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DESTRUCTION OF THE ADHESION ZONE BY COMBINED PULSED-VIBRATIONAL IMPACTS

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Abstract. The model of a string on an elastic foundation is applied to study adhesive zone delamination induced by combined vibrational - pulsed actions. The incubation time criterion is utilized in order to predict a critical delamination condition. The influence of the background ultrasonic field onto the value of threshold stress-pulse amplitudes is demonstrated using the example of several particular cases of external actions. A significant reduction of critical amplitudes is obtained with certain frequencies of background vibration field.

Keywords: adhesive zone; string on an elastic foundation; pulse-vibration effects.

1. Introduction

Acoustic waves of high frequency have a significant effect on the strength properties of the material [1,2]. The study of this process is an important and perspective issue for many realms of science and technology. Nowadays, different models of different complexity are constructed, depending on the consideration degree of issue`s detail [3], in order to describe the adhesive strength. The model of a string (or a beam) on an elastic foundation is one of the simplest representations of the delamination process. It reflects the dependence of adhesive zone characteristics on the external load and allows one to effectively evaluate critical parameters of the delamination process [4,5]. Here we confine ourselves to a one-dimensional model of a string on an elastic foundation [6]. We will show the effect of background harmonic vibrations on critical amplitudes of the principal force pulse. The results show the possibility of considerable decrease in threshold amplitude by a proper choice of the frequency of a background excitation.

2. Formulation of the problem

The problem is described by an equation

$$\frac{\partial^2 \mathrm{u}}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 \mathrm{u}}{\partial t^2} - \frac{\omega^2}{c^2}u = -f(x,t), \quad x \in (0,l), \quad t > 0, \tag{1}$$

where $u(x,t)$ is the displacement of the string with respect to the equilibrium position, c is the wave velocity (m/s), ω is the characteristics of the elastic foundation rigidity (s^{-1}), $f(x,t)$ is the external load, and l is the string length.

The zero initial conditions are considered:

$$u(x,0) = \frac{\partial u}{\partial t}(x,0) = 0. \tag{2}$$

We take two types of boundary conditions: a string with clamped ends and a string with free ends:

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$$u(0,t) = u(l,t) = 0. \tag{3}$$

$$\frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(l,t) = 0. \tag{4}$$

We search for the solution in the form of the expansion:

$$u(x,t) = \sum_{k=1}^{n} U_k(x)V_k(t). \tag{5}$$

We are going to retain a finite number of expansion terms because the main contribution is introduced only by the first expansion terms.

For the case of boundary conditions (3) we obtain:

$$u^{(1)}(x,t) = \frac{2c^2}{l}\sum_{k=1}^{n}\frac{\sin(\lambda_k x)}{\Omega_k}\int_0^t\left(\int_0^l f(\xi,\eta)\sin(\lambda_k \xi)d\xi\right)\sin(\Omega_k(t-\eta))d\eta. \tag{6}$$

For the case of boundary conditions (4) we obtain:

$$u^{(2)}(x,t) = \frac{c^2}{l\omega}\int_0^t\left(\int_0^l f(\xi,\eta)d\xi\right)\sin\big(\omega(t-\eta)\big)\,d\eta + $$

$$+\frac{2c^2}{l}\sum_{k=1}^{n}\frac{\cos(\lambda_k x)}{\Omega_k}\int_0^t\left(\int_0^l f(\xi,\eta)\cos(\lambda_k \xi)d\xi\right)\sin(\Omega_k(t-\eta))d\eta, \tag{7}$$

where $\lambda_k = \pi k/l$, $\Omega_k = \sqrt{\omega^2 + c^2\lambda_k^2}$, $k = 1,2,\dots$.

3. The case of a constant load

As an external load we take:

$$f(x,t) = P\left(H(t)\delta\left(x - \frac{l}{2}\right) + r\sin(\nu t)\right), \tag{8}$$

where P is the value of the load amplitude, $0 \le r$ is the ratio of the vibration amplitude to the amplitude P, ν is the forced-vibration frequency, $H(t)$ is the Heaviside function, and $\delta(x)$ is the Dirac function.

