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RESULTS OF COMPUTER MODELLING OF A COMPOSITE POROVISCOELASTIC PRISMATIC SOLID DYNAMICS

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Abstract. Boundary-value problems for piecewise homogeneous solids in terms of linear three-dimensional poroviscoelasticity are considered. Mathematical model of poroviscoelastic material is based on Biot's model of poroelasticity. Viscoelastic effects refer to a skeleton of porous material and are described through the correspondence principle. Standard linear solid model is employed. Viscosity parameter influence on dynamic responses of displacements, pore pressure and tractions is studied.

In order to study the boundary-value problem boundary integral equations (BIEs) method is applied, and to find their solutions boundary element method (BEM) for obtaining numerical solutions is used. The numerical scheme is based on the Green-Betty-Somilliana formula. The solution of the original problem is constructed in Laplace transforms, with the subsequent application of the algorithm for numerical inversion. Modified Durbin's algorithm of numerical inversion of Laplace transform is applied to perform solution in time domain.

The problem a poroviscoelastic prismatic solid clamped from one end and free at another is considered. The solid is composed of two subdomains. Heaviside-type load is applied to a free end of the solid. Numerical results for displacements and pore pressure, when subdomains are modelled with different viscoelastic properties, are presented.

Keywords: poroviscoelasticity; viscoelastic models; boundary element method; boundary integral equation; Laplace transform.

1. Introduction

Wave propagation in dispersed media is a great interest of many disciplines. Mechanics of advanced materials, such as poro-, visco- or poroviscoelastic materials, is relevant to such disciplines as geophysics, geo- and biomechanics, seismology, physical chemistry, petroleum engineering etc.

Study of wave propagation processes in saturated porous continua began from the works of Y.I. Frenkel and M. Biot [1, 2]. The implementation of the solid viscoelastic effects in the theory of poroelasticity was first introduced by Biot [3]. Common state of the art can be found in works of R. de Boer (1996) [4] and M. Schanz (2001) [5]. Recent years the dynamic interaction analyses involving poroelastic/poroviscoelastic media is extensively studied in literature. The governing equation for saturated poroviscoelastic media by introducing the Kelvin–Voigt model (one-dimensional solution) were developed by Shanz and Cheng [6].

Fundamental and singular solutions play a key role in BEM, the expressions of which in the case of poroelasticity are known only in the Laplace or Fourier domain. There are two major approaches to dynamic processes modeled by means of BEM: solving BIEs system directly in time domain or in Laplace or Fourier domain followed by the respective transform

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inversion [7]. Classical formulations for BIEs method with their discretized realization and traditional BEM are successful approaches for solving three-dimensional problem [8].

The present paper is dedicated to the development of 3d poroviscodynamic problems numerical modeling technique based on using the Boundary Element Method in Laplace domain in case of inhomogeneous media. Inhomogeneous media is modeled as a piecewise homogeneous.

2. Problem formulation

Basic poroelastic material is a two-phase material consisting of an elastic skeleton and compressible fluid or gas filler. Porous material of a volume V can be constructed as follows:

$$V = V^f + V^s, \tag{1}$$

where V is the total volume, V^f is the summary pore volume and V^s is the volume of the skeleton. It is assumed that filler can openly seep through the pores and all closed pores are assumed as a part of the skeleton. Then a correspondence principle is applied to the skeleton, so we extend poroelastic formulation to poroviscoelasticity.

