Materials Physics and Mechanics 44 (2020) 48-60 Received: September 29, 2019

STRAIN–HARDENING EFFECT ON CRITICAL STRAIN ASSESSMENT OF PIPE PLASTIC BENDING AT BUCKLING

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Abstract. In the current paper, the characteristic factor $\beta = [1 + 0.904(\sigma_b / \sigma_s) \cdot (1 - \sigma_s / \sigma_b)]$ is extracted from the analytical solution of the bending moment for a rectangular strain-hardening beam first. Thereafter, the characteristic factor β is transplanted into the previously proposed assessment of the critical buckling strain for rigid-perfectly plastic bending pipe by analogy method, thus an extended expression of the critical buckling strain for a pipe plastic bending including strain-hardening effect is developed, $\varepsilon_c = 0.19\frac{t}{r} \cdot (1 + \frac{t}{1.78r}) \cdot [0.096 + 0.904/(1-n)^2]$. Moreover, available test data is employed to check the suitability of the extended expression. The results show that the extended expression is reasonable to reveal the effect of Hollomon type strain-hardening behavior on the critical buckling strain of pipe plastic bending.

Keywords: plastic bending, buckling, critical strain, strain-hardening, pipe

List of symbols

M_{BR}: critical bending moment of a pipe

r: cross-sectional radius of a pipe

t: thickness of a pipe

E: Young's modulus of a pipe material

ν: Poisson's ratio of a pipe material

D_0: cross-sectional diameter of a pipe

M: bending moment of a pipe

κ: bending curvature of a pipe

$\kappa_1 = t/D_0^2$

ΔD: change in diameter of a pipe

σ_s: yielding strength of pipe material

P: internal pressure of a pipe

$P_0 = 2\sigma_0 t/D_0$

σ_{ac}: critical buckling stress of classical (elastic) solution for Donnell equation for circular shell

ε_{ac}: critical buckling strain of classical (elastic) solution for Donnell equation for circular shell

p_i: maximum internal design pressure

p_e: minimum external hydrostatic pressure

http://dx.doi.org/10.18720/MPM.4412020_6

E_s = 207GPa
F_y : effective specified minimum yield strength
σ_h: characteristic hoop stress
α_h : maximum yield to tensile ratio;
α_{gw} : girth weld factor
C : longitudinal curvature of the pipe during bending
C_c : critical longitudinal curvature C
γ : dimensionless parameter to reflect the ovalization of the cross section of the bending pipe
γ_c : critical value of γ
k : the intensity factor of metal
n : hardening exponent of metal
σ_b : ultimate strength
$R = \sigma_s / \sigma_b$: ratio of yield strength to ultimate strength
ε_u : uniform elongation
$M_0 = bh^2\sigma_s / 4$
b : width of a beam
h : height of the beam
M_p : bending moment of the rectangular strain- hardening beam
$\beta = [1 + 0.904(\sigma_b / \sigma_s) \cdot (1 - \sigma_s / \sigma_b)]$: factor characterizing strain -hardening effect on bending moment of a beam as compared to the ideal plastic one

1. Introduction

Buckling failure is a common phenomenon in pipe production and service conditions involving bending. This kind of failure is fatal due to the loss of load capacity. However, the assessment of critical strain of plastic bending pipe at buckling is still insufficient.

In general, buckling initiation can be used to define the failure of pipeline. During pipe bending, the cross - section shape of pipe changes from a round to an oval one gradually, and the bending load or moment increases at the beginning stage; however, the bending load or moment could no longer increase or even suddenly decreases when the pipe bending exceeds certain degree, which is defined as buckling failure of the bending pipe.

Early in 1927 [1], Brazier proposed an elastic solution to correlate the bending moment and critical state at the limit point. His work showed that when an initial straight pipe is bent uniformly, the tension and compression in longitudinal direction in the pipe resist the applied bending moment, at the same time the cross section of the pipe tends to ovalize or flatten elastically, which in turn reduces the flexural stiffness of the member as the bending curvature increasing. He showed that the flexural stiffness has a maximum value which is defined as the critical instable moment, $M_{BR} = 0.987Et^2r/(1-\nu^2)^{1/2}$, where r and t are the cross-sectional radius and thickness of a pipe, E is its Young's modulus and ν is the Poisson's ratio, respectively.

