

# ON SOME FEATURES OF IDENTIFICATION OF INHOMOGENEOUS PRESTRESSED STATE OF THERMOELASTIC HOLLOW CYLINDER WITH COATING

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**Abstract.** An inverse thermoelasticity problem of identification of inhomogeneous prestressed state of an infinitely long hollow cylinder with a coating is formulated. The characteristics of cylinder's material are described by piecewise continuous functions. A direct thermoelasticity problem is solved on the basis of the shooting method and inversion of solutions based on the Durbin method after applying the Laplace transform. The nonlinear inverse problem is solved by constructing an iterative process, at each stage of which the operator equations of the first kind are solved. The most informative time intervals for gaining the additional information are determined. The influence of prestress level, coupling parameter and coating thickness on the results of prestress reconstruction is analyzed.

**Keywords:** thermoelasticity, prestress, coating, cylinder, identification, inverse problem

## 1. Introduction

Protection of structural elements (like turbine blades, combustion chambers, piping systems and nozzle guide vanes), operating under conditions of combined thermomechanical loading, is usually provided by applying a thermal protective coating on their surfaces [1]. The main characteristic of thermal protective coatings is low thermal conductivity coefficient, due to which the temperature on the metal substrate surface is reduced down to 100-300°C. The production of materials with thermal protective coatings is a complex technological process. Due to the multi-stage technological operations, inhomogeneous residual stresses often occur in the final product and can lead to coating delamination.

The first results of investigations of dynamic thermoelasticity problems in the presence of homogeneous prestresses were given in [2]. For rigorous description of thermomechanical processes in prestressed bodies, it was necessary to involve an apparatus of nonlinear thermoelasticity. However, for a wide range of problems for prestressed thermoelastic bodies a simplified linearized theory is widely used [3], which based on the A.N. Guz model [4].

Since the main structural elements are often not available for direct observation and control, there is a need to develop a non-destructive method of identification of the prestressed state. Because of the considerable interest in the problem of prestressed state identification, the number of publications on this issue is steadily growing [5-11]. However, most diagnostic methods are aimed at investigating homogeneous prestressed state. Mathematically inhomogeneous prestressed state is manifested in the dependence of the differential thermoelasticity operators coefficients on the coordinates. The determination of the inhomogeneous prestressed state is only possible by use of the apparatus of coefficient inverse problems (CIP) of thermoelasticity [12,13] and requires some additional information.

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The research in the field of thermoelasticity CIP is mainly limited by slightly inhomogeneous materials [14].

Two types of mechanics inverse problems statements are widespread in practical applications. For the first type the additional information is assumed to be known at internal points of a body at some moment of time; for the second type the additional information is known only on the part of the boundary on certain time interval.

If additional information is only known on the body's boundary, the inverse problem is essentially nonlinear. As a rule, the solution of CIP is reduced to the solution of the corresponding extremal problems by use of gradient methods [13,15]. The use of gradient methods for minimization requires significant computation time and has a number of other drawbacks. As an alternative to gradient methods, an approach based on constructing an iterative process, assuming solving a linearized operator equation of the first kind at each iteration, has been used in recent years [16]. It should be noted that the problems of reconstructing material characteristics and inhomogeneous prestressed states in elastic and thermoelastic bodies were solved in [17-21]. However, an issue of identifying prestresses in bodies with coatings remained unexplored in these studies.

In this paper we present the equations of thermoelasticity for a prestressed cylinder based on the approach proposed in [12]. The coated cylinder is modeled as a thermoelastic cylinder with thermomechanical characteristics that are described by piecewise continuous functions of radial coordinate. After applying the Laplace transform, the direct problem of thermoelasticity is solved on the basis of the shooting method and inversion of transformants based on the Durbin method. In the inverse problem, on the basis of the algorithm developed in [12], we restore the functions with a first-kind discontinuity point on the interface of the coating with the cylinder. The analysis of the effect of the prestress level, the coupling parameter and the coating thickness on the results of the reconstruction of inhomogeneous prestresses is made. The developed approach allows recovering arbitrary functions characterizing prestressed states of cylinders.