Considering a boundary-value problem for Biot's model of fully saturated poroelastic continuum in Laplace domain in terms of 4 unknowns (displacements $\bar{u}_i$ and pore pressure $\bar{p}$), the set of differential equations take the following form [4]:

$$\begin{gathered} G\bar{u}_{i,jj} + \left(K + \frac{G}{3}\right)\bar{u}_{j,ij} - (\psi - \beta)\bar{p}_{,i} - s^2(\rho - \beta\rho_f)\bar{u}_i = -\bar{F}_i, \\ \frac{\beta}{s\rho_f}\bar{p}_{,ii} - \frac{\phi^2 s}{R}\bar{p} - (\psi - \beta)s\bar{u}_{i,i} = -\bar{a}, \quad x \in \Omega, \end{gathered} \tag{2}$$

$$\begin{gathered} \bar{u}'(x,s) = \tilde{u}', \quad x \in \Gamma^u, \quad \bar{u}' = (\bar{u}_1, \bar{u}_2, \bar{u}_3, \bar{p}), \\ \bar{t}_n'(x,s) = \tilde{t}_n', \quad x \in \Gamma^\sigma, \quad \bar{t}' = (\bar{t}_1, \bar{t}_2, \bar{t}_3, \bar{q}), \end{gathered} \tag{3}$$

where Γ^u and Γ^σ denotes boundaries for boundary conditions of 1st and 2nd kind respectively, G, K are elastic moduli, $\phi = V^f / V$ is porosity, $\bar{F}_i$, $\bar{a}$ are bulk body forces,

$$\beta = \frac{\kappa\rho_f\phi^2 s}{\phi^2 + s\kappa(\rho_a + \phi\rho_f)}, \; \psi = 1 - \frac{K}{K_s} \; \text{and} \; R = \frac{\phi^2 K_f K_s^2}{K_f(K_s - K) + \phi K_s(K_s - K_f)}$$

are constants reflecting interaction between skeleton and filler, κ is permeability. Further, $\rho = \rho_s(1-\phi) + \phi\rho_f$ is a bulk density; ρ_s, ρ_a, ρ_f are solid density, apparent mass density and filler density, respectively; K_s, K_f are elastic bulk moduli of the skeleton and filler, respectively. Apparent mass density $\rho_a = C\phi\rho_f$ was introduced by Biot to describe dynamic interaction between fluid and skeleton. C is a factor depending on the pores geometry and excitation frequency.

Poroviscoelastic solution is obtained from poroelastic solution by means of the elastic-viscoelastic correspondence principle, applied to skeleton's constants K and G in Laplace domain. Forms of functions $\bar{K}(s)$ and $\bar{G}(s)$ are depended on chosen viscoelastic model.

In present paper, we employed standard linear solid model [8]:

$$\bar{K}(s) = K^\infty \cdot \left[(\beta_0 - 1)\frac{s}{s+\gamma} + 1\right], \; \bar{G}(s) = G^\infty \cdot \left[(\beta_0 - 1)\frac{s}{s+\gamma} + 1\right]. \tag{4}$$

The equilibrium and instantaneous values of the relaxation function, associated with material moduli, are connected as follows:

$$\beta_0 = K^0 / K^\infty = G^0 / G^\infty. \tag{5}$$

Equilibrium and instantaneous values are denoted by «∞» and «0», respectively.

3. Boundary-element approach

Boundary-value problem (2), (3) can be reduced to the BIE system as follows [8, 9]:

$$\frac{1-\alpha_\Omega}{2}\upsilon_i(\mathbf{x},s)+\int_\Gamma\left(T_{ij}(\mathbf{x},\mathbf{y},s)\upsilon_i(\mathbf{y},s)-T_{ik}^0(\mathbf{x},\mathbf{y},s)\upsilon_i(\mathbf{x},s)-U_{ij}(\mathbf{x},\mathbf{y},s)t_i(\mathbf{y},s)\right)d\Gamma=0,\ \mathbf{x},\mathbf{y}\in\Gamma, \tag{6}$$

where U_{ij}, T_{ij} are the fundamental and singular solutions, T_{ij}^0 contains the isolated singularities, $\mathbf{x}\in\Gamma$ is an arbitrary point. Coefficient α_Ω equals 1 in the case of finite domain and –1 in the case of infinite domain.