Consider the point $x = l/2$. In this case we obtain for the first type of boundary conditions:

$$u^{(1)}\left(\frac{l}{2},t\right) = \frac{2c^2}{l}P\sum_{k=1}^{n}\left[A_k^{(1)}(1-\cos\Omega_k t) + rB_k^{(1)}w(\nu,\Omega_k,t)\right], \tag{9}$$

where $A_k^{(1)} = \left(\frac{1}{\Omega_k}\sin\frac{\pi k}{2}\right)^2$, $B_k^{(1)} = \frac{1}{2\lambda_k\Omega_k}\sin\frac{\pi k}{2}\left(1-\cos\frac{\pi k}{2}\right)$

For the second type of boundary conditions, we have:

$$u^{(2)}\left(\frac{l}{2},t\right) = \frac{2c^2}{l}P\left[\frac{1}{2\omega^2}(1-\cos\omega t) + \frac{rl}{4\omega}w(\nu,\omega,t) + \sum_{k=1}^{n}A_k^{(2)}(1-\cos\Omega_k t)\right], \tag{10}$$

where $A_k^{(2)} = \left(\frac{1}{\Omega_k}\cos\frac{\pi k}{2}\right)^2$

Function $w(\nu,\beta,t)$ is defined as follows:

$$w(\nu,\omega,t) = \begin{cases}\frac{1}{\nu^2-\beta^2}(\nu\sin\beta t - \beta\sin\nu t), & \nu \ne \beta\\ \frac{1}{\nu}\sin\nu t - t\cos\nu t, & \nu = \beta\end{cases} \tag{11}$$

To determine the critical value of the load necessary for the rupture of the adhesive zone, we use the criterion of incubation time, which was previously successfully used to solve similar problems of fracture mechanics [6,7]:

$$\max_{t\in[0,t_c]}\frac{1}{\tau}\int_{t-\tau}^{t}u\left(\frac{l}{2},\eta\right)d\eta = u_c, \tag{12}$$

where τ – incubation time (characteristics of the material).

Let`s put $l = 1$ m, $c = 10$ m/s, $\omega = 5$ s^{-1}, $n = 5$, $t_c = 25$ s, $\tau = 0.1$ s. Consider the dependence $\tilde{P} = \frac{2c}{lu_c}P$ on ν for different r for boundary conditions of the first and second type. As we can see on Fig.1 and Fig. 2, there are frequencies at which the load will be much less.

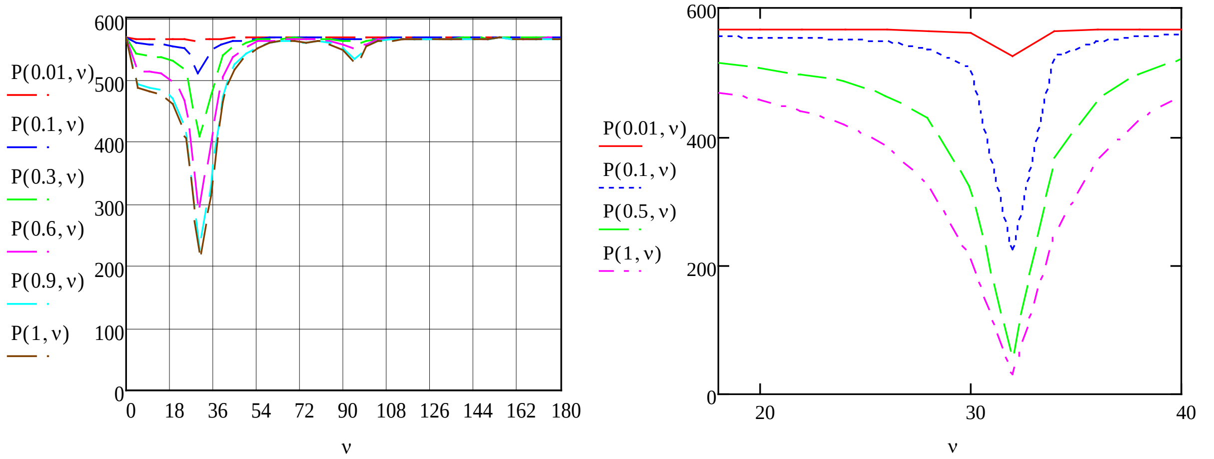

Fig. 1. The dependence $\tilde{P}$ on ν for $r = \{0.01; 0.1; 0.3; 0.6; 0.9; 1\}$ in the case of the fixed ends of the string.

