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ACCURACY IMPROVEMENT FOR COMBINED STATIC STRENGTH CRITERION FOR STRUCTURES UNDER COMPLEX LOADING

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Abstract. The article deals with the relevance of the stress strain state (SSS) in areas of potential destruction for the assessment of deformed structures strength. The article develops a calculation and experimental method for building a combined strength criterion equation involving strength coefficients, introduced by Pisarenko-Lebedev criterion equation with regard to the real SSS type. The proposed method has been implementedand experimental mechanical tests were carried out for the selected samples which follow the idea that the SSS type in the active zone is identical to the SSS of the analyzed structure in the area of interest. Results of the calculations and experimental tests demonstrate a significant decrease in the limit strength value for 50CrV (high quality) under biaxial tension in comparison with the conventional limit strength value.

Keywords: modeling, stress strain state, limit state equation, prismatic sample, finite element method, strength parameters, biaxial stretching

1. Introduction

Strength analysis for deformed structures under multidirectional loads has to be carried out with regard to the stress strain state (SSS) type in the areas of potential destruction [1 – 6]. A large number of vital parts and elements with stress concentrators are used in machines and units of the transport, energy and petrochemical industries. Stress concentration near structural irregularities of different size and shape (fillets, holes, roundings, slots) is typical. Examples of such parts are choke parts of capacitive equipment [5], suspension system elements (levers, axes) [6], compressor and turbine discs [7], wrought wheels of the rolling stock [8], etc. The SSS near these concentrators is biaxial, and its most dangerous case is biaxial tension that affects the parts lifecycle and is able to cause the destruction under quasi-static loading [1 – 6].

According to the experimental data, the material strength under biaxial tension is different from the one under uniaxial tension [1, 9 – 11]. In this case, the static strength of the part calculated with regard to the traditional criteria can be erroneous if its material is under biaxial loading. The result of this error can cause premature destruction of the structure in the area where the stress level is not maximum for the whole structure. It can also cause an unjustified increase in the specific amount of metal per structure [3]. Thus, the SSS type which is ignored by conventional criteria, affects the strength.

The SSS type can be used by two methods. The first method is based on the Pisarenko-Lebedev, Yagna-Buzhinsky, and Drucker-Pragger strength criteria [1, 3]. The method involves the preliminary calculation of material strength properties by destroying the samples under different loads in a quasi-static way. The properties are as follows: uniaxial

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tension, compression and shear (respectively, determination of the values of σ_t, σ_c and τ_s). These criteria combine various weight factors and two summands corresponding to the shear destruction (the first summand) and the cleavage destruction (the second summand). A factor limiting the accuracy of the combined strength criteria is the difference of the real SSS type of the structure and the SSS type of the samples tested before destruction when calculating σ_t, σ_c and τ_s. Moreover, calculations of these values and the need for various laboratory tools complicate the implementation of the method.

Let us analyze the Pisarenko-Lebedev limit state equation [1, 3] which is used for the assessment of structures strength. The condition under which the quasi-static destruction of the material accompanied by crack formation occurs is as follows

$$\alpha\sigma_i^{lim} + (1-\alpha)\sigma_1^{lim} A^{1-P} \le \sigma_t\,, \tag{1}$$

where σ_i^{lim} is the stress intensity in the area of potential destruction

$$\sigma_i^{lim} = \frac{1}{\sqrt{2}}\sqrt{\left(\sigma_1^{lim} - \sigma_2^{lim}\right)^2 + \left(\sigma_2^{lim} - \sigma_3^{lim}\right)^2 + \left(\sigma_3^{lim} - \sigma_1^{lim}\right)^2}\ , \tag{2}$$

σ_1^{lim}, σ_2^{lim}, σ_3^{lim} are primary stresses occurring in this area; P is the Smirnov-Alyaev coefficient [4, 5] characterizing the SSS type at the point under study. The coefficient is calculated by formula

$$P = \frac{\sigma_1^{lim} + \sigma_2^{lim} + \sigma_3^{lim}}{\sigma_i^{lim}}\,, \tag{3}$$

