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DEFINING RELATIONS OF MECHANICS OF DAMAGED MEDIA AFFECTED BY FATIGUE AND CREEP

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Abstract. A possible version of a model describing degradation of initial strength properties of structural alloys affected by fatigue and creep is introduced, based on the modern viewpoint of mechanics of damaged media (MDM).

Processes of viscoplastic deformation and damage accumulation in structural alloys are analyzed using numerical modeling. Specia attention is paid to the issues of modeling the processes of viscoplastic deformation and damage accumulation for complex deformation processes accompanied by the rotation of main stress and strain tensor areas.

New qualitative and quantitative features of the failure process are noted, which are connected with stressed-strained state history, damage degree, stress relaxation and other factors.

Keywords: modeling; mechanics of damaged media; complex deformation; fatigue life; sustained strength; failure; service life.

1. Introduction

The development of structures and apparatuses of modern power engineering, aircraft manufacturing, etc. is characterized by an increase in their operating parameters, reduction of metal consumption due to the rational design and application of new structural materials, a significant increase in the weight of non-stationary modes of cyclic thermal loading, a significant extension of temperature range of the engineering designs. These trends have led to the fact that currently one of the main tasks in designing structures and apparatuses of new technology is the solution of the problem of reliably estimating service life of newly designed apparatuses, as well as on-line diagnosing of worked-out life and predicting individual residual life of already existing structures. These tasks are particularly relevant for structures and apparatuses with service lives of several tens of years (nuclear power plants, petrochemical equipment, aviation gas turbine engines (GTE) and plants (GTP) of the new generation, etc.).

Classical methods of predicting service life using semi-empirical formulas require a large amount of experimental information and are valid only for a narrow range of loading conditions.

In the recent years, a new scientific direction of mechanics of damaged media for solving such problems has been successfully developed [1–5].

The current practice of using MDM equations for different mechanisms of working out of service life suggests that this approach is efficient enough for practical applications, and quite accurate in evaluating the process of working out of service life of structural elements

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and components of load-bearing structures.

In the present paper, a mathematical model of mechanics of damaged media is developed, which is used to assess durability of materials and structures under fatigue and creep loading. Laws of damage accumulation in structural alloys affected by fatigue and creep are analyzed, using numerical modeling and comparing the obtained results with experimental data.

2. Defining relations of MDM

The model of damaged media developed in [1] consists of three interrelated parts:

– defining relations describing viscoplastic behavior of the material, taking into account its dependence on the failure process;

– evolutionary equations describing the kinetics of damage accumulation;

– strength criterion of the damaged material.

In the elastic region, the relationship between spherical and deviatoric components of the of stress and strain tensors is established by using Hooke's law:

$$\sigma = 3K(e-\alpha T),\ \sigma'_{ij} = 2Ge'^{e}_{ij},\ e'^{e}_{ij} = e'_{ij} - e'^{p}_{ij} - e'^{C}_{ij},$$
$$\dot{\sigma} = 3K(\dot{e}-\dot{\alpha}T-\alpha\dot{T}) + \dot{K}\sigma/K,\ \dot{\sigma}'_{ij} = 2G\dot{e}'^{e}_{ij} + \dot{G}\sigma'_{ij}/G, \tag{1}$$

where $\sigma, \dot{\sigma}, e, \dot{e}$ are spherical components and σ'_{ij}, $\dot{\sigma}'_{ij}$, e'_{ij}, $\dot{e}'_{ij}$ are deviatoric components of stress tensor σ_{ij}, deformations e_{ij} and their velocities $\dot{\sigma}_{ij}$, $\dot{e}_{ij}$, respectively; T is temperature; T_0 is initial temperature; $K(T)$ is bulk modulus; $G(T)$ is shear modulus; $\alpha(T)$ is coefficient of linear expansion of the material.

