpha_1\alpha_2U_{1b} & (\alpha_2^2+\varepsilon_{1b}\alpha_1^2)U_{2b} & 0\\ 0 & 0 & -(\alpha_1^2+\alpha_2^2)^2U_{3b}\end{matrix}\right\|$$

$$\mathbf{U}_b=\mathbf{F}\mathbf{u}_b,\quad \mathbf{G}_b=\mathbf{F}\mathbf{g}_b\quad \mathbf{T}_b=\mathbf{F}\mathbf{t}_b$$

$$\mathbf{u}_b=\{u_{1b},u_{2b},u_{3b}\},\quad \mathbf{g}_b=\{g_{1b},g_{2b},g_{3b}\},\quad \mathbf{t}_b=\{t_{1b},t_{2b},t_{3b}\}$$

Here the normal stresses t_{3b} affect the plate from above and g_{3b} from below.

Similarly, the stresses g_{1b},g_{2b} and t_{1b},t_{2b} act in the tangent plane, and g_{2b} and t_{2b} at the same time act in the direction along the normals to the ends of the lithospheric plates.

The following notation is used as in [1, 2]:

$$\mathbf{U}_b=\mathbf{F_2}\mathbf{u}_b,\quad \mathbf{G}_b=\mathbf{F_2}\mathbf{g}_b,\quad \mathbf{T}_b=\mathbf{F_2}\mathbf{t}_b\quad b=\lambda,r$$

$$M_b=-D_{b1}(\frac{\partial^2u_{3b}}{\partial x_2^2}+\nu_b\frac{\partial^2u_{3b}}{\partial x_1^2}),\quad D_{b1}=\frac{D_b}{H^2},\quad D_{b2}=\frac{D_b}{H^3}$$

$$Q_b=-D_{b2}(\frac{\partial^3u_{3b}}{\partial x_2^3}+(2-\nu_b)\frac{\partial^3u_{3b}}{\partial x_1^2\partial x_2}),\quad u_{3b},\quad \frac{\partial u_{3b}}{H\partial x_2}$$

$$D_b=\frac{E_bh_b^3}{12(1-\nu_b^2)}\qquad \varepsilon_{53b}=\frac{(1-\nu_b^2)12H^4}{E_bh_b^3}\qquad \varepsilon_6^{-1}=\frac{(1-\nu)H}{\mu}$$

$$\varepsilon_{1b}=0.5(1-\nu_b),\quad \varepsilon_{2b}=0.5(1+\nu_b),\quad \varepsilon_{5b}=\frac{1-\nu_b^2}{E_bh_b}$$

$$g_{1b} = \mu_{0b}\left(\frac{\partial u_{1b}}{\partial x_3} + \frac{\partial u_{3b}}{\partial x_1}\right), \quad g_{2b} = \mu_{0b}\left(\frac{\partial u_{2b}}{\partial x_3} + \frac{\partial u_{3b}}{\partial x_2}\right)$$

$$\mu_{0b} = \frac{\mu_b}{H}, \quad x_3 = 0, \quad \mathbf{g} = \{g_{1b}, g_{2b}\}$$

The notations used: μ_b - shear modulus, ν_b - Poisson's ratio, E_b - Young's modulus, h_b - thickness, $\mathbf{g}_b$, $\mathbf{t}_b$ - vectors of contact stresses and external horizontal, $g_{1b}, g_{2b}, t_{1b}, t_{2b}$ and vertical, g_{3b}, t_{3b} effects, respectively, acting along the tangent to the boundary of the base and along the normal to it. in the domains Ω_b. $\mathbf{F}_2 \equiv \mathbf{F}_2(\alpha_1, \alpha_2)$, $\mathbf{F}_1 \equiv \mathbf{F}_1(\alpha_1)$ are two-dimensional and one-dimensional operators of the Fourier transform, respectively. The boundary conditions described in [2] are preserved here. Expressions for normal N_{x_2} and tangent $T_{x_1x_2}$ stress components to the median plane at the ends of the plates are respectively the relations:

$$T_{x_1x_2} = \varepsilon_7\left(\frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2}\right), \; N_{x_2} = \varepsilon_8\left(\frac{\partial u_2}{\partial x_2} + \nu\frac{\partial u_1}{\partial x_1}\right),$$

$$\varepsilon_7 = \frac{E}{2(1+\nu)H}, \quad \varepsilon_8 = \frac{E}{(1-\nu^2)H}$$