Similar to Brazier, Chwalla studied the flattening of pipe cross-section due to pipe bending in 1933 [2]. It diminishes the bending resistance of pipes progressively due to the production of a certain curvature, and the bending moment for an oval section is smaller than that of a round one. This nonlinear effect leads to instability [3]. The theoretical flexural limit moment of cylindrical shells in considering section flattening (ovalization) is with a high critical value [4]. Seide and Weingarten studied the bifurcation of bending cylindrical shell, a linear pre-buckling state and a Ritz-type bifurcation solution were assumed [5]. Their results indicated that the buckling stress of a finite length simply supported cylindrical shell bending is similar to the pure compression shell. Reddy observed the wave - like ripples on the compression side of bending pipes ahead of collapse with steel and aluminum specimens [6]. Kyriakides and Ju studied the instability of cylindrical aluminum shells in pure bending [7,8], the ratio of diameter to thickness of the cylindrical aluminum shells was 19.5 ~ 60.5, and the length to diameter ratio was 18.1 ~ 30.1; the appearance of wave - like ripples was observed on the compression side of the bent pipes once more before collapse. Libai and Bert [9], Tatting et al. [10], Stephens and Starnes [11] also investigated the dependence of bifurcation instability on Brazier's flattening effect.

In these decades, massive work has been done to study the stability of circular pipe bending due to the needs of safety assessment of pipeline and behavior of nano-pipe.

As to the buried pipeline, it might suffer from complex and changeable environmental conditions, which could induce deformation and strain or even lead to pipe failure, the critical value of strain for a bending pipe at buckling might be rationally taken as a parameter in pipeline design in nowadays [12-15]. However, the assessment of critical buckling strain is still on the way, though some formulae have been proposed [13-15]. Till now some formulae are either short of physical meaning or unreasonable. The prediction of classical analytical solution is far from experimental results due to its elastic feature though it is with clear physical meaning [13,14], other regressive or fitted formulae are short of physical meaning or unreasonable [15], see the analysis in next section in detail. This situation indicates that the assessment of the critical strain of plastic bending pipe at buckling is still an important and open problem.

In 2006, Khurram Wadee et al. proposed a variational model to formulate the deformation localization of bending round thin-walled pipes in elastic status [16]. The results are compared to a number of case studies including nano-pipe, but it is in elastic case. Philippe Le Grognec and Anh Le van studied the theoretical aspects of elasto-plastic buckling of plates and cylinders under uniform compression in 2009 [17]. 3-D plastic bifurcation theory assuming the J_2 plastic flow with von Mises yield criterion and a linear isotropic hardening are involved in the analysis. The critical loads, the buckling modes and the initial slope of the bifurcated branch are obtained for the rectangular plates under uni-axial or biaxial compression (-tension) and cylinders under axial compression. Poonaya et al. analyzed the plastic collapse of thin-walled round pipe bending in 2009 [18]. The oblique hinge lines along the longitudinal pipe within the length of the plastic deformed zone were introduced in the 3-D geometrical collapse mechanism analysis. The internal energy dissipation rates, the inextensional deformation and perfect plastic material behavior were assumed in the derivations. Gianluca Ranzi and Angelo Luongo proposed an approach to illustrate the cross-section change in the context of the generalized beam theory (GBT) in 2011 [19]. The semi-variational method was employed to formulate the problem.

In 2012, Christo Michael et al. studied the effects of ovality and variable wall thickness on collapse loads of pipe bending in-plane by finite element limit analyses with elastic-perfect plastic material model [20]. It showed that ovality affects collapse load more significantly than thinning in the pipe bending process. They proposed a regressive mathematical equation to include the oval effect for their finite element analysis results.

Currently, Gayan Rathnaweera et al. studied the performance of aluminum / Terocore hybrid structures in quasi-static three-point bending by experiment and finite element analysis [21]. They observed two failure modes in their study, i.e., the top surface failure (compression) from structures made of AA7075 T6 and the bottom surface failure (tensile) from structures with higher percentage volume of foam.