Effects of monotonous and cyclic deformation in the stress space are accounted for, using a Mises form of the yield surface:

$$F_s = S_{ij}S_{ij} - C_p^2 = 0,\ S_{ij} = \sigma'_{ij} - \rho^p_{ij} \tag{2}$$

To describe complex cyclic deformation modes in the stress space, a cyclic ‘memory’ surface is introduced:

$$F_\rho = \rho^p_{ij}\rho^p_{ij} - \rho^2_{\max} = 0, \tag{3}$$

where $\rho_{\max}$ is maximum module of variable ρ^p_{ij} in the loading history.

For the yield surface radius, the following evolutionary equation is assumed [1]:

$$\dot{C}_p = \left[q_\chi H(F_\rho) + a(Q_s - C_p)\Gamma(F_\rho)\right]\dot{\chi} + q_3\dot{T} \tag{4}$$

$$C_p = C_p^0 + \int_0^t \dot{C}_p dt,\ \dot{\chi} = \left(\frac{2}{3}\dot{e}^p_{ij}\dot{e}^p_{ij}\right)^{1/2},\ \chi_m = \int_0^t \dot{\chi}H(F_\rho)dt,\ \chi = \int_0^t \dot{\chi}dt \tag{5}$$

$$q_\chi = \frac{q_2A\psi_1 + (1-A)q_1}{A\psi_1 + (1-A)},\ Q_s = \frac{Q_2A\psi_2 + (1-A)Q_1}{A\psi_2 + (1-A)},\ 0 \le \psi_i \le 1,\ (i = 1,\ 2) \tag{6}$$

$$A = 1 - \cos^2\theta,\ \cos\theta = n^e_{ij}n^s_{ij},\ n^e_{ij} = \frac{\dot{e}'_{ij}}{(\dot{e}'_{ij}\dot{e}'_{ij})^{1/2}},\ n^s_{ij} = \frac{S_{ij}}{(S_{ij}S_{ij})^{1/2}}, \tag{7}$$

$$H(F_\rho) = \begin{cases} 1, F_\rho = 0 \wedge \rho^p_{ij}\dot{\rho}^p_{ij} > 0 \\ 0, F_\rho < 0 \vee \rho^p_{ij}\dot{\rho}^p_{ij} \le 0 \end{cases},\ \Gamma(F_\rho) = 1 - H(F_\rho), \tag{8}$$

where q_1, q_2, q_3 are modules of isotropic hardening corresponding to the monotonous radial loading paths (q_1), to a 90° kink of the deformation trajectory (q_2), to temperature variation of the yield surface radius (q_3); a is constant that determines the speed of the process of stabilization of the shape of the hysteresis loop of cyclic deformation of the material; Q_s is stationary value of the yield surface radius for the given $\rho_{\max}$ and T; χ and χ_m are lengths of plastic deformation trajectories of the material under cyclic and monotonic loading; C_p^0 is initial value of the yield surface radius.

The equation for the displacement of the yield surface is based on A.A. Ilyushin's hypothesis postulating that hardening depends on the history of deformation only over a certain nearest part of the path (delay of vector properties):

$$\dot{\rho}_{ij}^p = g_1^p \dot{e}_{ij}^p - g_2^p \rho_{ij}^p \dot{\chi} - g_3^p \rho_{ij}^p \dot{T}\,,\ \rho_{ij}^p = \int_0^t \dot{\rho}_{ij}^p dt\,, \tag{9}$$

where $g_1^p > 0$, $g_2^p > 0$ and $g_3^p > 0$ are modules of anisotropic hardening. The first and second members of the equation are responsible for the anisotropic part of strain hardening, and the third one for the variation of ρ_{ij} resulting from thermal loading with temperature T.

To characterize the behavior of the "memory" surface, it is necessary to formulate an evolution equation for $\rho_{\max}$:

$$\dot{\rho}_{\max} = \frac{(\rho_{ij}^p \dot{\rho}_{ij}^p) H(F_\rho)}{(\rho_{mn}^p \rho_{mn}^p)^{1/2}} - g_2 \rho_{\max} \dot{\chi} - g_3 \rho_{\max} \left\langle \dot{T} \right\rangle. \tag{10}$$