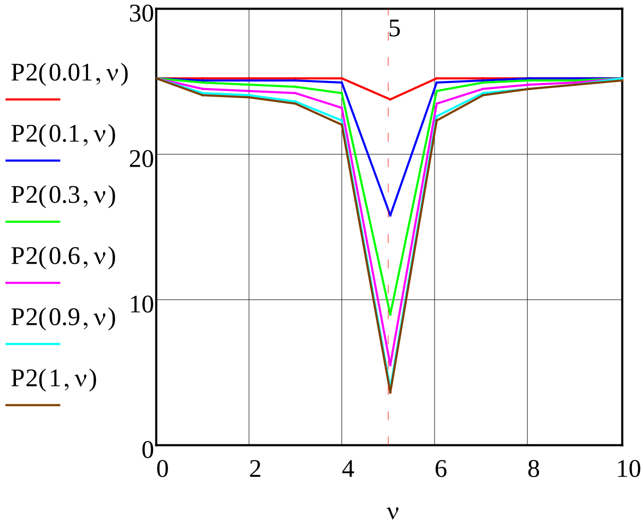

Fig. 2. The dependence $\tilde{P}$ on ν for $r = \{0.01; 0.1; 0.3; 0.6; 0.9; 1\}$ in the case of the free ends of the string.

4. The case of a constant load with a dying high-frequency impact

As an external load, we take

$$f(x,t) = P\left(H(t)\delta\left(x - \frac{l}{2}\right) + re^{-\alpha t}\sin(\nu t)\right), \tag{13}$$

where α characterizes the velocity of dying vibration impact. The rest of the notation is analogous to the previous case. The solution has the form (9) and (10), except that the function w also depends on α.

$$w(\nu,\omega,t,\alpha) = \begin{cases} J(t,\alpha,\nu+\beta,-\beta t) - J(t,\alpha,\nu-\beta,-\beta t), & \nu \neq \beta \\ J(t,\alpha,2\nu,-\nu t) - \frac{\cos \nu t}{2\alpha}(1-e^{-\alpha t}), & \nu = \beta \end{cases} \tag{14}$$

$$J(t,a,b,\varphi) = \int_0^t e^{-\alpha x}\cos(bx+\varphi)\,dx. \tag{15}$$

Parameters l, c, ω, t_c, τ have the same values as in a case of a constant load. Consider the dependence $\tilde{P} = \frac{2c}{lu_c}P$ on ν and α for different r for boundary conditions of the first (Fig. 3,4) and second type (Fig. 5,6). As we can see on Fig. 3 and Fig. 5, with an increase α, the value of the critical load also increases. Also from the Fig. 4 and Fig. 6, it can be seen that, there are frequencies at which the load will be much less.

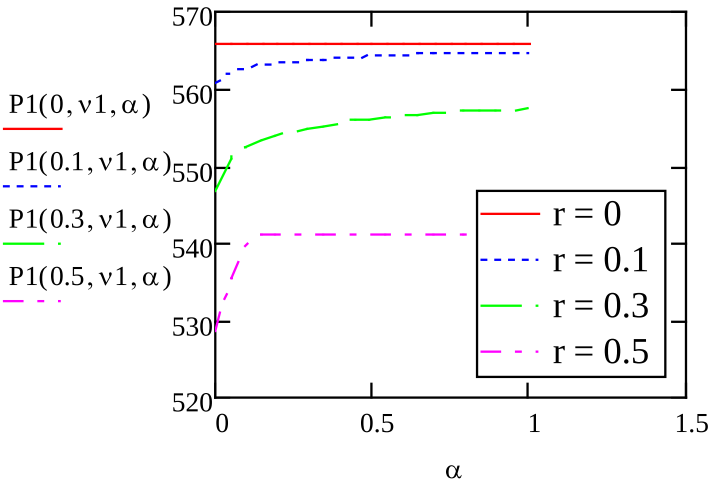

Fig. 3. The dependence $\tilde{P}$ on α for $r = \{0; 0.1; 0.3; 0.5\}$ and $\nu = (\Omega_{1+}\,\Omega_2)/2$ in the case of the fixed ends of the string.

