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INTERACTION DUE TO EXPANDING SURFACE LOADS IN THERMOPOROELASTIC MEDIUM

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Abstract. The present investigation is concerned with interaction due to expanding surface loads in thermoporoelastic medium whose surface is subject to loads that suddenly emanate from a point on the surface and expand radially at constant rate. The cases of loads shaped as a ring and disc are considered in detail to show the utility of the approach. These loads are chosen so that they exert a constant force on the surface as they expand. Laplace and Hankel transform technique is used to solve the problem. The expressions for displacement components, stress components, pore pressure and temperature change are obtained in the transformed domain. To obtain the resulting quantities in the physical domain, a numerical inversion technique is applied. Effect of porosity is shown on the resulting quantities. A particular case of interest is also deduced from the present investigation.

1. Introduction

Most of the modern engineering structures are generally made up of multiface porous continuum. The classical theory, which represents a fluid saturated porous medium as a single face material, is inadequate to represent the mechanical behaviour of such materials especially when the pores are filled with liquid. In this context, the solid and liquid phases have different motions. Due to these different motions, the different material properties and the complicated geometry of pore structures, the mechanical behaviour of fluid saturated porous medium is very complex and difficult. Researchers have tried to overcome this difficulty. For more details and for the more historical review on the subject of the multiphase continuum mechanics, the work of de Boer and Ehler [1] or a recently published monograph of de Boer [2] can be studied.

Biot [3] proposed a general theory of three dimensional deformations of a fluid saturated porous solid. Biot theory is based on the assumption of compressible constituents and till recently some of his results have been taken as standard references and basis for subsequent analysis in acoustics, geophysics and other such fields. Bowen [4], de Boer and Ehlers [5, 6] developed and used another interesting theory in which all the constituents of a porous medium are assumed to be in compressible.

Kumar and Hundal [7] discussed the wave propagation in a fluid saturated incompressible porous medium. They also discussed the one dimensional wave propagation in a non-homogeneous fluid saturated incompressible porous medium [8]. They also discussed

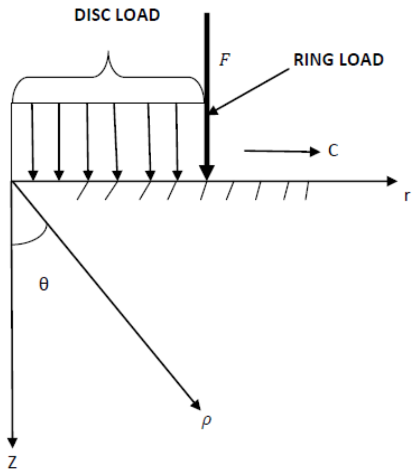

Fig. 1. Expanding ring and disk loads.

3. Formulation and solution of the problem

Since we are considering two-dimensional problem, so we assume the components of displacement vector $\vec{u}$ of the form

$$\vec{u} = (u_r, 0, u_z). \tag{5}$$

We define the non-dimensional quantities

$$r' = \frac{\omega^*}{c_1} r,\ z' = \frac{\omega^*}{c_1} z,\ u_r' = \frac{\omega^*}{c_1} u_r,\ u_z' = \frac{\omega^*}{c_1} u_z, p' = \frac{p}{\beta T_0}, c_1^2 = \frac{\lambda + 2\mu}{\rho}, t' = \omega^* t, T' = \frac{T}{T_0}$$

$$\sigma_{zz}' = \frac{\sigma_{zz}}{\beta T_0},\ \sigma_{zr}' = \frac{\sigma_{zr}}{\beta T_0}, \tag{6}$$

where ω^* is the constant having the dimensions of frequency.

Using dimensionless quantities defined by (6), in equations (1)-(3) and with the aid of (5) (after suppressing the prime), we obtain

$$\frac{\partial e}{\partial r} + a_1 (\nabla^2 - \frac{1}{r^2}) u_r - a_2 \frac{\partial p}{\partial r} - a_3 \frac{\partial T}{\partial r} = a_4 \frac{\partial^2 u_r}{\partial t^2}, \tag{7}$$

$$\frac{\partial e}{\partial z} + a_1 \nabla^2 u_z - a_2 \frac{\partial p}{\partial z} - a_3 \frac{\partial T}{\partial z} = a_4 \frac{\partial^2 u_z}{\partial t^2}, \tag{8}$$