The tensor components of plastic strain rate are governed by the gradientality law:

$$\dot{e}_{ij}^p = \lambda S_{ij}\,. \tag{11}$$

To describe creep processes, equipotential creep surfaces F_c having a common centre ρ_{ij}^c and different radii C_c are introduced in the stress space:

$$F_c^{(i)} = S_{ij}^c S_{ij}^c - C_c^2 = 0\,,\ S_{ij}^c = \sigma'_{ij} - \rho_{ij}^c\,,\ i = 0,1,2,\ldots. \tag{12}$$

Among these equipotential surfaces it is possible to find a surface of radius $\bar{C}_c$ corresponding to zero creep rate:

$$F_c^{(0)} = \bar{S}_{ij}^c \bar{S}_{ij}^c - \bar{C}_c^2 = 0\,,\ \bar{S}_{ij}^c = \bar{\sigma}'_{ij} - \rho_{ij}^c\,. \tag{13}$$

It is then assumed that

$$\bar{C}_c = \bar{C}_c(\chi_c, T)\,,\ \dot{\chi}_c = \left(\frac{2}{3}\dot{e}_{ij}^c, \dot{e}_{ij}^c\right)^{1/2},\ \chi^c = \int_o^t \dot{\chi}^c dt\,,\ \psi_c = \left[\frac{\left(S_{ij}^c S_{ij}^c\right)^{1/2} - \bar{C}_c}{C_c}\right],\ \lambda_c = \begin{cases} 0, \psi_c \le 0 \\ \lambda_c, \psi_c > 0 \end{cases}. \tag{14}$$

The evolution equation for the variation of coordinates of the center of the creep surface has the form:

$$\dot{\rho}_{ij}^c = g_1^c \dot{e}_{ij}^c - g_2^c \rho_{ij}^c \dot{\chi}_c\,, \tag{15}$$

where g_1^c and $g_2^c > 0$ are experimentally determined material parameters.

The gradientality law can be represented as:

$$\dot{e}_{ij}^c = \lambda_c(\psi_c, T) S_{ij}^c = \lambda_c \psi_c S_{ij}^c = \lambda_c \left(\frac{\sqrt{S_{ij}^c S_{ij}^c} - \bar{C}_c}{C_c}\right) S_{ij}^c\,, \tag{16}$$

where λ_c for each of the three sections of the creep curve is defined as [1]:

$$\lambda_c^I = \lambda_c^{(0)}\left(1 - \frac{e_{11}^c}{e_{11}^{c(1)}}\right) + \lambda_c^{(1)} \frac{e_{11}^c}{e_{11}^{c(1)}}\,,\ \lambda_c^{II} = \frac{3}{2}\frac{\dot{e}_{11}^{ycm}}{\left(\sigma'_{11} - \frac{3}{2}\rho_{11}^c - \bar{\sigma}_c\right)}\,,\ \lambda_c^{III} = \lambda_c^{II} / (1-\omega)^{r_c}\,. \tag{17}$$

At the final stages of the accumulation process the effect of damage degree on the physical-mechanical properties of materials is observed. This effect can be accounted for on the basis of degrading continuum, by introducing effective stresses:

$$\tilde{\sigma}'_{ij} = F_1(\omega)\sigma'_{ij} = \frac{G}{\tilde{G}}\sigma'_{ij} = \frac{\sigma'_{ij}}{(1-\omega)\left[1-\frac{(6K+12G)}{(9K+8G)}\omega\right]},\ \tilde{\sigma} = F_2(\omega)\sigma = \frac{K}{\tilde{K}}\sigma = \frac{\sigma}{4G(1-\omega)/(4G+3K\omega)}, \quad (18)$$

$$\tilde{\rho}_{ij}^k = F_1(\omega)\rho_{ij}^k,\ k = p, c. \quad (19)$$

The structure of the evolution equation of fatigue damage accumulation will be presented in the following form [1, 3, 4, 6]:

$$\dot{\omega}_p = \frac{\alpha_p+1}{r_p+1} f_p(\beta) Z_p^{\alpha_p} (1-\dot{\omega}_p)^{-r_p} \langle \dot{Z}_p \rangle,\ Z_p = \frac{W_p - W_a}{\left(W_p^f - W_a\right)},\ \langle \dot{Z}_p \rangle = \frac{\langle \dot{W}_p \rangle}{\left(W_p^f - W_a\right)},\ f_p(\beta) = \exp(k_p \beta), \quad (20)$$