Wrinkling is an accompanying phenomenon in pipe bending [22,23]. Guarracino pointed out that the growth of ripples on the compressed side of the pipe has a softening effect on the overall response of the bent pipe [22]. Lamam et al. studied the inelastic wrinkling and collapse of stainless steel (SS) 321 pipes with a D_0/t about 52 under combined bending and internal pressure experimentally [23], where D_0 and t are the diameter and thickness of the pipe. Their results indicated that the moment (M) – bending curvature (κ) response $M-\kappa$ can be characterized by an initial linear elastic regime during which the pipe underwent a small amount of ovalization. Then the elastic regime eases into a smooth moment knee caused by the onset of inelastic action. The knee is followed an essentially linear "hardening" regime, which continues to a relatively high curvature. The increase of the change rate in diameter ΔD indicates a growth of ovalization accompanied by a net plastic circumferential expansion of the pipe. Meanwhile, pockets of small amplitude wrinkles were observed on the compressed side of the pipe soon after the moment knee. Under zero pressure condition, the wrinkles were short lived, which developed on the compressed side of the pipe and soon thereafter one of them localized, it results in a sharp local inward kink. This is a sudden event that is associated with the sharp loss of rigidity at the end of the $M-\kappa$ response, which corresponds to the appearance of maximum moment. This phenomenon implies that the limit carrying capacity of the pipe could be characterized by $M-\kappa$ response at the buckling though small amplitude wrinkles were observed on the compressed side of the pipe [23].

While, in the presence of internal pressure the moment–curvature response $M-\kappa$ after the knee becomes stiffer and follows an early linear path that is higher than that of the pure bending test [23]. The bulging led to the drop in moment observed at the termination of the M–κ response indicating that the structure started collapsing. The collapse point of the pipe is defined at the curvature corresponding to the maximum moment. As the pressure increases the response after the knee maintains approximately the same slope but gradually moves down with pressure. The zero pressure case is seen to have a different post-yield slope than the rest due to the significance of the pipe cross section ovalization [23].

In summary, the limit carrying capacity of a bending pipe could be characterized by $M-\kappa$ response at the buckling regardless of the presence or absence of internal pressure.

In 2015, Ji and Zheng et al developed an analytical approach for assessing critical strain of plastic bending pipeline at buckling with the cross section ovalization and rigid-perfect plastic material models [24]. The available test data from Ref. [13,14] was employed to check the validity of the assessment, good agreement was obtained. However, strain-hardening effect of pipeline material was not included in such approach, which is a shortcoming of the work, see the analysis in next section in detail.

In this paper, an expression including strain-hardening effect on assessment of critical strain for plastic bending pipe at buckling is developed; the cross section ovalization model and the Hollomon type strain-hardening behavior of the pipe material are employed.

2. Typical approaches for assessing critical buckling strain of bending pipe

Elastic solution (classical solution). The classical (elastic) solution for Donnell equation of circular shell is [25,26],

$$\sigma_{cr} = \frac{E}{\sqrt{3(1-\nu^2)}}\frac{t}{r}, \tag{1}$$

in which, r, and t are the radius, and thickness of the circular shell, respectively; E is elastic modulus of the circular shell; σ is the uniform stress on the circular shell along its axial direction.

For common metallic material, such as steel, its Poisson's ratio is $\nu = 0.3$, while for aluminium and copper, their Poisson's ratio is $\nu = 0.34$ [27], Eq. (1) reduces,

$$\sigma_{cr} = 0.605E\frac{t}{r}, \text{ for steel;} \tag{2}$$

$$\sigma_{cr} = 0.614E\frac{t}{r}, \text{ for aluminium and copper.} \tag{2′}$$

The corresponding critical strain is

$$\varepsilon_{cr} = 0.605\frac{t}{r}, \text{ for steel;} \tag{3}$$

$$\varepsilon_{cr} = 0.614\frac{t}{r}, \text{ for aluminium and copper.} \tag{3′}$$

For a bending circular shell, it gives the same results as Eqs. (3) and (3′).

The experimental data shows that Eqs. (3) and (3′) give more significant overestimations than the test results [24].