Boundary surface of our homogeneous solid is discretized by quadrangular and triangular elements and triangular elements are assumed as singular quadrangular elements. The Cartesian coordinates of an arbitrary point of the element are expressed through the coordinates of the nodal points of this element, using shape functions of the local coordinates. Shape functions are quadratic polynomials of interpolation. We use reference elements: square $\xi=(\xi_1,\xi_2)\in[-1,1]^2$ and triangle $0\le\xi_1+\xi_2\le 1,\ \ \xi_1\ge 0, \xi_2\ge 0$, and each boundary element is mapped to a reference one by the following formula:

$$y_i(\xi)=\sum_{l=1}^{8}N^l(\xi)y_i^{\beta(k,l)},\qquad i=1,2,3, \tag{7}$$

where l is the local node number in element k, $\beta(k,l)$ is the global node number, $N^l(\xi)$ are the shape functions. Goldshteyn's displacement – stress mixed model is performed. To discretize the boundary surface eight-node biquadratic quadrilateral elements are used, generalized displacements and tractions are approximated by linear and constant shape functions, respectively.

Subsequent applying of collocation method leads to the system of linear equations. As the collocation nodes, we take the approximation nodes of boundary functions. Gaussian quadrature is used to calculate integrals on regular elements. However, if an element contains a singularity, algorithm of singularity avoiding or order reducing is applied. When singularity is excluded, we use an adaptive integration algorithm. An appropriate order of Gaussian quadrature is chosen from primarily known necessary precision, if it is impossible, the element is subdivided to smaller elements recursively.

Solving the system of linear equations leads to the solution of the initial boundary-value problem in Laplace domain. Durbin's method [10] with variable integrating step is used for numerical inversion of Laplace transform in order to obtain solution in time domain:

$$f(0)\approx\sum_{l=1}^{n}\left[\frac{(F_{l+1}+F_l)\Delta_l}{2\pi}\right] \tag{8}$$

$$\Delta_l=\omega_{l+1}-\omega_l \tag{9}$$

$$f(t)\approx\frac{e^{\alpha t}}{\pi t^2}\sum_{l=1}^{n}\left[\frac{F_{l+1}-F_l}{\Delta_l}\left(\cos(\omega_{l+1}t)-\cos(\omega_l t)\right)+\frac{G_{l+1}-G_l}{\Delta_l}\left(\sin(\omega_{l+1}t)-\sin(\omega_l t)\right)\right] \tag{10}$$

where $F_l=\mathrm{Re}\left[\bar{f}(\lambda+i\omega_l)\right]$, $F_{l+1}=\mathrm{Re}\left[\bar{f}(\lambda+i\omega_{l+1})\right]$, $G_l=\mathrm{Im}\left[\bar{f}(\lambda+i\omega_l)\right]$, $G_{l+1}=\mathrm{Im}\left[\bar{f}(\lambda+i\omega_{l+1})\right]$.

Our numerical approach was checked by comparison with analytical solution in problem of homogeneous isotropic prismatic solid under Heaviside type load [11].

4. Numerical example

The problem of composite poroviscoelastic prismatic solid, clamped at one end and subjected to a Heaviside type load $t_2 = 1N/m^2$ at another end, is considered. Point C is at the clamped face, point A is at the loaded face (Fig.1). The length of the beam is 3 m. Solid is divided into two subdomains the 1st one is on at the clamped end, the 2nd one is at loaded end. Basic poroelastic material is Berea sandstone. Material constants are: $K = 8\cdot10^9$ N/m^2, $G = 6\cdot10^9$ N/m^2, $\rho = 2458$ kg/m^3, $\phi = 0.66$, $K_s = 3.6\cdot10^{10}$ N/m^2, $\rho_f = 1000$ kg/m^3, $K_f = 3.3\cdot10^9$ N/m^2, $k = 1.9\cdot10^{-10}$ m^4/(N$\cdot$s).

The observation point is point B, which is situated on the axis x_2 in 1.5 m from loaded end, see Fig. 1. Boundary-element mesh visualization is present in Fig. 2.