For a deformable base described by the boundary problem, various models are used, described by the relations:

$$\mathbf{u}(x_1, x_2) = \boldsymbol{\varepsilon}_6^{-1}\frac{1}{4\boldsymbol{\pi}^2}\int_{-\infty}^{\infty}\int \mathbf{K}(\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2)\mathbf{G}(\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2)e^{-i\langle\alpha, x\rangle}d\boldsymbol{\alpha}_1 d\boldsymbol{\alpha}_2,$$

$$x \in \Omega_\lambda, \quad x \in \Omega_r, \quad x \in \Omega_\theta, \quad \langle\boldsymbol{\alpha}, x\rangle = \boldsymbol{\alpha}_1 x_1 + \boldsymbol{\alpha}_2 x_2$$

$$\Omega_\lambda(|x_1| \le \infty;\; x_2 \le -\theta),\; \Omega_r(|x_1| \le \infty;\; \theta \le x_2),\; \Omega_\theta(|x_1| \le \infty;\; -\theta \le x_2 \le \theta)$$

$$\mathbf{K} = \|K_{mn}\|,\; m, n = 1,2,3,\; \mathbf{K}(\alpha_1, \alpha_2) = O(A^{-1}),\; A = \sqrt{\alpha_1^2 + \alpha_2^2} \to \infty \tag{3}$$

$$\varepsilon_6^{-1} = \frac{(1-\nu)H}{\mu},\; \mathbf{G}(\alpha_1, \alpha_2) = \mathbf{F}_2(\alpha_1, \alpha_2)\mathbf{g},$$

$\mathbf{g}$ is the vector of tangential and normal stresses under the plates on the boundary of the base.

Some types of matrix functions of the $\mathbf{K}(\alpha_1, \alpha_2)$ bases, described by the symbol of the system of integral equations, are given in [13]. The form for an elastic layer with a fixed lower bound in the static case is given as an example:

$$\mathbf{K}(\alpha_1, \alpha_2) = \left\|\begin{matrix} \alpha_1^2 M + \alpha_2^2 N & \alpha_1\alpha_2(M-N) & i\alpha_1 P \\ \alpha_1\alpha_2(M-N) & \alpha_1^2 N + \alpha_2^2 M & i\alpha_2 P \\ -i\alpha_1 P & -i\alpha_2 P & K \end{matrix}\right\|. \tag{4}$$

$$M(u) = \frac{(1-\nu)(3-4\nu)(\mathrm{sh}4u + 4u)}{u^2\Delta}, \quad N(u) = \frac{2\mathrm{sh}2u}{u^3\mathrm{ch}2u},$$

$$P(u) = -\frac{(1-2\nu)(3-4\nu)\mathrm{sh}^2 2u - 4u^2}{u\,\Delta(u)}, \quad K(u) = \frac{(1-\nu)(3-4\nu)(\mathrm{sh}4u - 4u)}{\Delta(u)},$$

$$\Delta(u) = u\left[(3-4\nu)\mathrm{sh}^2 2u + 4u^2 + 4(1-\nu)^2\right], \quad u = \sqrt{\alpha_1^2 + \alpha_2^2}$$

The matrix (2) of the boundary value problem is block-diagonal, consisting of the second-order matrix on the diagonal representing the matrix operator or the vector operator, and the scalar operator that is separate on the diagonal. Since the operators are independent, this greatly facilitates the study of the boundary problem at the stage of external analysis [14], making it possible to use the results obtained in the works [1, 2].

3. External analysis of the boundary problem

The boundary problems for each block of a block structure are immersed in a topological space induced by the three-dimensional Euclidean space, after which they are reduced to functional equations by the application of the Stokes formula in a topological space. Functional equations corresponding to the listed operators of the boundary problem are presented. The functional equations of the scalar operator have the following form [1, 2]:

$$R_{3b}(-i\alpha_1,-i\alpha_2)U_{3b} \equiv (\alpha_1^2+\alpha_2^2)^2 U_{3b} = -\int_{\partial\Omega_b} \omega_{3b} + S_{3b}(\alpha_1,\alpha_2)$$

$$S_{3b}(\alpha_1,\alpha_2) = \varepsilon_{53b}\mathbf{F}_2(\alpha_1,\alpha_2)(t_{3b}+g_{3b}), \quad b=\lambda, r$$