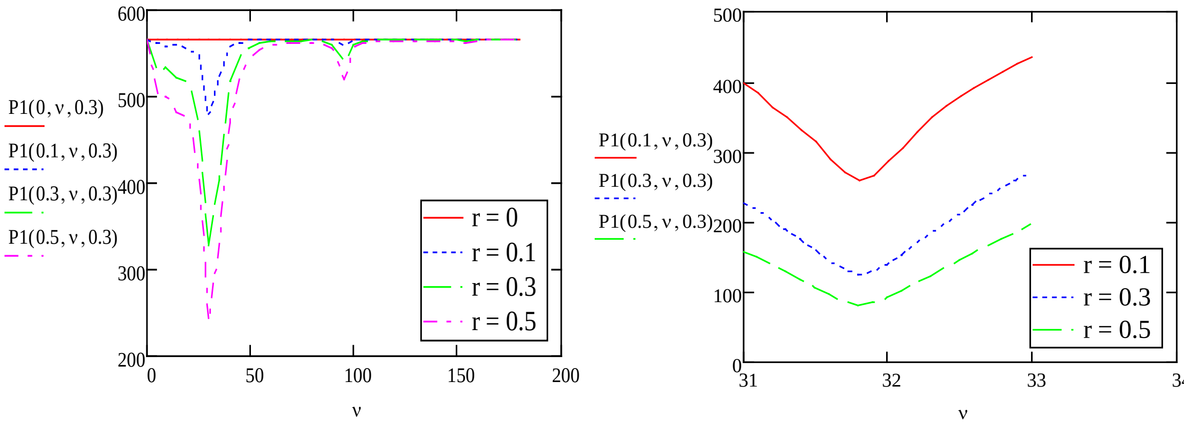

Fig. 4. The dependence $\widetilde{\mathrm{P}}$ on ν for $r = \{0; 0.1; 0.3; 0.5\}$ and $\alpha = 0.3$ in the case of the fixed ends of the string.

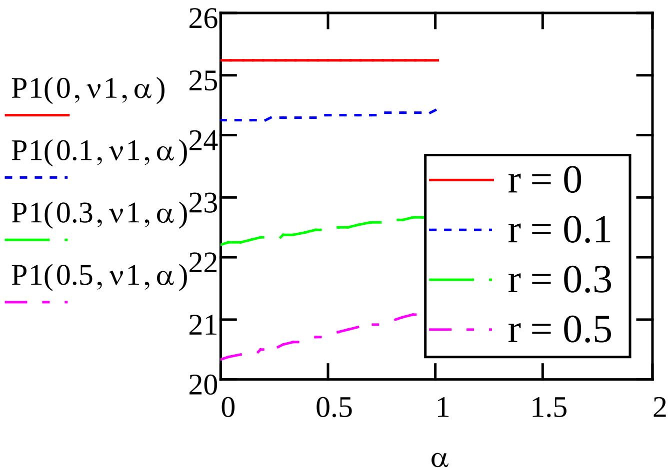

Fig. 5. The dependence $\widetilde{\mathrm{P}}$ on α for r = $\{0; 0.1; 0.3; 0.5\}$ and $\nu = 1.5\omega$ in the case of the free ends of the string.

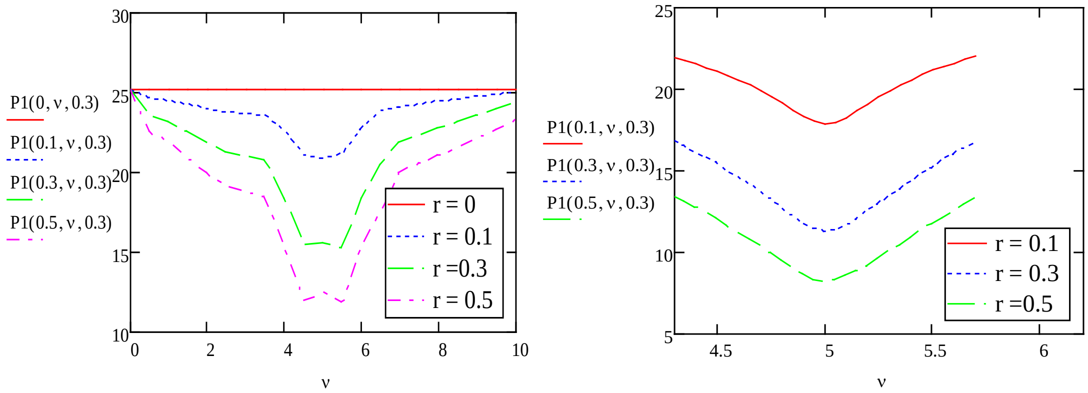

Fig.6. The dependence $\widetilde{\mathrm{P}}$ on ν for $r = \{0; 0.1; 0.3; 0.5\}$ and $\alpha = 0.3$ in the case of the free ends of the string.

5. The case of an impulsive loading with a vibration impact of finite duration

Let's now consider as an external load the following function

$$f(x,t) = P\big(H(t) - H(t-t_0)\big)\left(\delta\left(x - \frac{l}{2}\right) + r\sin(\nu t)\right), \tag{16}$$

where t_0 - impact time. The rest of the notation is analogous to the previous case.