## 2. Problem of identification of the inhomogeneous prestressed state of a thermoelastic cylinder

Consider an infinitely long hollow thermoelastic cylinder with an inner surface  $r = a$ . On the outer surface of the cylinder  $r = b$  there is a coating of a thickness  $h$ . The coated cylinder is subjected to a prestressed state which is characterized by the components of the prestress tensor  $\sigma_{rr}^0$  and  $\sigma_{\varphi\varphi}^0$ , which are related to each other by the equilibrium equation

$$\frac{d\sigma_{rr}^0}{dr} + \frac{\sigma_{rr}^0 - \sigma_{\varphi\varphi}^0}{r} = 0.$$

The inner surface of the cylinder is thermally insulated and free of mechanical stresses. The uniformly distributed mechanical  $p_0$  and thermal load  $q_0$  act on the outer surface of the coated cylinder ( $r = b + h$ ). The cylinder's material is characterized by the density  $\rho_s$ , the Lamé coefficients  $\lambda_s$  and  $\mu_s$ , the thermal conductivity coefficient  $k_s$ , the specific heat capacity  $c_s$ , the thermal stress coefficient  $\gamma_s$ , and the prestress  $\sigma_{rr}^{0s}$  and  $\sigma_{\varphi\varphi}^{0s}$  (s-substrate); the coating's material is described by the characteristics  $\rho_c$ ,  $\lambda_c$ ,  $\mu_c$ ,  $k_c$ ,  $c_c$ ,  $\gamma_c$ ,  $\sigma_{rr}^{0c}$  and  $\sigma_{\varphi\varphi}^{0c}$  (c-coating). Consider the material characteristics of the cylinder-coating system in the form of piecewise continuous functions of the form:

$$F(r) = \begin{cases} F_s(r), & \text{при } r \in [a, b]; \\ F_c(r), & \text{при } r \in [b, b + h] \end{cases}, \quad (1)$$

where  $F(r)$  conditionally denotes any of the material parameters  $\rho(r)$ ,  $\lambda(r)$ ,  $\mu(r)$ ,  $k(r)$ ,  $c(r)$ ,  $\sigma_{rr}^0(r)$ ,  $\sigma_{\varphi\varphi}^0(r)$ . We denote any of the material characteristics of the cylinder by  $F_s$ , and any of the coating ones by  $F_c$ .

Similarly, we introduce the notation for physical fields:

$$Q(r,t) = \begin{cases} Q^s(r,t), & \text{при } r \in [a,b]; \\ Q^c(r,t), & \text{при } r \in [b,b+h]. \end{cases} \quad (2)$$

Here,  $Q(r,t)$  is any of the functions describing the thermo-elastic process:  $u_r(r,t)$  - radial displacement,  $\theta(r,t)$  - temperature increment,  $T_{rr}(r,t)$ ,  $T_{\varphi\varphi}(r,t)$  - components of the Piola incremental stress tensor.

In accordance with the model proposed by A.N. Guz [4], the equations of coupled thermoelasticity for a prestressed cylinder under conditions of plane deformation ( $u_r = u_r(r,t)$ ,  $u_\varphi = 0$ ,  $u_z = \text{const}$ ) have the following form [12]:

$$\frac{\partial T_{rr}}{\partial r} + \frac{T_{rr} - T_{\varphi\varphi}}{r} = \rho \frac{\partial^2 u_r}{\partial t^2}, \quad (3)$$

$$\frac{1}{r} \frac{\partial}{\partial r} (k(r)r \frac{\partial \theta}{\partial r}) = c(r) \frac{\partial \theta}{\partial t} + T_0 \gamma(r) (1 + \frac{\sigma_{rr}^0}{\lambda + 2\mu}) (\frac{\partial^2 u_r}{\partial r \partial t} + \frac{1}{r} \frac{\partial u_r}{\partial t}), \quad (4)$$

$$T_{rr} = (\lambda + 2\mu) \frac{\partial u_r}{\partial r} + \lambda \frac{u_r}{r} + \sigma_{rr}^0 \frac{\partial u_r}{\partial r} - \gamma \theta, \quad (5)$$

$$T_{\varphi\varphi} = \lambda \frac{\partial u_r}{\partial r} + (\lambda + 2\mu) \frac{u_r}{r} + \sigma_{\varphi\varphi}^0 \frac{u_r}{r} - \gamma \theta, \quad (6)$$

$$\frac{\partial \theta}{\partial r}(a,t) = 0, \quad -k(b+h) \frac{\partial \theta}{\partial r}(b+h,t) = q_0, \quad (7)$$

$$T_{rr}(a,t) = 0, \quad T_{rr}(b+h,t) = p_0, \quad (8)$$

$$\theta(r,0) = u_r(r,0) = \frac{\partial u_r}{\partial t}(r,0) = 0. \quad (9)$$

Similarly, we can write the thermoelasticity initial-boundary problem for the case when the coating is set on the inner side of the cylinder.