where α_p, r_p, k_p are material parameters. $f_p(\beta)$ is parameter function of the volumetric character of the stress state $\beta = \sigma/\sigma_u$; $\sigma_u = \left(\sigma'_{ij}\sigma'_{ij}\right)^{1/2}$ is intensity of the stress tensor; $\dot{W}_p = \int_0^t \dot{W}_p dt$ is energy for the formation of scattered fatigue damage under low cycle fatigue (LCF); W_p^f is the value W_p at the time of formation of the macrocrack under LCF.

The evolution equation for creep will be formulated as:

$$\dot{\omega}_c = \frac{\alpha_c+1}{r_c+1} f_c(\beta) Z_c^{\alpha_c} (1-\omega_c)^{-r_c} \langle \dot{Z}_c \rangle,\ Z_c = \frac{W_c}{W_c^f},\ \langle \dot{Z}_c \rangle = \frac{\langle \dot{W}_c \rangle}{W_c^f},\ \dot{W}_c = \rho_{ij}^c \dot{e}_{ij}^c,\ f_c(\beta) = \exp(k_c \beta). \quad (21)$$

Summation of damage during fatigue and creep can be written as:

$$\dot{\omega} = H\left(\frac{W_p}{W_a} - 1\right)\dot{\omega}_p + \dot{\omega}_c, \quad (22)$$

where is H Heaviside function.

The condition when the damage degree reaches its critical value [1, 5]

$$\omega = \omega_f \le 1 \quad (23)$$

is taken as a criterion of the completion of the phase of development of scattered microcracks (the stage of formation of a macrocrack).

3. Numerical results

In work [7], the authors experimentally study the effect of the deformation trajectory type on fatigue life of the X10CrNiTi18-10 steel under a combined effect of alternating torsion and uniaxial tension-compression.

The experiments were varied: the amplitude of intensity of plastic deformation (e_u^p), the angle of the type of deformed state (ψ) and the angle of phase shift θ between the amplitude of axial deformation and torsion deformation.

Processing of the experimental results by regression analysis provided the regression equation of the form [1]:

$$\begin{aligned} \ln N_f &= 10.50 - 7.50 e_u^p + 1.71\cdot 10^{-2}\psi - 6.367\cdot 10^{-5}\psi^2 - 1.5839\cdot 10^{-2}\theta + 8.41\cdot 10^{-5}\theta^2 + \\ &+ 2.66\cdot 10^{-2} e_u^p \theta + 3.133\cdot 10^{-5}\psi^2\theta - 2.4\cdot 10^{-3}\psi\theta + 1.372\cdot 10^{-5}\psi\theta^2 - 2.04\cdot 10^{-7}\psi^2\theta^2. \end{aligned} \quad (24)$$

The analysis of experimental information shows a significant effect of ψ and θ on fatigue life of the steel.

Figure 1 shows the fatigue curves (dotted lines) obtained using the regressive equation (fatigue life under uniaxial tension-compression (curve 2), curve 1 corresponds to alternating torsion, curve 3 corresponds to the ‘square-type’ trajectory (a combined effect of uniaxial tension-compression and alternating torsion)).

In Figure 1, the solid line shows fatigue curves for various deformation trajectories. Points A_i, B_i, C_i $(i=1,2,3)$ in Fig. 1 correspond to the results of numerical evaluation of fatigue life for the same intensity amplitude of plastic strains e_u^p for the three deformation trajectories in question.