We obtain for the first type of boundary conditions:

$$u^{(1)}\left(\frac{l}{2},t\right) = \frac{2c^2}{l}\,P\sum_{k=1}^{n}\Big\{A_k^{(1)}[(1-\cos\Omega_k t)H(t) - (1-\cos\Omega_k(t-t_0))H(t-t_0)] - -rB_k^{(1)}[w(\nu,\Omega_k,t) - w_1(\nu,\Omega_k,t,t_0)]\Big\}. \tag{17}$$

For the second type of boundary conditions, we obtain:

$$u^{(2)}\left(\frac{l}{2},t\right) = \frac{2c^2}{l}\,P\Big\{\frac{1}{2\omega^2}[(1-\cos\omega t)H(t) - (1-\cos\omega(t-t_0))H(t-t_0)] + +\frac{rl}{4\omega}[w(\nu,\omega,t)H(t) - w_1(\nu,\omega,t,t_0)H(t-t_0)] + \sum_{k=1}^{n}A_k^{(2)}[(1-\cos\Omega_k t)H(t) - (1--\cos\Omega_k(t-t_0))H(t-t_0)]\Big\}, \tag{18}$$

where $A_k^{(1)}$, $B_k^{(1)}$, $A_k^{(2)}$, $w(\nu,\Omega,t)$ are defined the same way as in a case of a constant load and $w_1(\nu,\Omega,t,t_0)$:

$$w_1(a,b,t,\tau) = \begin{cases} -\dfrac{b\cos(bt)\cos(bt0)\sin(a\tau) - b\sin(bt) + b\sin(bt)\sin(b\tau)\sin(a\tau)}{b^2-a^2} + \\ \quad + \dfrac{a\cos(bt)\sin(b\tau)\cos(a\tau) - a\cos(b\tau)\sin(bt)\cos(a\tau)}{b^2-a^2} & a \neq b \\ \dfrac{\sin(at-2a\tau)+\sin(at)-2a\cos(at)(t-\tau)}{4a}, & a = b \end{cases} \tag{19}$$

Parameters l, c, ω, t_c have the same values as in a case of a constant load, let`s put $\tau = 0.2$ s. Consider the dependence $\tilde{P} = \frac{2c}{lu_c}P$ on ν and t_0 for different r for boundary conditions of the first (Fig. 7, 8) and second type (Fig. 9,10).

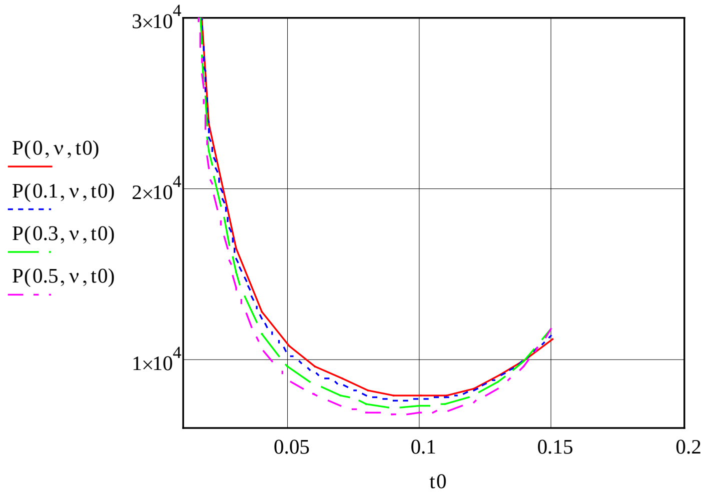

Fig. 7. The dependence $\tilde{P}$ on t_0 for $r = \{0; 0.1; 0.3; 0.5\}$ and $\nu = (\Omega_{1+}\,\Omega_2)/2$ in the case of the fixed ends of the string.