At the interface boundary between the coating and the cylinder  $r=b$ , by virtue of continuity, the following coupling conditions for radial stresses, temperature and heat flow must be met:

$$\begin{aligned} u_r^s(b,t) &= u_r^c(b,t), \quad T_{rr}^s(b,t) = T_{rr}^c(b,t), \\ \theta^s(b,t) &= \theta^c(b,t), \quad k_s(b) \frac{\partial \theta^s}{\partial r}(b,t) = k_c(b) \frac{\partial \theta^c}{\partial r}(b,t). \end{aligned} \quad (10)$$

Let us introduce dimensionless parameters and variables into (3)-(10):

$$\begin{aligned} h_0 &= b+h-a, \quad z = \frac{r-a}{h_0}, \quad z_0 = \frac{a}{h_0}, \quad h_1 = \frac{h}{h_0}, \quad H = 1-h_1, \quad \bar{s}(z) = \frac{\lambda+2\mu}{\mu_0}, \quad \bar{\lambda}(z) = \frac{\lambda(r)}{\mu_0}, \\ \bar{k}(z) &= \frac{k(r)}{k_0}, \quad \bar{c}(z) = \frac{c(r)}{c_0}, \quad \bar{\gamma}(z) = \frac{\gamma(r)}{\gamma_0}, \quad \bar{\rho}(z) = \frac{\rho(r)}{\rho_0}, \quad v = \sqrt{\frac{\mu_0}{\rho_0}}, \quad t_1 = \frac{h_0}{v}, \quad t_2 = \frac{h_0^2 c_0}{k_0}, \quad \tau = \frac{t}{t_2}, \\ W(z, \tau) &= \frac{\gamma_0 \theta}{\mu_0}, \quad U(z, \tau) = \frac{u_r}{h_0}, \quad \Omega_r(z, \tau) = \frac{T_{rr}}{\mu_0}, \quad \Omega_\varphi(z, \tau) = \frac{T_{\varphi\varphi}}{\mu_0}, \quad \Omega_r^0(z) = \frac{\sigma_{rr}^0}{\mu_0}, \quad \Omega_\varphi^0(z) = \frac{\sigma_{\varphi\varphi}^0}{\mu_0}, \end{aligned}$$

$$\delta_0 = \frac{\gamma_0^2 T_0}{\mu_0 c_0}, \quad \varepsilon = \frac{t_1}{t_2}, \quad p^* = \frac{p_0}{\mu_0}, \quad q^* = \frac{q h_0 \gamma_0}{k_0 \mu_0}, \quad \mu_0 = \frac{1}{h_0} \int_a^{b+h} \mu(r) dr, \quad k_0 = \frac{1}{h_0} \int_a^{b+h} k(r) dr,$$

$$c_0 = \frac{1}{h_0} \int_a^{b+h} c(r) dr, \quad \gamma_0 = \frac{1}{h_0} \int_a^{b+h} \gamma(r) dr, \quad \rho_0 = \frac{1}{h_0} \int_a^{b+h} \rho(r) dr, \quad \lambda_0 = \frac{1}{h_0} \int_a^{b+h} \lambda(r) dr.$$

After the transformation of (3)-(10) into dimensionless form, taking into account the conjugation conditions, the initial-boundary problem, takes the form:

$$\frac{\partial \Omega_r}{\partial z} + \frac{\Omega_r - \Omega_\phi}{z + z_0} = \varepsilon^2 \bar{\rho} \frac{\partial^2 U}{\partial \tau^2}, \quad (11)$$

$$\Omega_r = (\bar{s} + \Omega_r^0) \frac{\partial U}{\partial z} + \frac{\bar{\lambda}}{z + z_0} U - \bar{\gamma} W, \quad (12)$$

$$\Omega_\phi = \bar{\lambda} \frac{\partial U}{\partial z} + (\bar{s} + \Omega_r^0 + (z + z_0) \frac{d\Omega_r^0}{dz}) \frac{U}{z + z_0} - \bar{\gamma} W, \quad (13)$$

$$\frac{1}{z + z_0} \frac{\partial}{\partial z} (\bar{k}(z + z_0) \frac{\partial W}{\partial z}) = \bar{c} \frac{\partial W}{\partial \tau} + \delta \bar{\gamma} (1 + \frac{\Omega_r^0}{\bar{s}}) (\frac{\partial^2 U}{\partial z \partial \tau} + \frac{1}{z + z_0} \frac{\partial U}{\partial \tau}), \quad (14)$$

$$\frac{\partial W}{\partial z}(0, \tau) = 0, \quad -\bar{k}(1) \frac{\partial W}{\partial z}(1, \tau) = q^*, \quad (15)$$

$$\Omega_r(0, \tau) = 0, \quad \Omega_r(1, \tau) = p^*, \quad (16)$$

$$W(z, 0) = U(z, 0) = \frac{\partial U}{\partial \tau}(z, 0) = 0, \quad (17)$$

$$U^s(H, \tau) = U^c(H, \tau), \quad \Omega_r^s(H, \tau) = \Omega_r^c(H, \tau),$$

$$W^s(H, \tau) = W^c(H, \tau), \quad \bar{k}_s(H) \frac{\partial W^s}{\partial z}(H, \tau) = \bar{k}_c(H) \frac{\partial W^c}{\partial z}(H, \tau). \quad (18)$$