Here ω_{3b} are exterior forms participating in the presentation, that have the following form for the left λ, and the right, r plates:

$$\omega_\lambda = e^{i\langle\alpha,x\rangle}\Big\{ -\Big[\frac{\partial^3 u_{3\lambda}}{\partial x_2^3} - i\alpha_2 \frac{\partial^2 u_{3\lambda}}{\partial x_2^2} - \alpha_2^2 \frac{\partial u_{3\lambda}}{\partial x_2} + i\alpha_2^3 u_{3b} + 2\frac{\partial^3 u_{3\lambda}}{\partial x_1^2 \partial x_2} - 2i\alpha_2 \frac{\partial^2 u_{3\lambda}}{\partial x_1^2} \Big] dx_1 +$$

$$+\Big[\frac{\partial^3 u_{3\lambda}}{\partial x_1^3} - i\alpha_1 \frac{\partial^2 u_{3\lambda}}{\partial x_1^2} - \alpha_1^2 \frac{\partial u_{3\lambda}}{\partial x_1} + i\alpha_1^3 u_{3\lambda} \Big] dx_2 \Big\},$$

$$\omega_r = -e^{i\langle\alpha,x\rangle}\Big\{ -\Big[\frac{\partial^3 u_{3r}}{\partial x_2^3} - i\alpha_2 \frac{\partial^2 u_{3r}}{\partial x_2^2} - \alpha_2^2 \frac{\partial u_{3r}}{\partial x_2} + i\alpha_2^3 u_{3r} + 2\frac{\partial^3 u_{3r}}{\partial x_1^2 \partial x_2} - 2i\alpha_2 \frac{\partial^2 u_{3r}}{\partial x_1^2} \Big] dx_1 +$$

$$+\Big[\frac{\partial^3 u_{3r}}{\partial x_1^3} - i\alpha_1 \frac{\partial^2 u_{3r}}{\partial x_1^2} - \alpha_1^2 \frac{\partial u_{3r}}{\partial x_1} + i\alpha_1^3 u_{3r} \Big] dx_2 \Big\}$$

The functional equations of the boundary problem for the vector case presented for each plate are matrix and have the form:

$$\mathbf{R}_{12b}(-i\alpha_{1b},-i\alpha_{2b})\mathbf{U}_{12b} = -\int_{\partial\Omega_b} \boldsymbol{\omega}_{12b} + \mathbf{S}_{12b}(\alpha_1,\alpha_2), \quad \mathbf{U}_{12b} = \{ U_{1b}, U_{2b} \},$$

$$\boldsymbol{\omega}_{12b} = \{ \omega_{1b}, \omega_{2b} \}, \quad \mathbf{S}_{12b}(\alpha_1,\alpha_2) = -\varepsilon_{5b}\mathbf{F}_2(\alpha_{1b},\alpha_{2b})(\mathbf{g}_b + \mathbf{t}_b), \quad b=\lambda, r. \tag{5}$$

$$\mathbf{S}_{12b}(\alpha_1,\alpha_2) = \{ S_{1b}, S_{2b} \}$$

$$\mathbf{R}_{12b}(-i\alpha_{1b},-i\alpha_{2b}) = -\begin{Vmatrix} (\alpha_1^2+\varepsilon_{1b}\alpha_2^2) & \varepsilon_{2b}\alpha_1\alpha_2 \\ \varepsilon_{2b}\alpha_1\alpha_2 & (\alpha_2^2+\varepsilon_{1b}\alpha_1^2) \end{Vmatrix}$$

Here $\boldsymbol{\omega}_b$ is a vector of exterior forms that has the following representation:

$$\omega_{1\lambda} = -e^{i\langle\alpha,x\rangle}\{-(\varepsilon_{1\lambda}\frac{\partial u_{1\lambda}}{\partial x_2} + \varepsilon_{2\lambda}\frac{\partial u_{2\lambda}}{\partial x_1} - i\varepsilon_{1\lambda}\alpha_2 u_{1\lambda})dx_1 +$$

$$+(\frac{\partial u_{1\lambda}}{\partial x_1} - i\alpha_1 u_{1\lambda} - i\varepsilon_{2\lambda}\alpha_2 u_{2\lambda})dx_2 \ \},$$

$$\omega_{2\lambda} = -e^{i\langle\alpha,x\rangle}\{-(\varepsilon_{2\lambda}\frac{\partial u_{1\lambda}}{\partial x_1} + \frac{\partial u_{2\lambda}}{\partial x_2} - i\alpha_2 u_{2\lambda})dx_1 +$$

$$+(\varepsilon_{1\lambda}\frac{\partial u_{2\lambda}}{\partial x_1} - i\varepsilon_{1\lambda}\alpha_1 u_{2\lambda} - i\varepsilon_{2\lambda}\alpha_2 u_{1\lambda})dx_2 \ \}$$