$$b_1 \nabla^2 p - b_2 \frac{\partial p}{\partial t} - b_3 \frac{\partial T}{\partial t} - \frac{\partial e}{\partial t} = 0, \tag{9}$$

$$b_4 \nabla^2 T - b_5 \frac{\partial T}{\partial t} + b_6 \frac{\partial p}{\partial t} - \frac{\partial e}{\partial t} = 0, \tag{10}$$

where $a_1 = \frac{\mu}{\lambda+\mu},\ a_2 = \frac{\alpha \beta T_0}{\lambda+\mu},\ a_3 = \frac{\beta T_0}{\lambda+\mu},\ a_4 = \frac{\rho c_1^2}{\lambda+\mu},\ b_1 = \frac{k \omega^* \beta T_0}{\gamma_w \alpha c_1^2},\ b_2 = \frac{\alpha_p \beta T_0}{\alpha}, b_3 = \frac{Y T_0}{\alpha},$
$b_4 = \frac{K \omega^*}{\beta c_1^2}\ ,\ b_5 = \frac{Z T_0}{\beta}, b_6 = Y T_0$, and

$$e = \frac{\partial u_r}{\partial r} + \frac{1}{r} u_r + \frac{\partial u_z}{\partial z}, \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}. \tag{11}$$

To simplify the problem, we introduce the potential functions as:

$$u_r = \frac{\partial \Phi}{\partial r} - \frac{\partial \Psi}{\partial z}, u_z = \frac{\partial \Phi}{\partial z} + \frac{\partial \Psi}{\partial r} + \frac{\Psi}{r}. \tag{12}$$

Substituting the value of u_r and u_z from (12) in (7)-(10) gives

where m_1, m_2, m_3 are the roots of the characteristic equation (23) and $m_4 = \sqrt{A_5}$, where $A_5 = \xi^2 + a_4$,

and the coupling constants are given by

$$r_i = \frac{B_3(m_i^2-\xi^2)^2+(B_2B_6-B_3B_4)(m_i^2-\xi^2)}{(m_i^2-\xi^2)^2-(B_1+B_4)(m_i^2-\xi^2)+(B_1B_4-B_2B_5)}, \tag{27}$$

$$s_i = \frac{B_6(m_i^2-\xi^2)^2+(B_3B_5-B_1B_6)(m_i^2-\xi^2)}{(m_i^2-\xi^2)^2-(B_1+B_4)(m_i^2-\xi^2)+(B_1B_4-B_2B_5)}, \text{ (i=1, 2, 3).} \tag{28}$$

The displacement components $\tilde{\tilde{u}}_r$ and $\tilde{\tilde{u}}_z$ are obtained with the aid of (17)-(18) and (25)-(26) as

$$\tilde{\tilde{u}}_r = -E_1\xi e^{-m_1z} - E_2\xi e^{-m_2z} - E_3\xi e^{-m_3z} + E_4m_4e^{-m_4z}, \tag{29}$$

$$\tilde{\tilde{u}}_z = -E_1m_1e^{-m_1z} - E_2m_2e^{-m_2z} - E_3m_3e^{-m_3z} + E_4\xi e^{-m_4z}. \tag{30}$$

4. Boundary conditions

The boundary conditions at z = 0 are

$$\sigma_{zz} = -F_1F(r,t), \sigma_{zr} = 0,\ p = F_2F(r,t),\ \frac{\partial T}{\partial x_3} = F_3F(r,t), \tag{31}$$

where F_1 are the magnitudes of the forces, F_2 is the constant pressure applied on the boundary and F_3 is the constant temperature applied on the boundary,

$$F(r,t) = \begin{cases} \frac{F_0}{2\pi r}\delta(ct-r) & for\ ring\ load \\ \frac{F_0}{\pi(ct)^2}H(ct-r) & for\ disc\ load \end{cases}, \tag{32}$$

in which $\delta(\,)$ is the Dirac delta function and H is the Heaviside function and F_0 is the magnitude of the force. With these boundary conditions, the total force applied to the surface is the same for all the time and equal to amount F_0 which is initially applied at r=0. However, the normal stress decays as $\frac{1}{r}$ for the ring load and as $\frac{1}{t^2}$ for the disc as these loads expands.