The direct problem of thermoelasticity consists in determining the functions  $U(z, \tau)$ ,  $W(z, \tau)$  from (11)-(18) for known thermomechanical characteristics  $\bar{s}(z)$ ,  $\bar{\lambda}(z)$ ,  $\bar{k}(z)$ ,  $\bar{c}(z)$ ,  $\bar{\gamma}(z)$ ,  $\bar{\rho}(z)$  and prestresses  $\Omega_r^0(z)$ ,  $\Omega_\phi^0(z)$ .

For arbitrary laws of variation of thermomechanical characteristics and prestresses, the problem (11)-(18), after applying the Laplace transform, can be solved only numerically. Proceeding in a similar way to [12], in order to solve the obtained system of differential equations in transforms, we use the shooting method modified for the case of piecewise continuous functions. To find the actual space of solutions, we used the Durbin method [22].

In the inverse problem, it is required to determine the functions  $\Omega_r^0(z)$ ,  $\Omega_\phi^0(z)$  from (11)-(18) with known thermomechanical characteristics  $\bar{s}(z)$ ,  $\bar{\lambda}(z)$ ,  $\bar{k}(z)$ ,  $\bar{c}(z)$ ,  $\bar{\gamma}(z)$ ,  $\bar{\rho}(z)$  for some additional information.

a) Temperature data

$$W(1, \tau) = f_1(\tau), \quad \tau \in [a_1, b_1]. \quad (19)$$

b) Displacement data

$$U(1, \tau) = f_2(\tau), \quad \tau \in [a_2, b_2], \quad (20)$$

measured on the outer surface of the cylinder with the coating  $z = 1$  on time intervals  $[a_1, b_1]$  and  $[a_2, b_2]$  that are informative in terms of identifying and close to the reference point.

With the known  $\Omega_r^0(z)$ , the prestresses  $\Omega_\phi^0(z)$  can be easily found from the equilibrium equation  $\frac{d\Omega_r^0}{dz} + \frac{\Omega_r^0 - \Omega_\phi^0}{z + z_0} = 0$ .

The inverse problem (11)-(20) is a nonlinear problem that can be solved on the basis of an iterative process, as in [12]. It should be noted that problems of reconstruction of thermomechanical characteristics and prestresses characterized by continuous functions were solved on the basis of such approach. In this paper we restore functions that have a point of discontinuity of the first kind on the interface between the coating and the cylinder on the basis of the algorithm developed in [12].

### 3. A scheme for solving the inverse problem

Consider a procedure for restoring the prestress  $\Omega_r^0(z)$  of a cylinder-coating system. The function  $\Omega_r^0(z)$ , similarly to [12,19], is represented in the form  $\Omega_r^0(z) = \beta \bar{g}(z)$ , where

$\beta = \frac{\max_{r \in [a, b+h]} \sigma_{rr}^0}{\mu_0}$  is the prestress level,  $\bar{g}(z)$  is the law of inhomogeneity distribution which must be restored.

The iterative process of restoring a piecewise continuous function  $\bar{g}(z)$  consists of two stages.

At the first stage, the initial approximation is determined in the form of a piecewise constant function based on minimization of the residual functional. In case of the additional information (19), the residual functional has the form:

$$J_1 = \int_{a_1}^{b_1} (f_1(\tau) - W(1, \tau)) d\tau, \quad (21)$$

and in case of the additional information (20), it will be represented as

$$J_2 = \int_{a_2}^{b_2} (f_2(\tau) - U(1, \tau)) d\tau. \quad (22)$$

Using the initial approximation  $\bar{g}^{(0)}(z)$ , the corresponding displacement  $U^{(0)}(z, \tau)$  and temperature  $W^{(0)}(z, \tau)$  are found from the solution of the direct problem (11)-(18).

In the second step, the corrections are found from the solution of the Fredholm integral equations of the first kind.