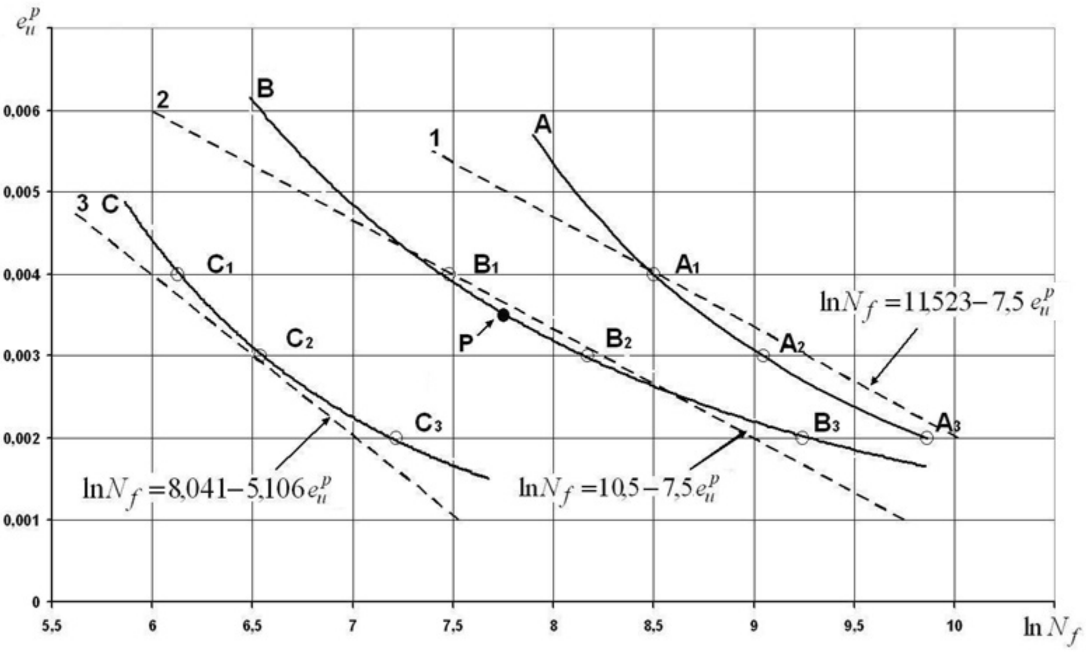

Fig. 1. Experienced and fatigue calculated curves.

The results of the comparison of the calculated and experimental data show that:

– the kind of fatigue curves is strongly nonlinear in its nature;

regression relations of the kind of (24) do not describe the experimental data in the region of "small" and "large" durability, so formula (24) should be used with caution;

– under the combined action of uniaxial tension–compression and alternating torsion (trajectory of the type "square") with the same amplitude of plastic deformation e_u^p, the durability is reduced compared to the uniaxial tension-compression is more than 6 times, and under alternating torsion over 14 times.

– intensity or full plastic deformation, the length of the trajectory of plastic strain are not the criteria of equivalence for processes with low cycle fatigue, and for disproportionate loading lead to a significant overestimation of the calculated durability as compared with the actual one.

In the second example, the results of analyzing the processes of long-term strength of the VZH-159 heat-resistant alloy are presented [8].

Figure 2 depicts the creep curves for:

– temperature $T = 750^o C$ and stresses $\sigma_{11} = 350$ MPa and 450 MPa, respectively (Fig. 2a);

– temperature $T = 850^o C$ and stresses $\sigma_{11} = 120$ MPa and 150 MPa, respectively (Fig. 2b);

– temperature $T = 750^o C$ and transition from stress level $\sigma_{11} = 350$ MPa to stress level - $\sigma_{11} = 450$ MPa (Fig. 2c);

– temperature $T = 850^{o}C$ and transition from stress level $\sigma_{11} = 120$ MPa to stress level – $\sigma_{11} = 150$ MPa (Fig. 2d).