$$\omega_{1r} = e^{i\langle\alpha,x\rangle}\{-(\varepsilon_{1r}\frac{\partial u_{1r}}{\partial x_2} + \varepsilon_{2r}\frac{\partial u_{2r}}{\partial x_1} - i\varepsilon_{1r}\alpha_2 u_{1r})dx_1 +$$

$$+(\frac{\partial u_{1r}}{\partial x_1} - i\alpha_1 u_{1r} - i\varepsilon_{2r}\alpha_2 u_{2r})dx_2 \ \},$$

$$\omega_{2r} = e^{i\langle\alpha,x\rangle}\{-(\varepsilon_{2r}\frac{\partial u_{1r}}{\partial x_1} + \frac{\partial_{2r}}{\partial x_2} - i\alpha_2 u_{2r})dx_1 +$$

$$+(\varepsilon_{1b}\frac{\partial u_{2r}}{\partial x_1} - i\varepsilon_{1b}\alpha_1 u_{2r} - i\varepsilon_{2r}\alpha_2 u_{1r})dx_2 \ \}$$

Having carried out the operations of external analysis [14] for these functional equations, including factorization of the coefficient of the functional equation (matrix-function and simple function), calculation of Leray's residue forms, construction of pseudodifferential equations, extraction of the integral equations necessary for the boundary problems from them, and the solution of these equations. The solutions found are introduced into the exterior forms of the functional equations of each plate, after which they are coupled with the base, forming new topological space, which is called factor — topological. In the process of carrying out this part of the investigation, described in detail in the above-mentioned works, abbreviated notations for the parameters of the stress-strain state of the block structure used below are introduced. Namely, notations for the scalar operator are as follows:

$$\mathbf{Y}_{\lambda 0} = \{y_{1\lambda 0}, y_{2\lambda 0}\}, \quad \mathbf{Z}_{\lambda 0} = \{z_{1\lambda 0}, z_{2\lambda 0}\}, \quad \mathbf{Y}_{\mathbf{r}0} = \{y_{1r0}, y_{2r0}\}, \quad \mathbf{Z}_{\mathbf{r}0} = \{z_{1r0}, z_{2r0}\},$$

$$\mathbf{F}_1 g = \mathbf{F}_1(\alpha_1)g, \quad \mathbf{F}_2 g = \mathbf{F}_2(\alpha_1,\alpha_2)g,$$

$$y_{1\lambda 0} = D_\lambda^{-1}\mathbf{F}_1 M_\lambda, \quad y_{2\lambda 0} = D_\lambda^{-1}\mathbf{F}_1 Q_\lambda, \quad y_{1r0} = D_r^{-1}\mathbf{F}_1 M_r, \quad y_{2r0} = D_r^{-1}\mathbf{F}_1 Q_r,$$

$$z_{1\lambda 0} = \mathbf{F}_1\frac{\partial u_{3\lambda}}{\partial x_2^\lambda}, \quad z_{2\lambda 0} = \mathbf{F}_1 u_{3\lambda}, \quad z_{1r0} = \mathbf{F}_1\frac{\partial u_{3r}}{\partial x_2^r}, \quad z_{2r0} = \mathbf{F}_1 u_{3r},$$

$$\mathbf{K}_{\lambda 0} = \{k_{1\lambda 0}, k_{2\lambda 0}\}, \quad \mathbf{K}_{\mathbf{r}0} = \{k_{1r0}, k_{2r0}\}, \quad k_{1\lambda 0} = \varepsilon_{53\lambda}\mathbf{F}_2(\alpha_1,\alpha_{2-})(t_{3\lambda} + g_{3\lambda}) = \varepsilon_{53\lambda}S_{3\lambda}(\alpha_1,\alpha_{2-}),$$

$$k_{2\lambda 0} = \varepsilon_{53\lambda}S'_{3\lambda}(\alpha_1,\alpha_{2-}), \quad k_{1r0} = \varepsilon_{53r}\mathbf{F}_2(\alpha_1,\alpha_{2+})(t_{3r} + g_{3r}) = \varepsilon_{53r}S_{3r}(\alpha_1,\alpha_{2+}),$$

$$k_{2r0} = \varepsilon_{53r}S'_{3r}(\alpha_1,\alpha_{2+})$$

Similarly for the vector operator:

$$\begin{gathered}\mathbf{Y}_\lambda=\{y_{1\lambda},y_{2\lambda}\},\quad \mathbf{Z}_\lambda=\{z_{1\lambda},z_{2\lambda}\},\quad \mathbf{Y}_\mathbf{r}=\{y_{1r},y_{2r}\},\quad \mathbf{Z}_\mathbf{r}=\{z_{1r},z_{2r}\},\\ \mathbf{F}_1 g=\mathbf{F}_1(\alpha_1)g,\quad \mathbf{F}_2 g=\mathbf{F}_2(\alpha_1,\alpha_2)g,\\ y_{1\lambda}=\mathbf{F}_1 T_{x_1x_2\lambda},\quad y_{2\lambda}=\mathbf{F}_1 N_{x_2\lambda},\quad y_{1r}=\mathbf{F}_1 T_{x_1x_2r},\quad y_{2r}=\mathbf{F}_1 N_{x_2r},\\ z_{1\lambda}=\mathbf{F}_1 u_{1\lambda},\quad z_{2\lambda}=\mathbf{F}_1 u_{2\lambda},\quad z_{1r}=\mathbf{F}_1 u_{1r},\quad z_{2r}=\mathbf{F}_1 u_{2r}\\ \mathbf{K}_\lambda=\{k_{1\lambda},k_{2\lambda}\},\quad \mathbf{K}_\mathbf{r}=\{k_{1r},k_{2r}\},\\ k_{1\lambda}=\varepsilon_{5\lambda}[\ S_{1\lambda}(\alpha_1,\alpha_{2-})+\alpha_{2-}\alpha_1^{-1}S_{2\lambda}(\alpha_1,\alpha_{2-})\ ],\\ k_{2\lambda}=-\varepsilon_{5\lambda}[\ -(1+\varepsilon_{1\lambda})S_{1\lambda}(\alpha_1,\alpha_{2-})+\varepsilon_{2\lambda}\alpha_1 S'_{1\lambda}(\alpha_1,\alpha_{2-})+\varepsilon_2\alpha_{2-}S'_{2\lambda}(\alpha_1,\alpha_{2-})\ ],\\ k_{1r}=\varepsilon_{5r}[\ S_{1r}(\alpha_1,\alpha_{2+})+\alpha_{2+}\alpha_1^{-1}S_{2r}(\alpha_1,\alpha_{2+})\ ],\\ k_{2r}=-\varepsilon_{5r}[\ -(1+\varepsilon_{1r})S_{1r}(\alpha_1,\alpha_{2+})+\varepsilon_{2r}\alpha_1 S'_{1r}(\alpha_1,\alpha_{2+})+\varepsilon_{2r}\alpha_{2+}S'_{2r}(\alpha_1,\alpha_{2+})\ ]\end{gathered} \tag{6}$$

In order to connect lithospheric plates with the three-dimensional base, which has three-dimensional displacement and stress vectors at the boundary, it is necessary to represent the parameters of the stress-strain state of lithospheric plates in the same form. For this the combined vector of exterior forms and the parameter of external loads are introduced:

$$\boldsymbol{\omega}_b=\{\omega_{1b},\omega_{2b},\omega_{3b}\},\quad \mathbf{S}_b=\{S_{1b},S_{2b},S_{3b}\} \tag{7}$$

Then the solutions for each plate can be represented in the form:

$$\begin{gathered}\mathbf{u}_\lambda(x_1,x_2,0)=\mathbf{F}_2^{-1}(x_1,x_2)[\mathbf{R}_\lambda(-i\alpha_1,-i\alpha_2)]^{-1}\left(-\int_{\partial\Omega_\lambda}\omega_\lambda+\mathbf{S}_\lambda\right)\\ \mathbf{u}_r(x_1,x_2,0)=\mathbf{F}_2^{-1}(x_1,x_2)[\mathbf{R}_r(-i\alpha_1,-i\alpha_2)]^{-1}\left(-\int_{\partial\Omega_r}\omega_r+\mathbf{S}_r\right)\end{gathered} \tag{8}$$

By matching all three components of the displacement of the lithospheric plate, both normal and tangential, with the displacements of the upper boundary of the base, the following relations are obtained:

$$\begin{gathered}\mathbf{P}_\lambda\mathbf{u}(x_1,x_2,0)+\mathbf{P}_\theta\mathbf{u}(x_1,x_2,0)+\mathbf{P}_r\mathbf{u}(x_1,x_2,0)=\varepsilon_6^{-1}\mathbf{F}_2^{-1}\mathbf{K}(\alpha_1,\alpha_2,0)[\ \mathbf{G}_\lambda(\alpha_1,\alpha_2)+\\ +\mathbf{G}_r(\alpha_1,\alpha_2)\ ],\quad \mathbf{G}_\lambda(\alpha_1,\alpha_2)=\mathbf{F}_2\mathbf{P}_\lambda\mathbf{g}(x_1,x_2),\quad \mathbf{G}_r(\alpha_1,\alpha_2)=\mathbf{F}_2\mathbf{P}_r\mathbf{g}(x_1,x_2)\\ \mathbf{P}_p\mathbf{u}=\mathbf{F}_2^{-1}[\ \mathbf{R}_p(-i\alpha_1,-i\alpha_2)\ ]^{-1}\left\langle\ -\int_{\partial\Omega_p}\boldsymbol{\omega}_p+\mathbf{S}_b(\alpha_1,\alpha_2)\ \right\rangle,\quad p=\lambda,r\end{gathered} \tag{9}$$

Here $\mathbf{P}_\lambda,\mathbf{P}_r\ \ \mathbf{P}_\theta$, are projections on the left, the right half-planes and on the median interval which are carriers of the corresponding plates and describe the interval $|x_2|\le\theta$.

By introducing relations (8) into the left parts (9) and applying Fourier transforms, the following relations are obtained:

$$\begin{gathered}[\ \mathbf{R}_\lambda(-i\alpha_1,-i\alpha_2)\ ]^{-1}\left\langle\ -\int_{\partial\Omega_\lambda}\boldsymbol{\omega}_\lambda+\mathbf{S}_\lambda\ \right\rangle+\mathbf{U}_\theta+\\ +[\ \mathbf{R}_r(-i\alpha_1,-i\alpha_2)\ ]^{-1}\left\langle\ -\int_{\partial\Omega_r}\boldsymbol{\omega}_r+\mathbf{S}_r\ \right\rangle-\\ -\mathbf{K}(\alpha_1,\alpha_2,0)[\ \mathbf{G}_\lambda(\alpha_1,\alpha_2)+\mathbf{G}_r(\alpha_1,\alpha_2)\ ]=0,\\ \mathbf{U}_\theta=\mathbf{F}_2\mathbf{P}_\theta\mathbf{u}(x_1,x_2)\end{gathered}$$

The vector – functions $\mathbf{G}_\lambda(\alpha_1,\alpha_2)$, $\mathbf{G}_r(\alpha_1,\alpha_2)$, which are Fourier transforms of functions with carriers in half-planes, are regular functions of the parameters α_2 with fixed

α_1 in the lower and upper half-planes respectively. In this regard, vector functions can be designated, being regular in the parameter α_2 in the lower (minus sign) and in the upper (plus sign) half-planes:

$$\mathbf{G}_\lambda(\alpha_1,\alpha_2)=\mathbf{G}_-(\alpha_1,\alpha_2),\quad \mathbf{G}_r(\alpha_1,\alpha_2)=\mathbf{G}_+(\alpha_1,\alpha_2).$$

4. Analisis of solutions

By introducing these notations into the previous relation, a Wiener-Hopf matrix functional equation is obtained for $\theta>0$,

$$\mathbf{MG}_+=\mathbf{G}_-+\mathbf{V}+\mathbf{K}_1^{-1}\mathbf{U}_\theta,\quad \mathbf{M}=\mathbf{K}_1^{-1}\mathbf{K}_2,\quad \mathbf{K}_2=\boldsymbol{\varepsilon}_r\mathbf{R}_r^{-1}-\varepsilon_6^{-1}\mathbf{K},\quad \mathbf{K}_1=\varepsilon_6^{-1}\mathbf{K}-\boldsymbol{\varepsilon}_\lambda\mathbf{R}_\lambda^{-1}$$

$$\mathbf{V}=\mathbf{K}_1^{-1}(\mathbf{R}_\lambda^{-1}\int_{\partial\Omega_\lambda}\boldsymbol{\omega}_\lambda+\mathbf{R}_r^{-1}\int_{\partial\Omega_r}\boldsymbol{\omega}_r-\boldsymbol{\varepsilon}_\lambda\mathbf{R}_\lambda^{-1}\mathbf{T}_\lambda-\boldsymbol{\varepsilon}_r\mathbf{R}_r^{-1}\mathbf{T}_r),\quad \mathbf{U}_\theta=\mathbf{F}_2\mathbf{P}_\theta\mathbf{u}(x_1,x_2) \tag{10}$$

and for $\theta=0$

$$\mathbf{MG}_+=\mathbf{G}_-+\mathbf{V}\ , \tag{11}$$

which, along with the presence of the indeterminates, also contains their functionals in the form $\mathbf{G}_\pm(\alpha_1,\alpha_{2\pm})$ as indeterminates. These functionals have to be determined from a certain system of algebraic equations [1, 2].