Using Laplace and Hankel transforms defined by (17) and (18) on (32), we obtain

$$\tilde{\tilde{F}}(\xi,s) = \begin{cases} \frac{F_0}{2\pi}\frac{1}{\sqrt{\xi^2+\frac{s^2}{c^2}}} & for\ ring\ load \\ \frac{F_0}{\pi c\xi}\left(\sqrt{\left(\xi^2+\frac{s^2}{c^2}\right)}-\frac{s}{c}\right) & for\ disc\ load \end{cases}. \tag{33}$$

Applying Laplace and Hankel transforms defined by (17) and (18) on (31) and with the aid of (6) and $F_i{}' = \frac{F_i}{\beta T_0}$ (i =1, 2, 3) and $F_4{}' = \frac{F_4c_1}{\omega^* T_0}$, we obtain

$$\tilde{\tilde{\sigma}}_{zz} = -F_1\tilde{\tilde{F}}(\xi,s), \tilde{\tilde{\sigma}}_{zr} = 0, \tilde{\tilde{p}} = F_2\tilde{\tilde{F}}(\xi,s),\ \frac{\partial T}{\partial x_3} = F_3\tilde{\tilde{F}}(\xi,s), \text{ at } z = 0, \tag{34}$$

where

$$\tilde{\tilde{\sigma}}_{zz} = R_1\xi\tilde{\tilde{u}}_r + R_2\frac{d\tilde{\tilde{u}}_z}{dz} - \alpha\tilde{\tilde{p}} - \tilde{\tilde{T}}, \tag{35}$$

$$\tilde{\tilde{\sigma}}_{zr} = R_3\left[\frac{d\tilde{\tilde{u}}_r}{dz} - \xi\tilde{\tilde{u}}_z\right] \tag{36}$$

7. Numerical results and discussion

With the view of illustrating the theoretical results and for numerical discussion we take a model for which the values of the various physical parameters are taken from Jabbari and Dehbani [18]:

$E = 6 \times 10^5,\ \nu = 0.3,\ T_0 = 293,\ K_s = 2 \times 10^{10},\ K_w = 5 \times 10^9, K = 0.5,$
$\alpha_s = 1.5 \times 10^{-5}, \alpha_W = 2 \times 10^{-4}, c_s = 0.8\ , c_w = 4.2, \rho_s = 2.6 \times 10^6, \rho_w = 1 \times 10^6,$
$\alpha = 1, F_1 = F_2 = F_3 = F_4 = 1.$

The values of normal stress σ_{zz}, tangential stress σ_{zr}, pore pressure p and temperature change T for incompressible fluid saturated thermoporoelastic medium (FM) and empty porous thermoelastic medium (EM) are shown due to ring and disc loads. The computation are carried out for two values of dimensionless velocity c=0.5 and c=1.0 at z=1 in the range $0 \le r \le 10$.

The solid and small dashed lines without central symbols corresponds to the variations at c=0.5, whereas the solid and small dashed lines with central symbols corresponds to the variations at c=1.0. Curves by solid lines with and without central symbols correspond to the case of FM whereas small dashed lines with and without central symbols corresponds to the case of EM.

Figures 2 and 3 show the variation of normal stress component σ_{zz} due to ring and disc loads w.r.t. distance r for both FM and EM. The value of σ_{zz} starts with initial increase and then become close to zero in an oscillatory manner for FM as r increases and in case of EM, its value first increases rapidly and then oscillates as r increases for both values of c due to both types of loads.

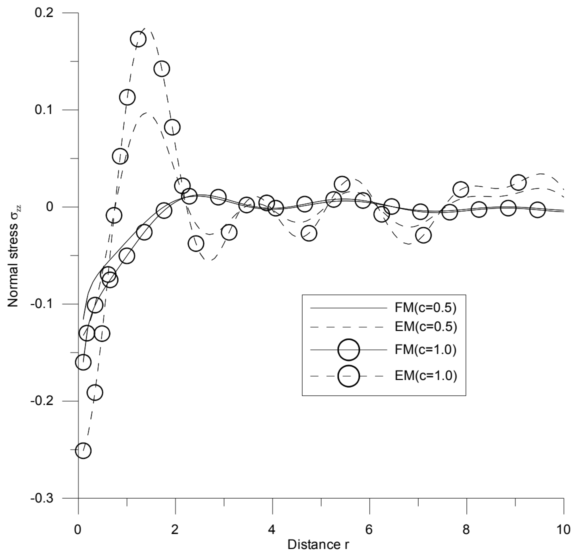

Fig. 2. Variation of normal stress σ_{zz} w.r.t. distance r due to ring load.