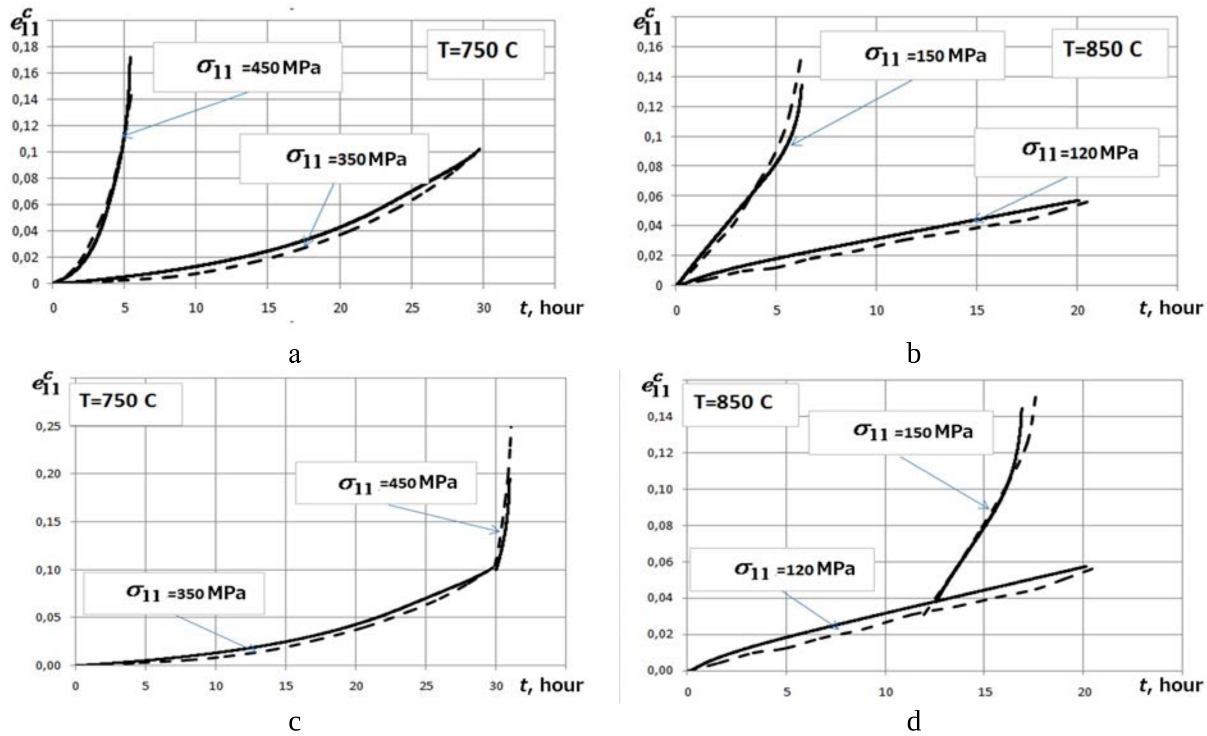

Fig. 2. Curves of creep of high-temperature alloy XH58MBJu.

Here, the solid lines represent the results of numerical modeling, using the defining relations of MDM, and the dotted ones show the corresponding experimental results. The qualitative and quantitative agreement between the experimental and numerical results is evident.

Figures 3–6, corresponding to the experiment depicted in Fig. 2d, show:

– effective stresses $\tilde{\sigma}_{11}$ as a function of process duration t (Fig.3);

– effective radius of zero level creep surface $\tilde{\tilde{C}}_c$ as a function of t (Fig. 4);

– creep failure energy W_c as a function of process duration (Fig. 5);

– damage degree ω as a function of process duration t (Fig. 6).

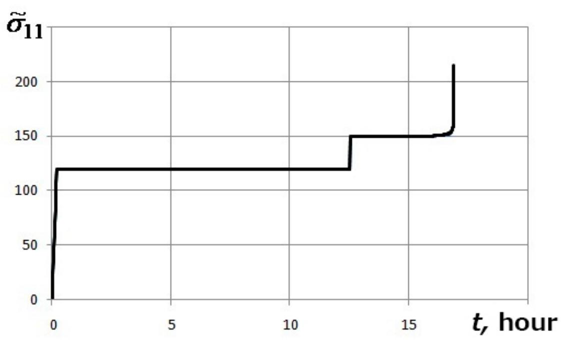

Fig. 3. The dependence of effective voltage versus $\tilde{\sigma}_{11}$ time of the process t.