Thus, in the case of thermal loading ( $p^* = 0$ ,  $q^* = 1$ ), to find the correction  $\delta \bar{g}^{(n-1)}$ , it is necessary to solve the following equation:

$$\int_0^1 \delta \bar{g}^{(n-1)} R_1(z, \tau) dz = G(\tau), \quad \tau \in [a_1, b_1]. \quad (23)$$

In the case of mechanical loading ( $p^* = 1$ ,  $q^* = 0$ ), it is necessary to solve the equation:

$$\delta_0 \int_0^1 \delta \bar{g}^{(n-1)} R_2(z, \tau) dz = P(\tau), \quad \tau \in [a_2, b_2]. \quad (24)$$

Here, the kernels and the right-hand sides of equations (23), (24) have the form:

$$R_1(z, \tau) = \int_0^\tau \left( \frac{\partial^2 U^{(n-1)}}{\partial z \partial \tau_1}(z, \tau_1) + \frac{1}{z + z_0} \frac{\partial U^{(n-1)}}{\partial \tau_1}(z, \tau_1) \right) W^{(n-1)}(z, \tau - \tau_1) d\tau_1,$$

$$R_2(z, \tau) = \int_0^\tau \left( \frac{\partial U^{(n-1)}}{\partial z}(z, \tau_1) \left( \frac{\partial U^{(n-1)}}{\partial z}(z, \tau - \tau_1) - U^{(n-1)}(z, \tau - \tau_1) \right) + \frac{U^{(n-1)}(z, \tau_1)}{z + z_0} U^{(n-1)}(z, \tau - \tau_1) \right) d\tau_1$$

$$G(\tau) = \int_0^\tau q^*(\tau - \tau_1) (f_1(\tau_1) - W^{(n-1)}(1, \tau_1)) d\tau_1, \quad P(\tau) = \int_0^\tau p^*(\tau - \tau_1) (f_2(\tau_1) - U^{(n-1)}(1, \tau_1)) d\tau_1.$$

The equations (23), (24) for the determination of corrections represent the Fredholm equations of the first kind with completely continuous operators; for their inversion we used the A.N. Tikhonov regularization method [23].

After finding the corrections, we obtain the corrected functions  $\bar{g}^{(n)} = \bar{g}^{(n-1)}(z) + \delta \bar{g}^{(n-1)}(z)$  that give an approximate solution of the inverse problem (11)-(20) when the exit conditions are satisfied.

The following inequalities serve as the exit conditions:

$$J_1 \leq \eta, \quad (25)$$

$$J_2 \leq \eta. \quad (26)$$

#### 4. Results of computational experiments

In the present work, the internal radius of the cylinder was assumed to be equal  $a = 0.5$  cm; the external one was assumed  $b = 1$  cm, the coating thickness in various computational experiments varied within  $0.088 < h < 0.214$  (cm), which in dimensionless form correspond to the interval  $0.15 < h_1 < 0.3$ . A copper was used as a cylinder material and  $Al_2O_3$  as a coating, as it has a low thermal conductivity.

Following the above scheme, computational restoration experiments were conducted. The exit from the iterative process in all the experiments was done according to the conditions (25) or (26) for  $\eta = 10^{-4}$ .

As it is well known, the results of solving an inverse problem strongly depend on the choice of the most informative time periods for retrieving additional information. The most informative are the time intervals close to the reference point, in which the additional information changes most strongly. In thermoelastic processes, due to the energy dissipation, the temperature and displacements come to a steady state over time. Therefore, measurements of the additional information in this mode are not very informative. The time required to reach such a mode depends both on the loading method and on the thermomechanical characteristics of the coating and cylinder'

s materials. In the course of analyzing the additional information, the most informative time intervals were revealed both for the thermal and in the mechanical methods of loading. It was found out that the measurement of temperature is the most informative on the interval  $[a_1, b_1] = [0, 6]$  at 4 observation points inside it, and measurement of displacement on the interval  $[a_2, b_2] = [0, 1.2]$  is the most informative at 6 observation points inside it.

When solving inverse problems, it is important to investigate the sensitivity of the input information. Calculations showed that the changes in additional information, i.e. temperature and displacement, measured on the outer surface of the cylinder, are greatly influenced only by prestresses with  $\beta \geq 8 \cdot 10^{-4}$ . In subsequent calculations it was accepted  $\beta = 10^{-3}$ .

Following the above scheme, we carried out computational experiments to restore the function  $\bar{g}(z)$ . The figures below show the results of reconstruction of the dimensionless function; while the solid line depicts the graph of the original function, the dots show the restored one.