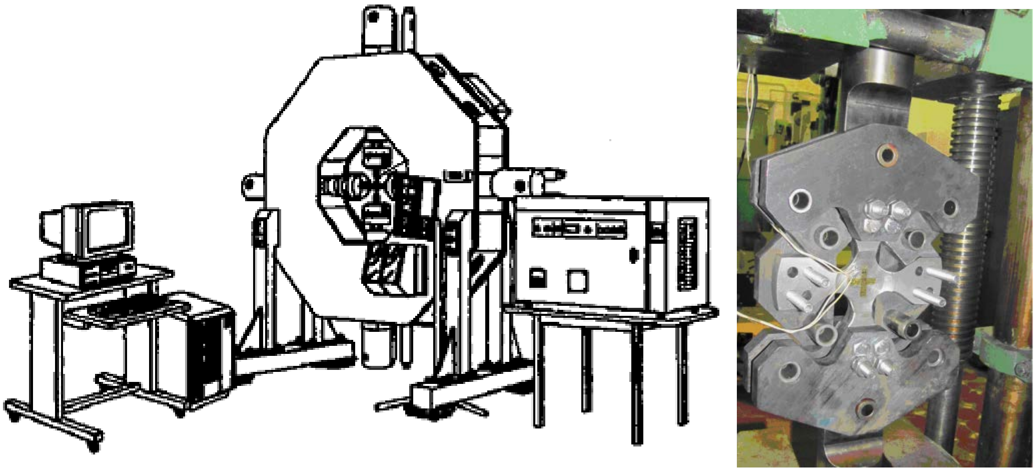
(for biaxial tension $P = 2$, for uniaxial tension $P = 1$, for uniaxial compression $P = -1$, for uniaxial shearing $P = 0$); α and A are empirical constants characterizing the material strength independent of the SSS (P) and the SSS level (σ_i^{lim}) of the material in the area of destruction (hereinafter referred to as strength coefficients) calculated by formulas

$$\alpha = \frac{\sigma_t}{\sigma_c},\quad A = \frac{\varphi - \sqrt{3}\alpha}{1-\alpha},\quad \varphi = \frac{\sigma_t}{\tau_s}. \tag{4}$$

Equation (1) takes into account the structural irregularity of the material, the ability to resist shearing and normal stresses. As mentioned above, in some cases, the accuracy of the criterion (1) is not sufficient due to differences between the real SSS type in the area of potential destruction and the SSS type for the samples tested before destruction when calculating σ_t, σ_c, τ_s and α and A. In [10 – 11], the authors argue that P calculated by equation (3) affects the area of destruction: with an increase in P (according to Smirnov-Alyaev, the rigidity of the SSS type), the limit stress intensity values and the first primary stress decrease.

The second method is based on the results of laboratory experiments carried out using special samples with the SSS type in the area of destruction similar to the SSS type in the area of potential destruction of the structure element. The results were accounted for in the deformation criteria [3]. The second method was implemented using special test equipment with several power drives (Fig. 1-a) creating multidirectional forces affecting the sample and special leverage mechanisms (Fig. 1-b) transforming a uniaxial force impact of the one-drive machine by levers linked to the sample into multidirectional forces affecting the sample. These samples, machines and mechanisms improve the accuracy of strength simulation and calculations but limit opportunities for carrying out the experiments.

The present article describes calculations and experiments carried out to improve the strength criterion which combines the elements of two methods. The combined criterion takes into account the real SSS type of the structure element in the area of potential destruction using typical one-drive testing machines. The use of experimental data on destruction of laboratory samples improved the calculations whose stress state at the moment of destruction simulates the stress state of the real structure in the working zone.



a b

Fig. 1. Biaxial loading unit (a) and leverage mechanisms (b) for static tests under biaxial tension

2. Methods

The mathematical simulation method and modern engineering technologies can be used to solve the task set in the article. The FEM can be used for solving the task of body mechanics. To improve the strength criteria for structures under complex loading and strength calculation accuracy, strength coefficients α and A can be calculated by equation (1) with regard to the real type of the SSS of the loaded area of the structure. The general scheme of the method is presented in Fig. 2.