In general, the value of radius-thickness ratio r/t for a practical pipeline, is about 30 ~ 50, which results in a higher $\varepsilon_{cr} = 1.21\% \sim 2.02\%$ from Eq. (3), it exceeds the usual elastic limit strain of the pipeline steel so much, says, about 0.2% [24]. This phenomenon indicates that the prediction of Eq. (3) exceeds its actual application scope for a practical pipeline seriously, so it isn't valid for practical pipeline.

Other expressions. Some other empirical approaches have been proposed to predict the critical strain of bending pipe at buckling, such as [24],

Sherman (1976): $\varepsilon_c = 16(t/D)^2$;

Stephens (1991): $\varepsilon_c = 2.42(t/D)^{1.59}$.

However, the comparison of their results with experiments indicates the non-reasonability of these formulae [24].

Available codes in industry.

(1) CSAZ662-07

CSAZ662-07 code C.C6.3.3.3 gives an assessment of local critical buckling strain including primary loads, secondary loads, or both [24],

$$\varepsilon_c = 0.5\frac{t}{D} - 0.0025 + 3000\left(\frac{(p_i - p_e)D}{2tE_s}\right), \quad \frac{(p_i - p_e)D}{2tF_y} \le 0.4, \quad p_i \ge p_e, \tag{4}$$

$$\varepsilon_c = 0.5\frac{t}{D} - 0.0025 + 3000\left(\frac{0.4F_y}{E_s}\right)^2, \quad \frac{(p_i - p_e)D}{2tF_y} \ge 0.4, \tag{5}$$

in which, ε_c is the ultimate compressive strain capacity of the pipe; t is the wall thickness of the pipe, mm; D is outside diameter of pipe, mm, p_i is maximum internal design pressure, MPa; p_e is minimum external hydrostatic pressure, MPa; E_s = 207GPa, F_y = effective specified minimum yield strength, MPa.

(2) DNV-OS-F101

DNV-OS-F101 clause 507 supplies the characteristic compressive bending strain capacity, $\varepsilon_{M,c}$ as [24],

$$\varepsilon_{M,c} = 0.78\left(\frac{t}{D} - 0.01\right)\left(1 + 5\frac{\sigma_h}{\sigma_s}\right)R^{-1.5}\alpha_{gw}, \tag{6}$$

$$\sigma_h = P\cdot\left(\frac{D-t}{2t}\right), \tag{7}$$

in which, t and D represent the thickness and diameter of the pipe; σ_h is the characteristic hoop stress; σ_s is the yield strength of material; R is ratio of yielding strength to ultimate strength; α_{gw} girth weld factor (=1 for specimen with no weld); P is the internal pressure.

However, studies show that the accuracy of predictions of both two codes is also limited [24].

3. Cross section ovalization model for assessing critical buckling strain of pipe plastic bending

Ji and Zheng et al proposed an analytical assessment for critical strain of pipe plastic bending at buckling, which considered the cross section ovalization during bending and rigid-perfect plastic material models [24].

In the derivations, the energy rates of cross section ovalization and the oval pipe bending were established, which were combined to derive the macro bending moment of pipe. Furthermore, the maximum of macro bending moment of the pipe at buckling is yielded, and the assessment for critical buckling strain of the plastic bending pipe is then obtained.

The longitudinal curvature of the pipe during bending is expressed by C, which could be used to characterize the instant status of the bending pipe [24].

For a thin-wall circular pipe [24], $t << r$, if a standard ellipse is employed to characterize its ovalized cross-section shape due to bending, a dimensionless parameter γ could be introduced to reflect the ovalization of the cross section, thus the lengths of the longer and shorter half axis of the ellipse could be written as, $a = r(1+\gamma)$ and $b = r(1-\gamma)$, respectively. The dimensionless parameter γ depends on the longitudinal curvature C of the bending pipe, which could be seen in [24] for details. Besides, the material of the pipe behaves as a rigid - perfect plastic one.

Instability of the bending pipe occurs when the curve of bending moment M with respect to γ reaches to the peak. Thus the critical value of γ_c is derived, it obtains $\gamma_c = 0.11$ [24].