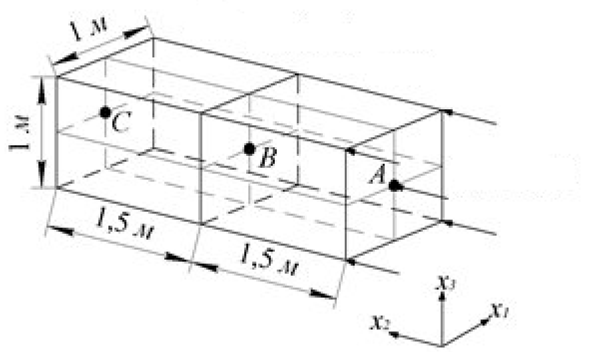

Fig. 1. Problem statement.

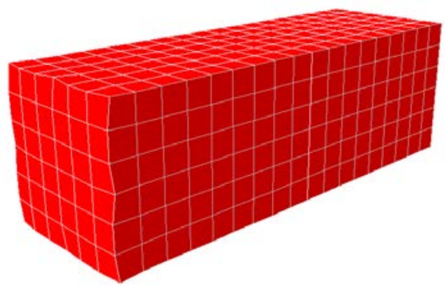

Fig. 2. Boundary-element mesh.

Elastic skeleton parameters of poroelastic medium are taken as equilibrium modules of viscoelastic model. Viscoelastic model parameters are: $\beta_0 = 4$, γ_1 and γ_2 are viscosities in each subdomain, respectively.

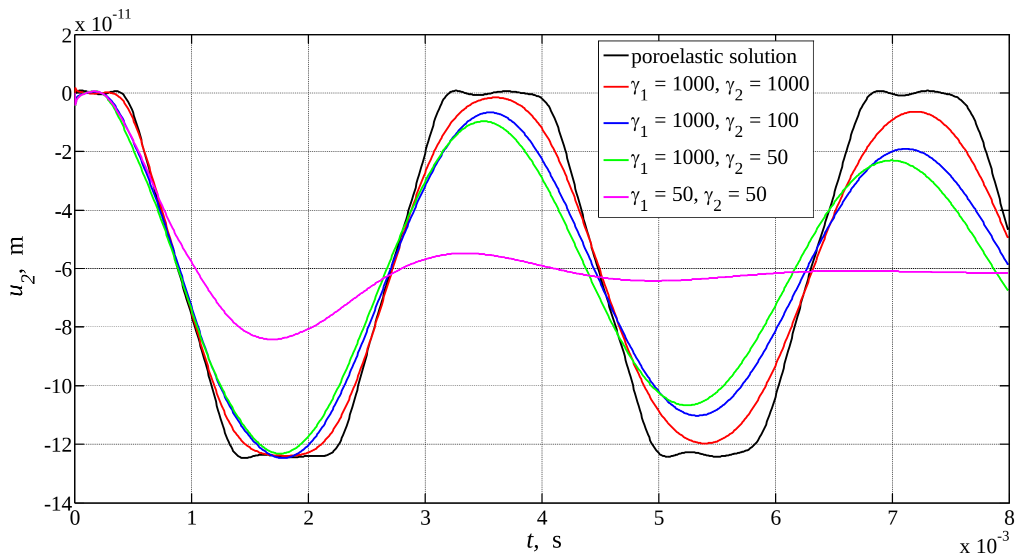

Fig. 3. Displacement u_2 at point B in case of standard linear solid model.