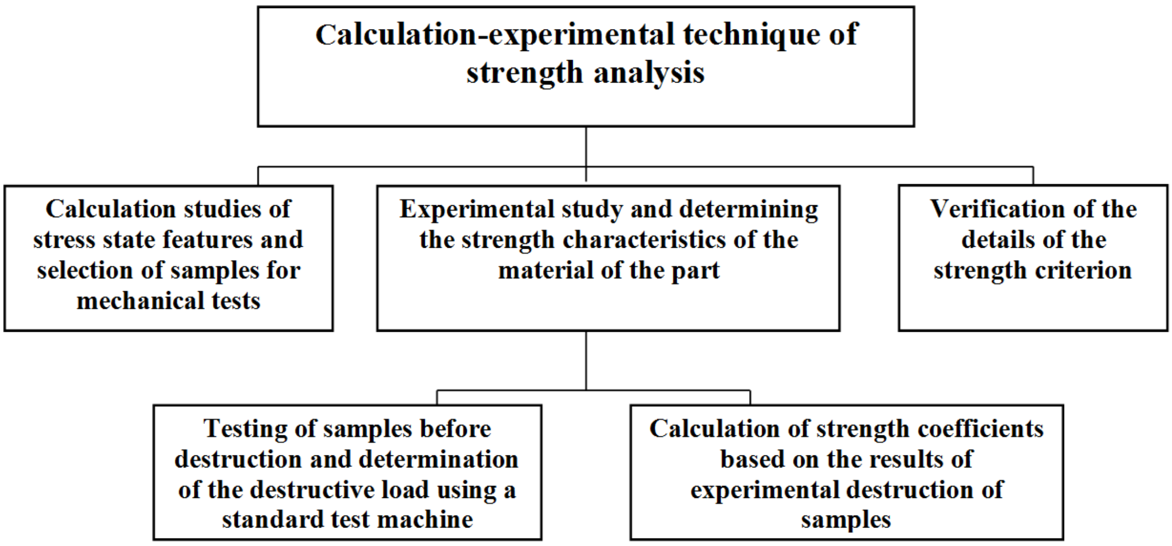


Fig.2. The scheme of improved strength calculations with regard to the real SSS type based on the combined strength criterion

Equation (1) shows that α and A are coefficients of the formula which describethe structural strength of amaterial with certain calculation values of σ_1^{lim}, $\sigma_i^{\ lim}$ and P^{lim} (parameters σ_1^{lim}, $\sigma_i^{\ lim}$) at the moment of destruction at P^{lim}. From this perspective, the Pisarenko-Lebedev criterion can be transformed into the following equation:

$$\alpha\sigma_i^{lim} + (1-\alpha)\sigma_1^{lim} A^{1-P^{lim}} \le \sigma_t , \tag{5}$$

where α and A can be calculated from equation (5) with regard to its other parameters σ_1^{lim}, $\sigma_i^{\ lim}$ and P^{lim}. That implies that α and A calculated from (1) do not have a conventional physical meaning resulting from the Pisarenko-Lebedev criterion. They depend on the SSS type of the material. Thus, α and A are coefficients of the empirical formula (hereinafter referred to as strength coefficients) considering the results for samples tested before destruction. They simulate the SSS type of the area of assessment.

The present article as well as the Pisarenko-Lebedev equation assumes the independence of α and A on the stress intensity σ_i and the first primary stress σ_1 in the area of potential destruction. Equation (5) approximates the real limit state equation built for a specific (or a relatively narrow range of modifications) value of P describing the SSS type. This type is determined by structural properties of samples tested before destruction. The area of destruction has to be characterized by P equal or close to P for the area of the structure.