Furthermore, it derives the critical longitudinal curvature C_c of the bending pipe by completing the complicated integral calculations in [24],

$$C_c = 0.2131\frac{t}{r^2} = 0.8524\frac{t}{D^2}. \tag{8}$$

Correspondingly, the apparent strain of the outer-fiber-line for the bending pipe at buckling is derived [24]:

$$\varepsilon_c = \left[r(1-\gamma) + \frac{t}{2} \right] \cdot C_c = r(0.89 + \frac{t}{2r}) \cdot 0.2131\frac{t}{r^2} = 0.19\frac{t}{r} \cdot (1 + \frac{t}{1.78r}). \tag{9}$$

Eq. (9) is the expression of apparent strain of the outer-fiber-line of the bending pipe at buckling geometrically. In the derivation, a rigid-perfect plastic material model and cross section ovalization are involved.

The factor 0.19 in Eq. (9) is close to the most experimental results [24].

In Ref. [24], the available test data from Ref. [13,14] was employed to check the validity of the predictions of cross section ovalization model for plastic bending pipe, good agreement was obtained.

Obviously, strain-hardening effect of pipeline material was not included in the above proposed model, which is the main shortcoming of the work.

4. Extension of critical buckling strain assessment to include strain-hardening effect on plastic bending pipe

Function of strain-hardening on critical buckling strain of plastic bending pipe. Strain hardening or deformation strengthening ability of metallic material is one of the most important properties of metals, and the most commonly used relationship describing this ability is the Hollomon formula [28],

$$\sigma = k\varepsilon^n, \quad (10)$$

in which, σ and ε present the true stress and true strain, respectively; k is the intensity factor; n is the hardening exponent of metal.

Okatsu et al. conducted buckling experiments for small scale linepipes of Hollomon type material with different strain-hardening exponent n [29]. Their results showed that higher strain-hardening exponent n obviously corresponds to higher critical buckling strain ε_c for pipes with different diameter to thickness ratio, D/t, see Fig. 1 [29]. Their results also indicated that high deformability linepipe ″JFE - HIPER″ is with superior resistance to buckling, which is developed by multiphase micro - structural control from X52 to X100 grades. The stress-strain curves in the longitudinal direction are round-house type for all pipes, and high n - value (low Y/T ratio).

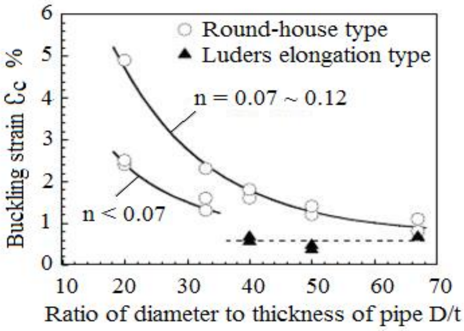

Fig. 1. ε_c *vs* D/t for small scale linepipes [29]

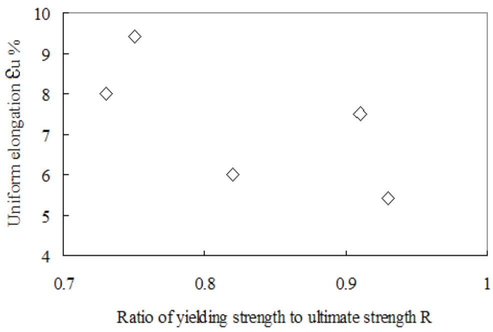

Fig. 2. ε_u *vs* R for X70, X80, and X90 steels [30]

With the development of pipeline steel from X65 to X100, the ratio of yielding strength to ultimate strength, $R = \sigma_s / \sigma_b$, increases from 0.80 to 0.90 ~ 0.93 or higher. Excessive ratio of yielding strength to ultimate strength leads to a decrease in the strain - hardening property of the steel pipe, which is harmful to the safe service of the pipe structure in a large displacement environment. Meanwhile, the increase of the yielding strength to ultimate strength ratio results in a decrease in the uniform elongation, ε_u.

Ji L.K. et al. studied the tensile behaviors of X70, X80, and X90 steels. Tensile specimens were cut longitudinally [30]. The rectangular tensile specimens with 50 - mm gage length, 38.1 - mm width, and full wall thickness were used to carry out their tensile tests on SHT 4106 machine according to ASTM A370. The tensile performance of the five specimens is shown in Fig. 2. It can be seen from Fig. 2 that the tendency of uniform elongation ε_u for the line pipe steels decreases with the increasing of the yielding strength to ultimate strength ratio, R. This phenomenon has been reported for various structural steels as well [30].