Firstly, computational experiments were performed with the thermal method of loading ( $p^* = 0, q^* = 1$ ). The results of the function  $\bar{g}(z)$  reconstruction for the  $Al_2O_3 / Cu$  system

turned out to be unsatisfactory – the reconstruction error at some points exceeded 20%. That was found out to be caused by the small thermomechanical coupling parameter of these materials –  $\delta_0 = 0.03$ . At the same time, with a large coupling parameter  $\delta_0 = 0.4$ , which is true only to a small number of materials, the reconstruction error did not exceed 7%, and no more than 14 iterations were required to fulfill the exit condition (25).

Figures 1-2 show the result of restoring  $\bar{g}(z)$  with the coupling parameter  $\delta_0 = 0.4$ .

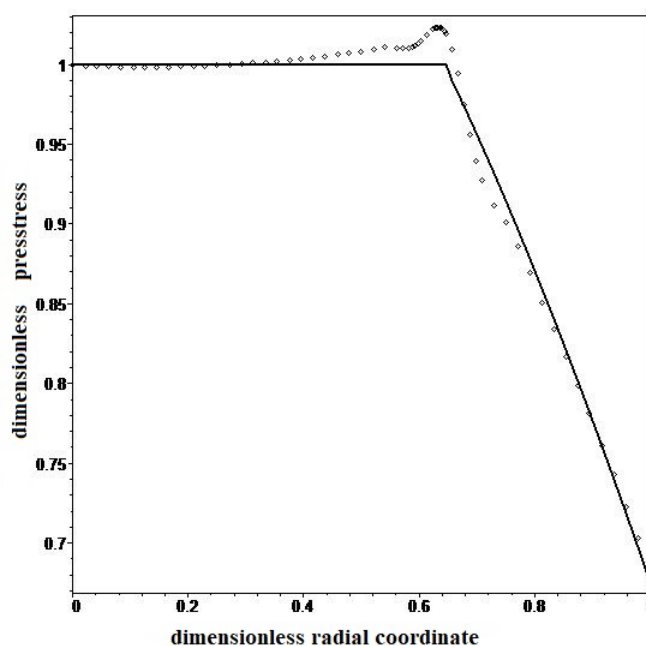
Figure 1 presents the result of reconstruction of  $\bar{g}_1(z) = \begin{cases} 1, & 0 \leq z \leq H; \\ 1.25 \cos(z), & H < z \leq 1 \end{cases}$  ( $H = 0.7$ ) in

the case when the outer surface of the cylinder is coated. Figure 2 shows the result of restoration of  $\bar{g}_2(z) = \begin{cases} z+1, & 0 \leq z \leq H; \\ 0.56 + e^{-z}, & H < z \leq 1 \end{cases}$  ( $H = 0.3$ ) in case when the inner surface of the cylinder is coated.

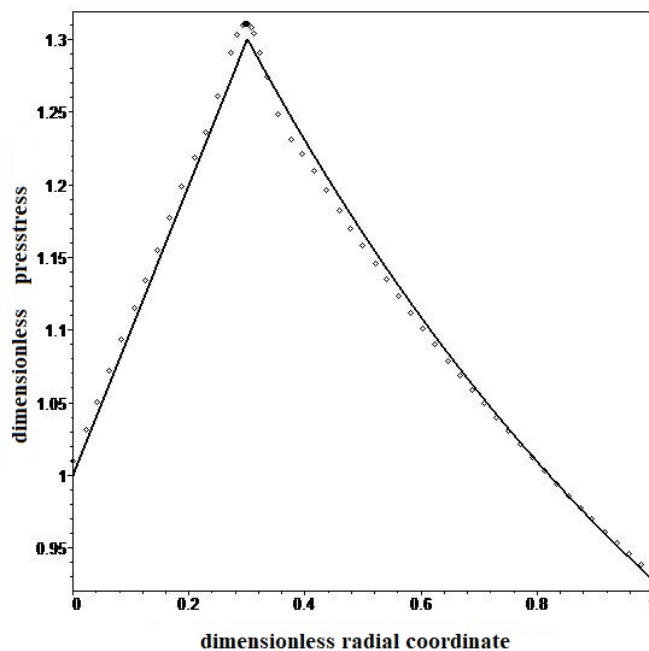
From Figs. 1,2, it can be seen that the largest reconstruction error (7%) occurred in the vicinity of the interface between the coating and the cylinder at  $H - \xi < z < H + \xi$  ( $\xi = 0.06$ ), which is caused by the features of the computational scheme.