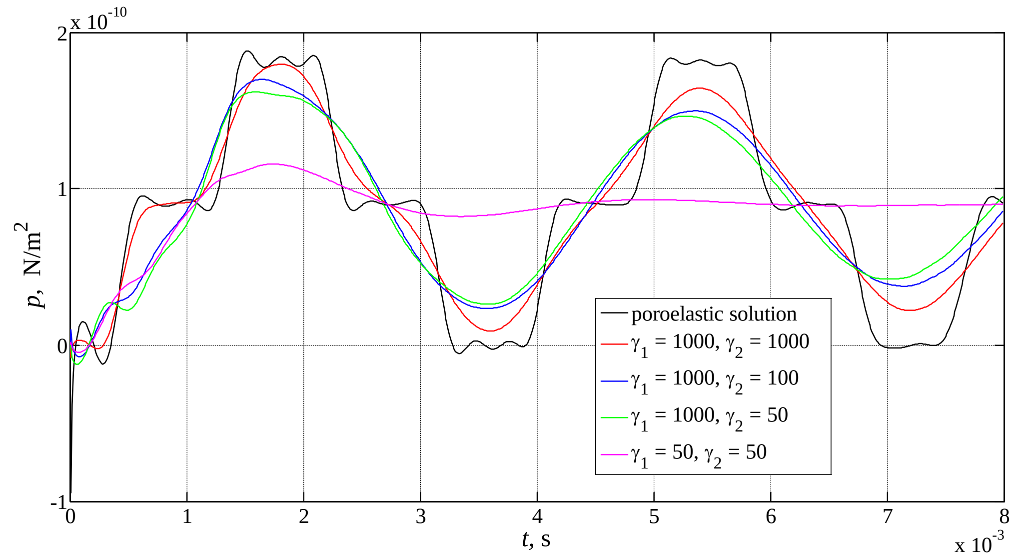

Fig. 4. Pore pressure p in case of standard linear solid model.

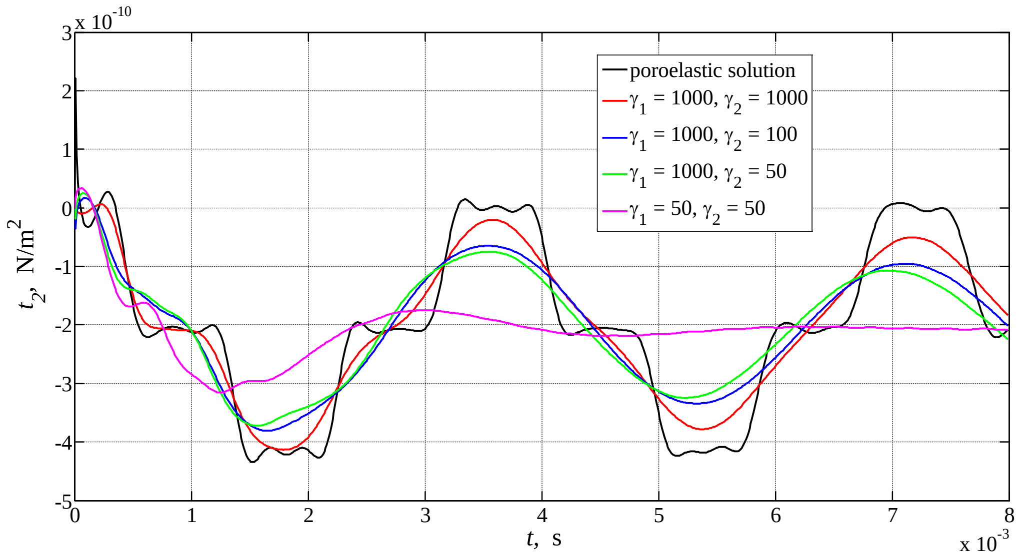

Fig. 5. Tractions t_2 at point *B* in case of standard linear solid model.

5. Conclusions

In present work we obtained a solution of the problem of a Heaviside-type load acting on a composite prismatic poroviscoelastic solid. The poroviscoelastic media modelling is based on Biot's theory of fully saturated poroelastic material in combination with the elastic-viscoelastic corresponding principle. Numerical solutions for displacement u_2, pore pressure p and traction t_2 of three dimensional dynamic poroviscoelasticity in the case of standard linear solid model are presented on Figs. 3 – 5, respectively. An influence of viscoelastic model parameters on dynamic responses of boundary functions is studied.

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