In addition, the deformability of the steel pipe decreases as the structural size D/t increases. Due to the use of high-strength pipeline steel, the wall of the steel pipe is thinned, which further limits the ultimate plastic deformation ability of the pipeline. As to the two-phase structure steel, its features of low yielding strength to ultimate strength ratio, high uniform elongation and strain strengthening exponent ensure the safety of pipeline structures in service, especially under strain-controlled load condition.

In fact, strain-hardening exponent is an important material parameter of pipe, which reflects the strengthening behavior of material during deformation. The value of strain-hardening exponent equals to the maximum uniform strain of material in principle [31], which represents the ability of the material to perform strain hardening so as to make deformation uniformly before necking. Hu et al proposed a relationship to correlate the strain-hardening exponent n and the ratio of yielding strength to ultimate strength for Hollomon type material [32],

$$n = 1 - \left(\sigma_s / \sigma_b\right)^{1/2}. \tag{11}$$

Hu et al. also collected many experimental data to check the reasonability of Eq. (11) [32], which are shown in Fig. 3. It can be seen from Fig. 3 that good agreement is obtained.

These results indicate that the strain-hardening exponent n is the rate of the increase of strength or hardness of Hollomon type material during deformation process. It reveals the internal relationship between strength and plasticity. Higher σ_s / σ_b corresponds to lower n.

Jaske C.E. also studied the correlation of strain - hardening exponent n and σ_s / σ_b, the variation of strain-hardening exponent n *vs* σ_s / σ_b for typical pipeline steels with Hollomon type is shown in Fig. 4 [33].

From Figure 3 and Figure 4, the result of higher σ_s / σ_b corresponds to lower critical buckling strain ε_c reasonably.

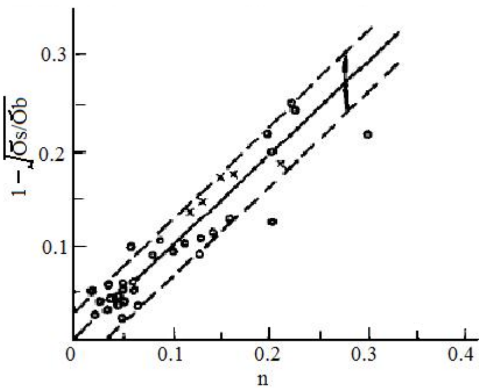

Fig. 3. Variation of σ_s / σ_b with n [32]

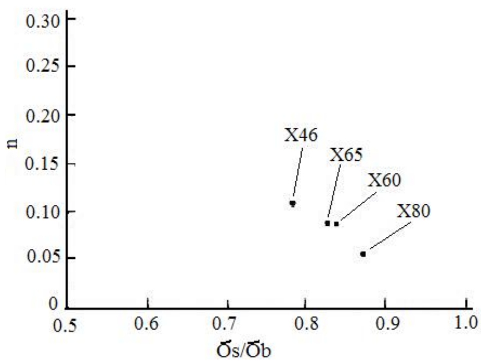

Fig. 4. n *vs* σ_s / σ_b for typical pipeline steels [33]

Extraction of the characteristic factor β from bending moment of a rectangular strain-hardening beam to specifically characterize the strain-hardening effect on plastic bending beam. The effect of hardening exponent on pipe bending was once studied by Murata et al. with both FEM (Finite Element Method) and experiment [34]. In their FEM, the tube is axial symmetry, and therefore a quarter part of tube was analyzed, the strain-hardening behavior of pipe material is Hollomon type. The hardening exponent changed with n = 0.1, 0.3, 0.5 and 1.0. A commercial finite element code ELFEN was employed to conduct the 3D explicit analysis for the press bending process of circular, and shell element was employed. ELFEN is a widely used code for the analysis of metal forming developed by Rockfield Software Limited, Swansea [34]. The results showed that the increasing of hardening exponent significantly resists the flatness of pipe cross section during bending due to its action for deformation uniformity.