Then experiments were carried out to reconstruct the function  $\bar{g}(z)$  under the mechanical loading method ( $p^* = 1$ ,  $q^* = 0$ ). In this case, the reconstruction error decreased significantly and did not exceed 9%. Also, no more than 12 iterations were required to fulfill the exit condition (26).

In Figs 3,4 we present the result of  $\bar{g}_3(z)$  recovery, which has the form of an increasing function  $0.15 + 3z^3$  for the coating, and a constant equal to 0.15 for the cylinder. The influence of the relative thickness  $h_1 = 0.3$  (Fig. 3),  $h_1 = 0.15$  (Fig. 4) of the coating on the results of reconstruction was investigated. From Figs. 3,4 it follows that the reconstruction error increases significantly with the decrease of  $h_1$ . For  $h_1 \leq 0.06$  the reconstruction of  $\bar{g}(z)$  becomes impossible due to a large error.

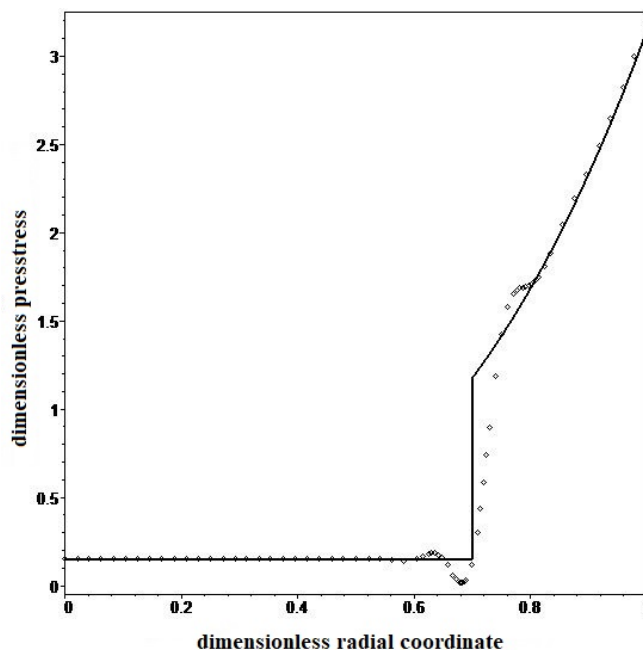


**Fig. 1.** The result of recovery of  $\bar{g}_1(z)$ ; thermal loading type



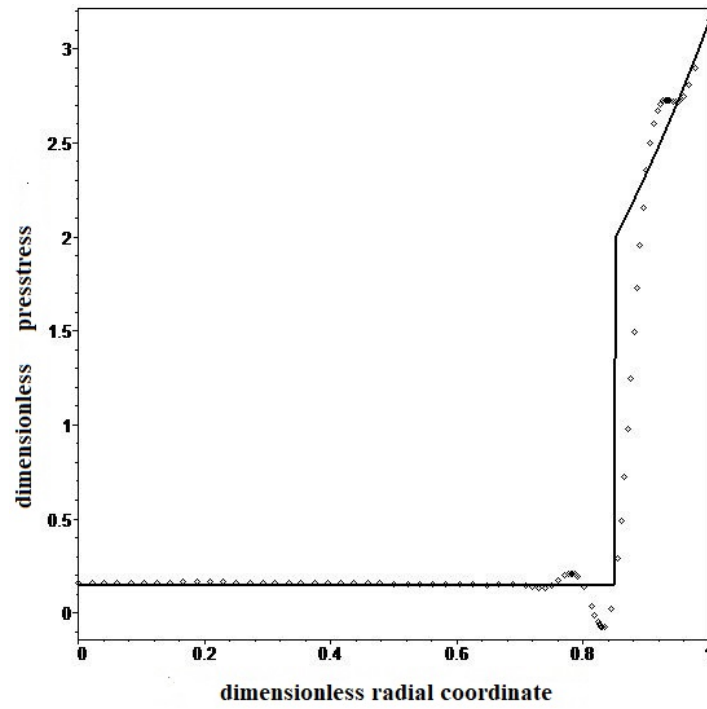
**Fig. 2.** The result of reconstruction of  $\bar{g}_2(z)$ ; thermal loading type

In Figs. 5,6 we present the results of  $\bar{g}(z)$  recovery for the case when the coating is deposited on the inner surface of the cylinder. Figure 5 shows the result of the reconstruction of the piecewise continuous functions  $\bar{g}_4(z) = \begin{cases} -0.5\ln(0.1+2z), & 0 \leq z \leq 0.3; \\ 0.1, & 0.3 < z \leq 1 \end{cases}$ . In Fig. 6 we show the result of the restoration of the piecewise constant function  $\bar{g}_5(z) = \begin{cases} 0.2, & 0 \leq z \leq 0.3; \\ 1, & 0.3 < z \leq 1 \end{cases}$ . The maximum reconstruction error is observed in the vicinity of the coating-cylinder interface  $0.24 < z < 0.36$  and does not exceed 6%.