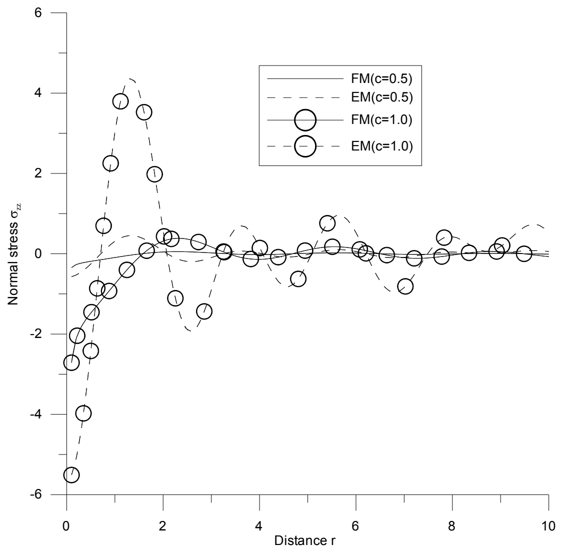

Fig. 3. Variation of normal stress σ_{zz} w.r.t. distance r due to disc load.

The behavior of tangential stress component σ_{zr} due to ring and disc loads w.r.t distance r for both FM and EM is shown in Figs. 4 and 5. The value of σ_{zr} for FM starts with initial decrease and then oscillates as r increases and in case of EM, its value first decreases in the range $0 \le r \le 1.2$ and then oscillates about the origin as r increases further for all values of c .

Figures 6 and 7 depict the variation of pore pressure p w.r.t distance r for FM due to ring and disc loads. The value of p for FM increases in the range $0 \le r \le 2.5$ and then become close to zero in oscillatory manner as r increases for all values of c due to both the loads.

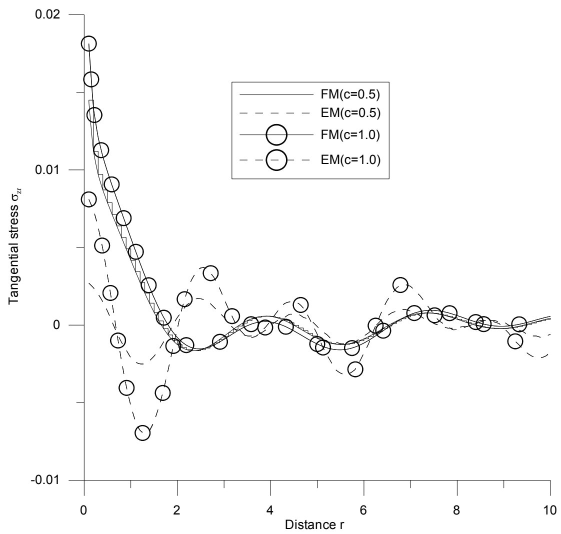

Fig. 4. Variation of tangential stress σ_{zr} w.r.t. distance r due to ring load.

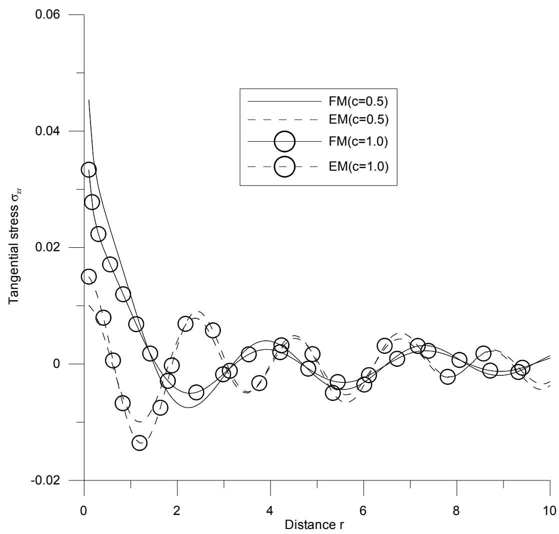

Fig. 5. Variation of tangential stress σ_{zr} w.r.t. distance r due to disc load.