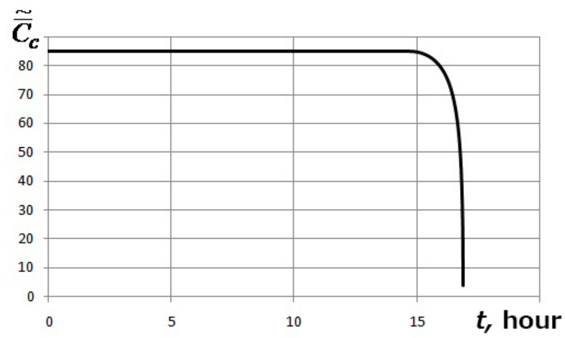

Fig. 4. The dependence of effective radius of the surface creep of the zero level $\tilde{\tilde{C}}_c$ from t.

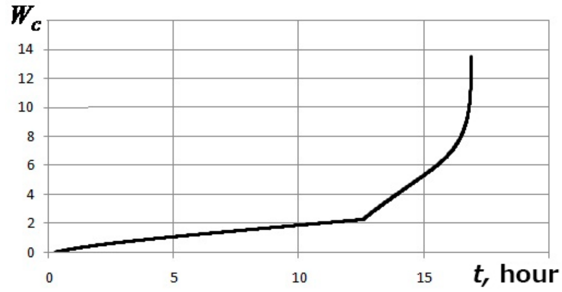

Fig. 5. The dependence of the energy failure in creep time W_c of the process t.

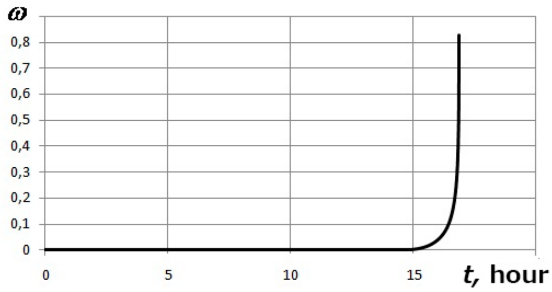

Fig. 6. The dependence of damage ω the time of the process t.

In general, when analyzing the obtained numerical results, it can be noted that the developed model of MDM qualitatively and quantitatively describes the main effects observed in nonstationary creep of structural materials (metals and their alloys) and degradation of initial strength properties of materials according to the long-term strength degradation mechanism.

4. Conclusion

A mathematical model of MDM has been developed, that describes processes of complex viscoplastic deformation and damage accumulation in structural alloys affected by fatigue and creep. The developed model makes it possible to account for:

- the dependence of the physical-mechanical properties of the material on temperature;
- the effect of the existing relations between mechanical and thermal strain rates;
- the effect of strain rates;
- the effects of complex loading;
- the effect of the volumetric character of the stressed state on damage accumulation rates;
- the existence of two stages of fatigue damage accumulation;
- nonlinearity of the fatigue damage accumulation process and nonlinearity of summation of damage when changing loading modes.

The adequacy of the defining relations of MDM has been assessed by comparing the results of numerical computations with the available test data on low-cycle fatigue and long-term strength, which corroborated the adequacy of the modeling and determining material parameters.

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References

[1] I.A. Volkov, L.A. Igumnov, *Introduction to Continuum Damage Mechanics* (Moscow, Phizmatlit, 2017), p.304 (in Russian).
[2] S. Murakami, T. Imaizumi // *J. Mec. Theor. Appl.* **1** (1982) 743.
[3] J. Lemaitre // *Trans. 5th Int. Conf. SMRiT* **L5/1b** (1979).
[4] J. Lemaitre // *ASME* **1(107)** (1985) 90.
[5] J.L. Chaboche // *Inter. J. of Plasticity* **3(5)** (1989) 247.
[6] S.R. Bodner, U.S. Lindholm // *ASME* **2(100)** (1976) 51.
[7] N.S. Mozharovsky, S.I. Shukaev // *Problems of strength* **10** (1988) 47.
[8] S.A. Kapustin, D.A. Kazakov, Yu.A. Churilov, A.I. Galushchenko // *Problems of Strength and Plasticity* **100** (2008) 100.