Yang investigated the strain-hardening effect on bending moment of a rectangular strain-hardening beam [35], an analytical expression was developed. The strain-hardening behavior of the material is Hollomon type.

Yang′s analytical result showed that the bending moment of the rectangular strain-hardening beam could be written as [35]:

$$M_p \approx bh^2\sigma_s \cdot [1 + 0.904(\sigma_b / \sigma_s) \cdot (1 - \sigma_s / \sigma_b)] / 4 = M_0 \cdot [1 + 0.904(\sigma_b / \sigma_s) \cdot (1 - \sigma_s / \sigma_b)], \tag{12}$$

in which $M_0 \approx bh^2\sigma_s / 4$ represents the bending moment of the beam with ideal plasticity property and yielding strength σ_s; b and h represent the width and the height of the beam, respectively.

Obviously, Eq. (12) indicates that a specifically characteristic factor β can be extracted from the expression, which reflects the effect of strain-hardening behaviour on bending moment of the beam as compared to the ideal plastic beam, i.e.

$$\beta = [1 + 0.904(\sigma_b / \sigma_s) \cdot (1 - \sigma_s / \sigma_b)]. \quad (13)$$

Extension of critical buckling strain assessment to include strain-hardening effect for plastic bending pipe at buckling. On the other hand, recalling to the two cases of elastic buckling condition, i.e., the 1st one is the classical (elastic) solutions for critical moment and strain of circular shell at bending buckling [36,37],

$$M'_{cc} = \pi E t^2 r / [3(1-\nu^2)]^{1/2} = 0.577\pi E t^2 r / (1-\nu^2)^{1/2}, \quad (14)$$

$$\varepsilon'_{cc} = (r/R) / [3(1-\nu^2)]^{1/2} = 0.577(r/R) / (1-\nu^2)^{1/2}, \quad (15)$$

in which the cross-section shape of the elastic circular shell keeps completely round.

While the 2nd one is the Brazier solution, which involves the cross-section shape of the elastic circular shell changing into an ellipse during bending, in this case the critical moment and strain of circular shell at bending buckling are,

$$M''_{bc} = 0.314\pi E t^2 r / (1-\nu^2)^{1/2}, \quad (16)$$

$$\varepsilon''_{bc} = 0.366(r/R) / (1-\nu^2)^{1/2}. \quad (17)$$

Compare Eqs. (14) and (15) with Eqs. (16) and (17), it yields two significant conclusions: 1) the numerical factors in the classical (elastic) solutions for critical moment and strain are identically all bigger than those in Brazier solutions, which is due to the consideration of out-round of the pipe cross-section shape in Brazier mode. This phenomenon clearly indicates that pipe with better roundness of cross-section shape will have bigger critical buckling moment and strain at the same time during bending; 2) the numerical factors are almost the same for the critical moment and strain at buckling.

Meanwhile, the previous section states that the action of strain hardening is to make deformation uniformly before necking, and it may retain the roundness of cross-section shape of pipe during bending. Therefore, as to the Hollomon type strain-hardening material and bending moment problem, the specifically characteristic factor β extracted from bending moment of a rectangular strain- hardening beam could be employed and transplanted into the expression for assessing the critical bending moment and strain of bending pipe at buckling to reflect effect of strain-hardening effect due to the similarity of the problem.

On the other hand, recalling to the three cases of elastically bending buckling condition of pipe, i.e., the 1st one that is the classical (elastic) solution of the bending pipe retaining the cross-section shape perfect round, the Brazier′s solution which involves the cross-section shape changing into an elliptical one during of pipe bending, and Li′s solution considering the cross section ovalization of pipe due to elastic bending [24], the corresponding critical moment and strain of above models for bending buckling are shown in Table 1.

From Table 1, it results in a significant consequence that the numerical factors in the classical (elastic) solutions for critical moment and strain are all greater than those in Brazier solution and Li′s solution, which is due to the consideration of out-round of the pipe cross-section shape in Brazier and Li models. This phenomenon obviously reveals that pipe with better roundness of cross-section shape exhibits greater critical buckling moment and strain at the same time during bending, and ratio of the numerical factor ε_c to M_c are all not far from 1.0 in the above three examples.