Papers [1, 2] also discuss the methods of determining functionals that enter exterior forms. The factorization approach presented in [13] was used to study the singularities of the solution of the functional equation.

The study of the properties of the solutions of this matrix functional equation also led to the results described in [1, 2].

We note that the latter systems of equations contain the corresponding equations derived for the scalar and two-dimensional vector cases obtained in [1, 2], respectively. For this it is sufficient to exclude two components of tangential contact stresses in the scalar case. In this case, for the design of normal contact stresses functional equations of the following type are needed:

For $\theta>0$,

$$\begin{aligned}
&[\ \varepsilon_{53r}(\alpha_1^2+\alpha_2^2)^{-2}+\varepsilon_6^{-1}K_1(\alpha_1,\alpha_2)\ ]G^+(\alpha_1,\alpha_2)=\\
&=-[\ \varepsilon_{53\lambda}(\alpha_1^2+\alpha_2^2)^{-2}+\varepsilon_6^{-1}K_1(\alpha_1,\alpha_2)\ ]G^-(\alpha_1,\alpha_2)+U_{3\theta}(\alpha_1,\alpha_2)\\
&+(\alpha_1^2+\alpha_2^2)^{-2}[\ A_\lambda k_{1\lambda}+B_\lambda k_{2\lambda}+A_r k_{1r}+B_r k_{2r}+\\
&+\varepsilon_{53\lambda}T^+(\alpha_1,\alpha_2)+\varepsilon_{53r}T^-(\alpha_1,\alpha_2)\ ],\quad \theta>0,
\end{aligned} \tag{12}$$

$$U_{3\theta}(\alpha_1,\alpha_2)=\int_{-\infty}^{\infty}\int_{-\theta}^{\theta}u_3(x_1,x_2)e^{i\langle\alpha,x\rangle}dx_1dx_2$$

For $\theta=0$,

$$\begin{aligned}
&[\ \varepsilon_{53r}(\alpha_1^2+\alpha_2^2)^{-2}+\varepsilon_6^{-1}K_1(\alpha_1,\alpha_2)\ ]G^+(\alpha_1,\alpha_2)=\\
&=-[\ \varepsilon_{53\lambda}(\alpha_1^2+\alpha_2^2)^{-2}+\varepsilon_6^{-1}K_1(\alpha_1,\alpha_2)\ ]G^-(\alpha_1,\alpha_2)+\\
&+(\alpha_1^2+\alpha_2^2)^{-2}[\ A_\lambda k_{1\lambda}+B_\lambda k_{2\lambda}+A_r k_{1r}+B_r k_{2r}+\\
&+\varepsilon_{53\lambda}T^+(\alpha_1,\alpha_2)+\varepsilon_{53r}T^-(\alpha_1,\alpha_2)\ ],\quad \theta=0
\end{aligned} \tag{13}$$

In the second case, the normal stress component in the contact zone is excluded. In this case, for the design of the vector of tangential contact stresses more lengthy functional equations are obtained, which, in the vector form, are as follows [13]

For $\theta>0$,

$$\mathbf{MG}_+=\mathbf{G}_-+\mathbf{V}+\mathbf{K}_1^{-1}\mathbf{U}_\theta,\quad \mathbf{U}_\theta=\mathbf{F}_2\mathbf{P}_\theta u(x_1,x_2) \tag{14}$$

For $\theta = 0$,

$$\mathbf{MG}_{+} = \mathbf{G}_{-} + \mathbf{V} \tag{15}$$

For $\theta > 0$, that is, when the ends of the plates are separated by a distance 2θ, contact stresses at the edges of the plates in (12) have the properties of [13] of the form

$$g_{3\lambda}(x_1,x_2) = \boldsymbol{\sigma}_{1\lambda}(x_1,x_2)(-x_2-\theta)^{-1/2}, \quad x_2 < -\theta$$

$$g_{3r}(x_1,x_2) = \boldsymbol{\sigma}_{1r}(x_1,x_2)(x_2-\theta)^{-1/2}, \quad x_2 > \theta$$

In the second case the resulting solutions of the functional equations after substitution into stress formulas have for $\theta > 0$ the properties for $x_2 \to \theta$: for (14)

$$\mathbf{g}_{\lambda}(x_1,x_2) = \boldsymbol{\sigma}_{1\lambda}(x_1,x_2)(-x_2-\theta)^{-1/2}, \quad x_2 < -\theta,$$

$$\mathbf{g}_{r}(x_1,x_2) = \boldsymbol{\sigma}_{1r}(x_1,x_2)(x_2-\theta)^{-1/2}, \quad x_2 > \theta$$