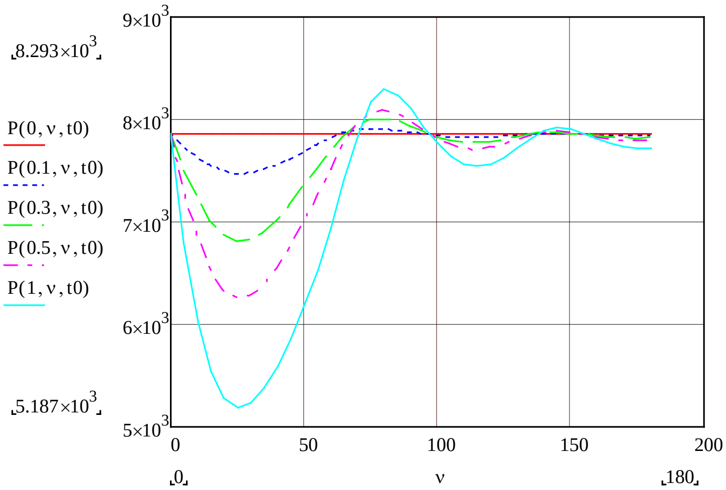

Fig. 8. The dependence $\widetilde{\mathrm{P}}$ on ν for $r = \{0; 0.1; 0.3; 0.5\}$, $t_0 = \frac{\tau}{2}$ and in the case of the fixed ends of the string.

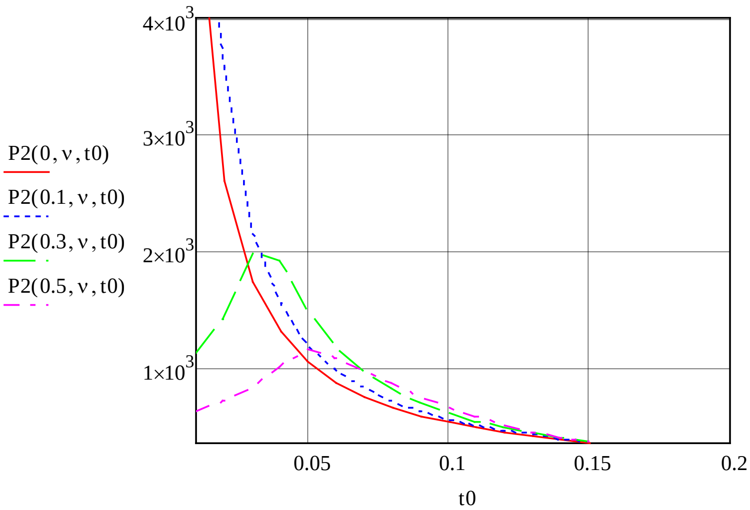

Fig. 9. The dependence $\widetilde{\mathrm{P}}$ on t_0 for $\mathrm{r} = \{0; 0.1; 0.3; 0.5\}$ and $\nu = 1.5\omega$ in the case of the free ends of the string.

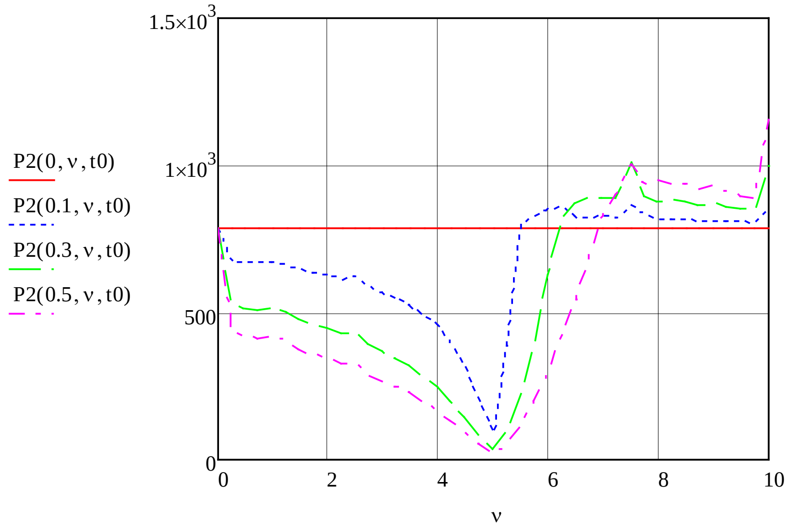

Fig. 10. The dependence $\widetilde{\mathrm{P}}$ on ν for $r = \{0; 0.1; 0.3; 0.5\}$ $t_0 = \frac{\tau}{2}$ and in the case of the free ends of the string.

6. Conclusions

A model of vibration of a string on the Winkler foundation, which is applicable in a simplified consideration of adhesive problems, is considered. The problem is considered from the viewpoint of the strength characteristics of the mechanical system, and, as a result, an analytical expression was found for the lowest load causing rupture of the adhesive joint. It was obtained that, imposing a background vibration on the already applied load, it is possible to decrease considerably the value of the critical force. This means, that it is possible to influence the strength properties of adhesive joints by superimposing background periodic fields. It makes sense to consider other models of the adhesion zone (for instance, a beam on an elastic base) in future studies

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