After the value of P has been determined by the FEM used for solving the tasks of body mechanics to determine α and A from equation (5), it is necessary to match the type of the SSS of the samples and the type of the SSS of the structure. To this end, it is sufficient to test two samples before destruction. The samples have to have different sizes and SSS characteristics at the moment of destruction. Strength coefficients α and A can be calculated by the following algorithm:

- matching geometrical parameters of two different structural samples of special shapes with the values of P^{lim} which are close to the values of P for the area of the structure under study– $P^{\ 1\ lim}$ and $P^{\ 2\ lim}$;
- testing structural variants of the samples before destruction;
- calculating σ_1^{lim} and σ_i^{lim} which are the characteristics of the SSS level of equation (5) in the working area of the samples;
- calculating values of α and A from the system of equations

$$\begin{aligned} &\alpha\sigma_{\mathrm{i}}^{1\lim} + (1-\alpha)\sigma_1^{1\lim}\mathrm{A}^{1-\mathrm{P}^{1\lim}} = \sigma_t \\ &\alpha\sigma_{\mathrm{i}}^{2\lim} + (1-\alpha)\sigma_1^{2\lim}\mathrm{A}^{1-\mathrm{P}^{2\lim}} = \sigma_t , \end{aligned} \tag{6}$$

where values of $\sigma_1^{1\ lim}$, $\sigma_i^{1\ lim}$ and $P^{1\ lim}$ correspond to the experimentally determined moment of destruction of the first sample, $\sigma_1^{2\ lim}$, $\sigma_i^{2\ lim}$ and $P^{2\ lim}$ correspond to the experimentally determined moment of destruction of the second sample. In this case, equations of type (5) corresponding to two selected structural variants form the system of two non-linear algebraic equations with unknown variables α and A which can be solved by the successive approximation method. The use of experimental data on destruction of laboratory samples obtained under the SSS of the real structure can improve calculations accuracy. The need for strength parameters depending on primary stresses complicate the strength criterion. The author and his co-authors used a prismatic shape sample for the assessment of the strength of the structure under the complex stress state [12]. That helped obtain strength data with regard to the required coefficient P using a standard test machine compressing the sample. The principle of multidirectional test forces formation in the sample and the sample structure have been described in [22].

3. Calculation results

To improve the strength criterion with regard to deformation characteristics by the method presented in Fig. 2, strength coefficients α and A were calculated for 50CrV (high quality) steel which is in the most rigid SSS (using the Smirnov-Alyaev terms) under biaxial tension ($1< P <2$). The SSS with P =1.85 was used.

The state at P = 1,85 corresponds to the type of the SSS of the experimental model of the pressure vessel's choke part whose destruction area is on its external surface in the area adjacent to the weld of the choke (Fig. 3-b) [19]. In [19], the experimental model of the choke part consisted of a spherical frame and a welded choke (148 mm in internal diameter, and 216 mm in external diameter) made from the same material. The model was loaded with internal pressure in a quasi-static way. The destruction pressure of the model was p_{pf} = 77.7 MPa. As a result of the model destruction, a thorough crack cutting the choke wall along the meridian line formed. The crack is maximum in the area adjacent to the weld of the choke on its external surface (see Fig. 3-b). The view of the fracture surface corresponded to the quasi-brittle nature of destruction [19].

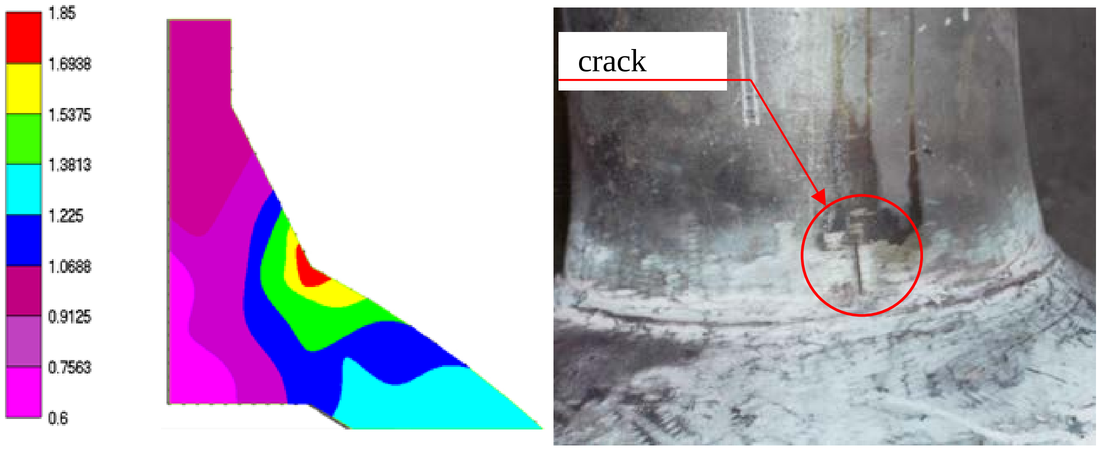


Fig. 3. Distribution of the calculated values of coefficient P (a) at the moment of destruction along the cross-section of the choke part (b)