Vectors of contact stresses at the edges of the plates have the following representation for (10)

$$\mathbf{g}_{\lambda}(x_1,x_2) = \boldsymbol{\sigma}_{1\lambda}(x_1,x_2)(-x_2-\theta)^{-0,5+i\gamma} + \boldsymbol{\sigma}_{2\lambda}(x_1,x_2)(-x_2-\theta)^{-0,5-i\gamma}, \quad x_2 < -\theta,$$

$$\mathbf{g}_{r}(x_1,x_2) = \boldsymbol{\sigma}_{1r}(x_1,x_2)(x_2-\theta)^{-0,5+i\gamma} + \boldsymbol{\sigma}_{2r}(x_1,x_2)(x_2-\theta)^{-0,5-i\gamma}, \quad x_2 > \theta \quad \gamma > 0$$

The vectors $\boldsymbol{\sigma}_{1\lambda}$, $\boldsymbol{\sigma}_{1r}$ are continuous in both parameters. The parameter γ is determined by the mechanical characteristics of the base, the method for finding it is described in [13]. For example, for the case (4) it is $\gamma = arcth\dfrac{1-2\nu}{2(1-\nu)}$, where ν is the Poisson's ratio of base material.

The solution of the functional equation leads to the following behavior of stress.

For, $\theta = 0$ that is, when the ends of the plates are fully brought together, the contact stresses at the edges of the plates have the representation for (13):

$$\mathbf{g}_{3\lambda}(x_1,x_2) \to \boldsymbol{\sigma}_{2\lambda}(x_1,x_2)x_2^{-1} + \boldsymbol{\sigma}_{3\lambda}(x_1,x_2)ln|x_2| + \boldsymbol{\sigma}_{4\lambda}(x_1,x_2)sgnx_2,$$

$$\mathbf{g}_{3r}(x_1,x_2) \to \boldsymbol{\sigma}_{2r}(x_1,x_2)x_2^{-1} + \boldsymbol{\sigma}_{3r}(x_1,x_2)ln|x_2| + \boldsymbol{\sigma}_{4r}(x_1,x_2)sgnx_2;$$

for (15) are

$$\mathbf{g}_{\lambda}(x_1,x_2) \to \boldsymbol{\sigma}_{2\lambda}(x_1,x_2)x_2^{-1} + \boldsymbol{\sigma}_{3\lambda}(x_1,x_2)ln|x_2| + \boldsymbol{\sigma}_{4\lambda}(x_1,x_2)sgnx_2,$$

$$\mathbf{g}_{r}(x_1,x_2) \to \boldsymbol{\sigma}_{2r}(x_1,x_2)x_2^{-1} + \boldsymbol{\sigma}_{3r}(x_1,x_2)ln|x_2| + \boldsymbol{\sigma}_{4r}(x_1,x_2)sgnx_2.$$

The solution of the vector functional equation (11) leads to the following behavior of stress vectors represented by

$$\mathbf{g}_{\lambda}(x_1,x_2) \to \boldsymbol{\sigma}_{4\lambda}(x_1,x_2)x_2^{-1} + \boldsymbol{\sigma}_{5\lambda}(x_1,x_2)ln|x_2| + \boldsymbol{\sigma}_{64\lambda}(x_1,x_2)sgnx_2,$$

$$\mathbf{g}_{r}(x_1,x_2) \to \boldsymbol{\sigma}_{4r}(x_1,x_2)x_2^{-1} + \boldsymbol{\sigma}_{5r}(x_1,x_2)ln|x_2| + \boldsymbol{\sigma}_{6r}(x_1,x_2)sgnx_2.$$

All vectors $\boldsymbol{\sigma}_{n\lambda}(x_1,x_2)$ и and $\boldsymbol{\sigma}_{nr}(x_1,x_2), \quad n = 3,...,6$ are continuous in both parameters.

5. Conclusions.

The stress concentration in the boundary zone is investigated. The block-element method is used, which allows strictly mathematical study of the boundary problems posed. Comprehensive analisis of the stress concentration features in the problem is performed. The result is not equal to the sum of solutions to previous problems, because coefficients $\boldsymbol{\sigma}_{n\lambda}(x_1,x_2)$ and $\boldsymbol{\sigma}_{nr}(x_1,x_2), \quad n = 3,...,6$ are related to all the parameters of the matrix – functions (2) and (3) in a complex way.

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