Table 1. Solutions of critical moments and strains for initially circular pipe bending at buckling corresponding 3 elastic models [24]

Elastic model	Classical (elastic) solution	Brazier solution	Li's solution
M_c	$M_c = 0.577\pi Et^2 r/(1-\nu^2)^{1/2}$	$M_c = 0.314\pi Et^2 r/(1-\nu^2)^{1/2}$	$M_c = 0.388\pi Et^2 r/(1-\nu^2)^{1/2}$
ε_c	$\varepsilon_c = 0.577(r/R)/(1-\nu^2)^{1/2}$	$\varepsilon_c = 0.366(r/R)/(1-\nu^2)^{1/2}$	$\varepsilon_c = 0.461(r/R)/(1-\nu^2)^{1/2}$
Ratio of numerical factor ε_c to M_c	1.000	1.166	1.188
Shape of pipe cross-section during bending	Perfect round	Ellipse	Ovalization

Meanwhile, the previous section indicates that the function of strain hardening is to ensure deformation uniformly before necking, and it may retain the roundness of cross-section shape of pipe during bending. Therefore, as to the Hollomon type strain-hardening material and the bending moment problem, the characteristic factor β separated from bending moment of a rectangular strain-hardening beam could be transplanted into the representation for assessing the critical buckling moment and strain of a bending tube to reveal the effect of strain-hardening effect due to the similarity of the problem.

Referring that Eq. (9) is the estimation of the critical buckling strain of the outer-fiber-line of a rigi -perfectly plastic bending tube due to cross section ovalization, which is a complete geometric one without considering the action of strain-hardening.

Therefore, Eq. (9) could be extended to contain the action of deformation uniformity of strain-hardening effect by the specifically characteristic factor β reasonably, thus it yields

$$\varepsilon_c = 0.19\frac{t}{r}\cdot(1+\frac{t}{1.78r})\cdot\beta = 0.19\frac{t}{r}\cdot(1+\frac{t}{1.78r})\cdot[1+0.904(\sigma_b/\sigma_s)\cdot(1-\sigma_s/\sigma_b)] = \\ = 0.19\frac{t}{r}\cdot(1+\frac{t}{1.78r})\cdot(0.096+0.904\sigma_b/\sigma_s). \qquad (18)$$

In the light of Eq. (10), Eq. (18) becomes,

$$\varepsilon_c = 0.19\frac{t}{r}\cdot(1+\frac{t}{1.78r})\cdot[0.096+0.904/(1-n)^2]. \qquad (19)$$

Eq. (19) is the extended expression of the critical buckling strain assessment containing the strain - hardening of plastic bending pipe.

Ishikawa N. et al. collected the variation of ε_c with respect to n for some pipes with different D/t = 40 to 44 and 62 [38]. These data is redrawn in Fig. 5 to check the validity of Eq. (19), and it takes the average value 42 for the D/t = 40 to 44. Figure 5 represents the reasonability of Eq. (19) obviously.

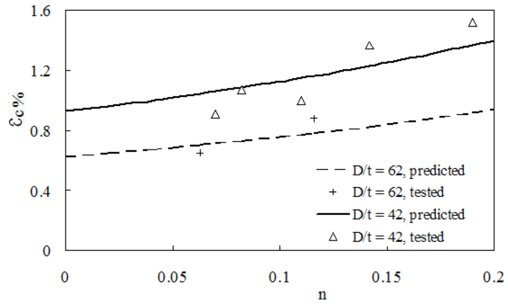

Fig. 5. Variation of ε_c *vs* n for Hollomon type pipes different D/t

5. Conclusion

By extracting the characteristic factor $\beta = \left[1 + 0.904\sigma_b/\sigma_s\left(1 - \sigma_s/\sigma_b\right)\right]$ from the analytical solution of the bending moment for a rectangular strain - hardening beam, the Hollomon type strain - hardening effect on plastic bending moment of beam can be characterized specifically. Furthermore, by analogy method the critical strain assessment for strain - hardening pipeline plastic bending at buckling is developed. The result shows the reasonability of the developed expression for the critical buckling strain assessment of pipe with Hollomon type strain-hardening behavior.

Acknowledgements. *No external funding was received for this study.*

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