The calculated value of P at the moment of destruction of the pressure vessel's choke model was determined by numerical analysisin MSC Patran and MSC Nastran. The numerical solution was based on the experimental results obtained when testing the model before destruction with regard to the elastoplastic properties of the deformed material. The mathematical elastoplastic deformation model described by Prandl-Reiss equations [20, 21] was used.

The comparison of the calculated value p_{pc} = 75 MPa and the real destruction pressure p_{pf} = 77.7 MPa shows that the calculation error does not exceed 5%.

With regard to the SSS characteristics at the moment of destruction, P in the FE-network along the cross-section of the choke part and in the area of destruction of the choke (on the external surface in the weld adjacent zone) was calculated using (3). Fig. 3-a shows the distibution of the calculated values of P along the cross-section of the choke part. In the area of destruction, the maximum value of P was P =1.85.

Model development is the first implementation stage for the method which can improve the accuracy of the strength criterion. This part of the article is a methodological description of the stage.

4. Results of experiments and calculations

To determine the conventional values of mechanical characteristics of 50CrV (high quality) steel, standard round samples were tested for tension. Static machining of five samples identified that 50CrV (high quality) steel has strength limit σ_t= 1270 MPa and relative extension δ = 7.5 %.

According to the method (see Fig. 1), to determine the values of α and A introduced by equation (5), it is necessary to destroy two structural samples of different size and value of $\sigma_1^{1\ lim}$, $\sigma_i^{1\ lim}$ and $\sigma_1^{2\ lim}$, $\sigma_i^{2\ lim}$ at the moment of destruction. The samples have to have equal (or close) values of P in their working zones. Sampling was described in [4, 23]. Two series of samples (series 1 and series 2) with three samples in each series were produced. Their sizes had to ensure P =1.85 in the working zone [23].

Prismatic samples of series 1 and series 2 were tested using Instron 5989 Testing System. The load diagrams presented in Fig. 4 show averaged values of force characteristics of three loaded samples in each series. The relative mean square deviation of these values did not exceed 5%.

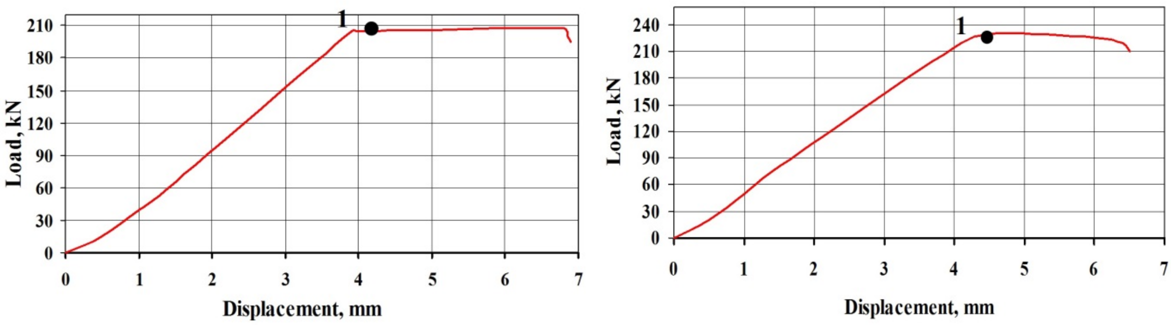


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Fig. 4. Averaged load diagrams for prismatic samples series 1 (a) and series 2 (b): point 1 – crack formation moment in the working zone of the prismatic samples