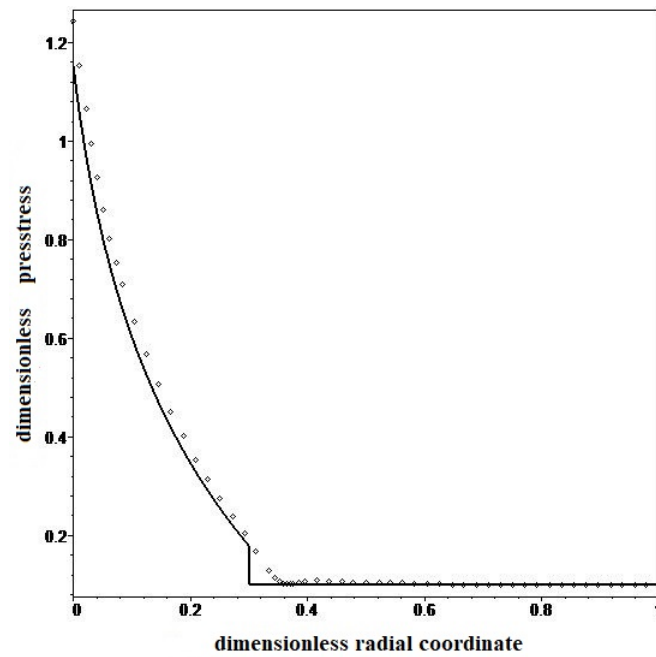


**Fig. 3.** The result of restoration of  $\bar{g}_3(z)$  with a coating thickness equal to  $h_1 = 0.3$ ; mechanical loading type

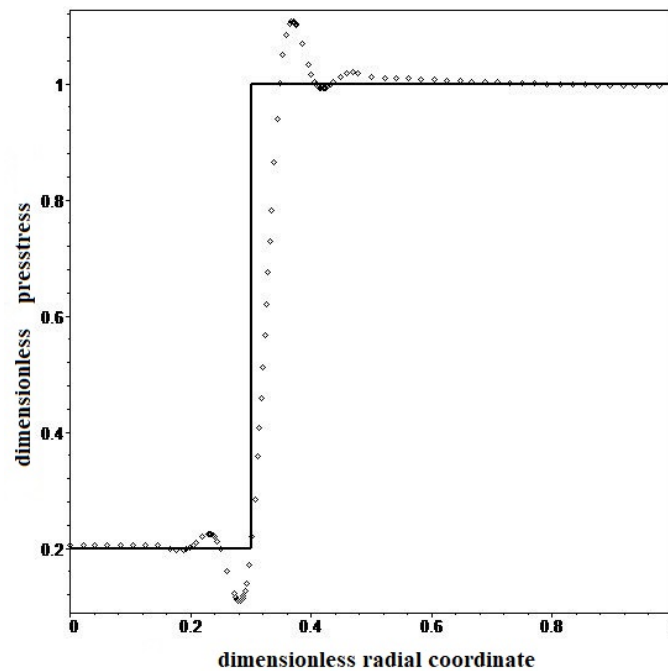




**Fig. 4.** The result of restoration of  $\bar{g}_3(z)$  with a coating thickness equal to  $h_1 = 0.15$ ; mechanical loading type



**Fig. 5.** The result of restoration of  $\bar{g}_4(z)$ ; mechanical loading type



**Fig. 6.** The result of restoration of  $\bar{g}_5(z)$ ; mechanical loading type

## 5. Conclusions

The method for determining inhomogeneous prestressed state of an infinitely long hollow thermoelastic coated cylinder is presented. The solution of the inverse problem is constructed on the basis of an iterative process, at each step of which the corrections are determined by solving the Fredholm integral equations of the first kind. The computational experiments of reconstruction of the inhomogeneous prestressed state of the coated cylinder are carried out. It is found out that the maximum reconstruction error occurs in the vicinity of the coating-cylinder connection. The reconstruction results for the thermal cylinder loading depend on the coupling parameter value. With a small coupling parameter, a large error arises in the prestressed state reconstruction. For any loading type, the results of reconstruction of the prestressed state depend on the coating thickness. With a decrease in the coating thickness, the reconstruction error considerably increases.

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