Destructive forces were F_1 = 205 kN, F_2 = 235 kN. F_1 and F_2 were used as initial values for the numerical FE analysis of the samples at the moment of destruction. The accuracy of calculated deformation models was proved by calculations and full-sized experiments [4, 23]. Values $\sigma_1^{1\ lim}$, $\sigma_i^{1\ lim}$ were calculated for the first prismatic sample ($P^{1\ lim}$ = 1.9). Values $\sigma_1^{2\ lim}$, $\sigma_i^{2\ lim}$ were calculated for the second prismatic sample ($P^{2\ lim}$ =1.8). Fig. 5-a shows the results of the numerical analysis of prismatic samples series 1 at the moment of destruction (corresponds to the moment of crack formation). Fig. 5-b shows the distribution of the component of the first primary stress.

The results of the numerical analysis of deformation for prismatic samples series 2 at the moment of destruction are shown in Fig. 6-a (the distribution of stress intensity) and in Fig. 6-b (the distribution of the component of the first primary stress).

The calculations and experimental results are presented in Table 1. The experimental results show that the influence of biaxial tension in the area of destruction can be significant for the samples of series 1. The limit value of the first primary stress $\sigma_1^{1\ lim}$ corresponding to the moment of destruction is 892 MPa which is a quarter less than the value of strength limit σ_t $\sigma_1^{1\ lim} = \sigma_t$ = 1270 MPa determined under uniaxial tension (P = 1). The results are close to the result of experimental studies carried out by Y.A. Vilimok, K.A. Nazarov, and A.K. Evdokimov [14]. Using special test equipment, the researchers identified the same decrease (by more than a quarter) in $\sigma_1^{1\ lim}$ under biaxial tension of X10CrNiTi18-10 steel with $P \approx 2$.

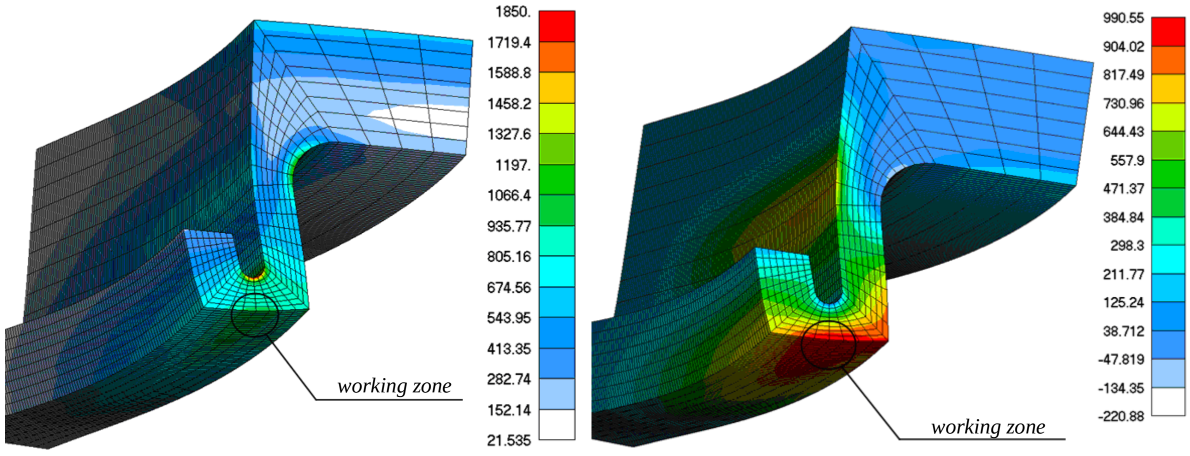

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Fig. 5. Calculated distribution of stresses at the moment of destruction of samples series 1 (the view of a quarter of the sample cut by two axial planes):
a – distribution of stress intensity $\sigma_i^{1\ lim}$, b – distribution of the first primary stress $\sigma_1^{1\ lim}$

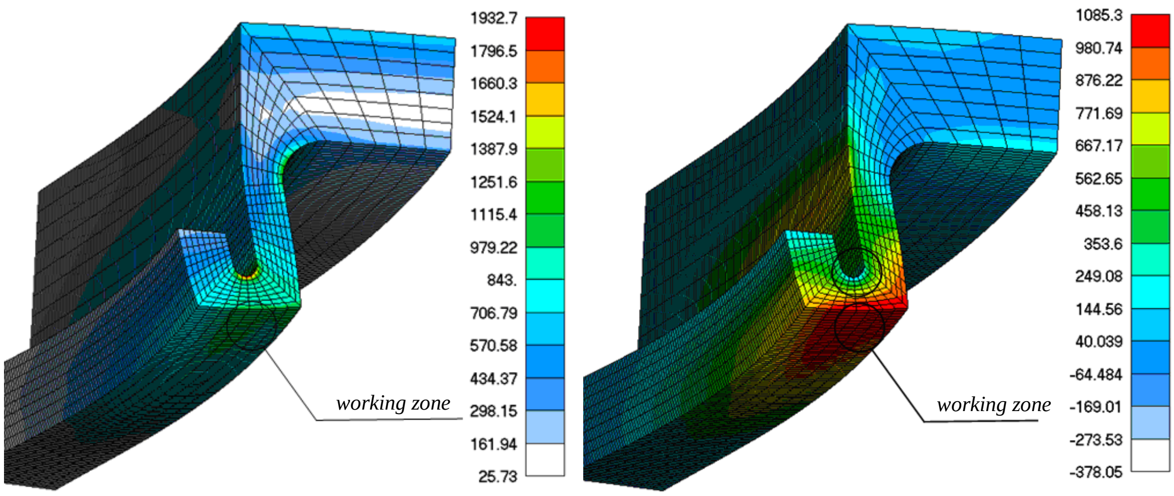

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Fig. 6. Calculated distribution of stresses at the moment of destruction of samples series 2 (the view of a quarter of the sample cut by two axial planes):
a – distribution of stress intensity $\sigma_i^{2\ lim}$, b – distribution of the first primary stress $\sigma_1^{2\ lim}$

Table 1. Calculated characteristics of the SSS of the samples at the moment of destruction

No. of series	Working zone			
	σ_1^{lim}, MPa	σ_2^{lim}, MPa	σ_i^{lim}, MPa	*P*
1	985	743	892	1,9
2	1050	615	917	1,8

To calculate α and A introduced by equation (5), let us insert the calculated values of the SSS characteristics (see Table 1) into the system of equations (6)

$$\begin{cases} \alpha 892 + (1-\alpha)985 A^{1-1,9} = 1270 \\ \alpha 917 + (1-\alpha)1050 A^{1-1,8} = 1270 \end{cases}. \qquad (7)$$

Solving equations (7) by the successive approximation method, one can obtain the following values:

$\alpha = 0.73; \; A = 0.40.$ (8)

Equation (5) can be used to improve the calculations of static strength of the structure made from 50CrV (high quality) steel and described by $P \approx 1.85$.

If we take into account that α and A in equation (1) do not depend on P, for 50CrV (high quality) steel strength parameters (according to the conventional terminology) α and A calculated by (4) will be $\alpha = 0.6$, $A = 0.75$ at any P. It contradicts the hypothesis that the strength depends on the SSS type (P). Therefore, α and A have to be adjusted. Otherwise, at given P, a rigid zone of the structure will fail (it happened to the choke part in the area adjacent to the weld where the stress intensity value was not maximum for the whole structure). If $\alpha = 0.73$ and $A = 0.4$, one can see that the influence of P on α and A can be significant. Thus, condition (1) has to be the approximation of the real limit state equation built for a specific (or a relatively narrow range of modifications) value of P describing the SSS type.

5. Conclusions

The proposed method for building a combined strength criterion involves using experimental data on destruction of laboratory samples whose stress state at the moment of destruction simulates the stress state of the structure. The use of these data can improve calculations accuracy for strength coefficients in the Pisarenko-Lebedev strength equation.

The calculations and experiments identified that for 50CrV (high quality) steel, the limit value of stress intensity under biaxial tension ($P \geq 1.8$) decreases by a quarter in comparison with the strength value determined under uniaxial tension ($P = 1$).

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