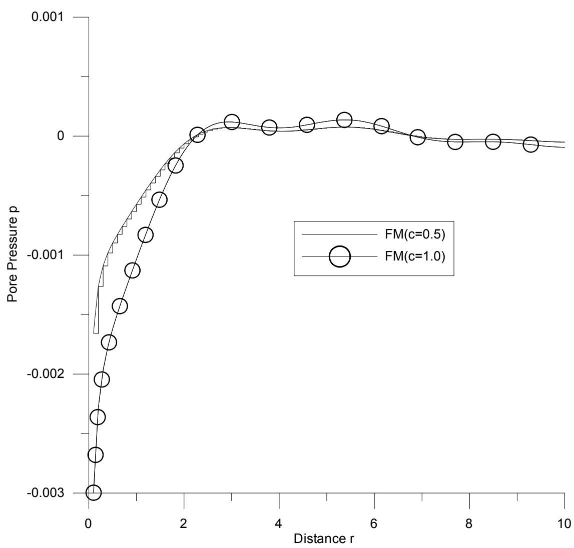

Fig. 6. Variation of pore pressure p w.r.t. distance r due to ring load.

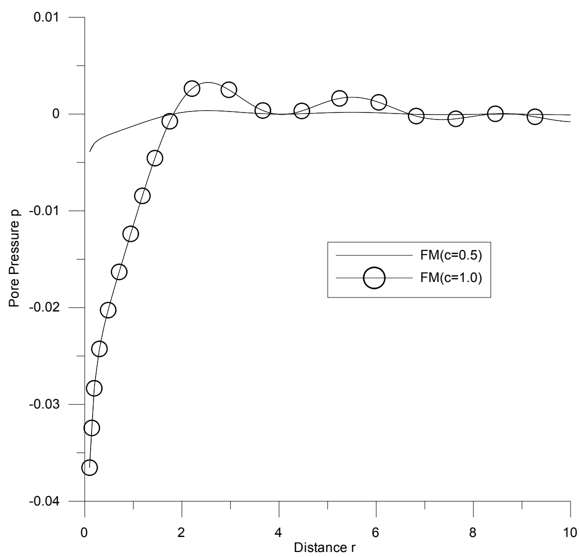

Fig. 7. Variation of pore pressure p w.r.t. distance r due to disc load.

Figures 8 and 9 show the variation of temperature T due to ring and disc loads w.r.t. distance r for both FM and EM. The value of T starts with initial decrease and then become close to zero in an oscillatory manner for FM as r increases and in case of EM, its value first increases in the range $0 \leq r \leq 1.6$ and then oscillates about the origin as r increases further for all values of c due to both types of loads.

8. Conclusion

In this chapter, deformation in thermoporoelastic medium is involved due to ring and disc loads. An appreciable porosity effect is observed on normal stress, tangential stress, pore pressure and temperature change on the application of ring and disc loads. Near the application of the load, the porosity effect increases the value of σ_{zz} and decreases the value of σ_{zr} and away from the source these values oscillates with the difference in their magnitude values.

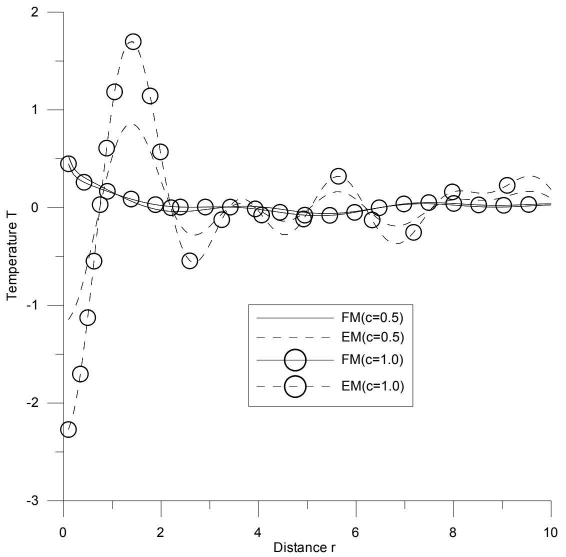

Fig. 8. Variation of temperature T w.r.t. distance r due to ring load.

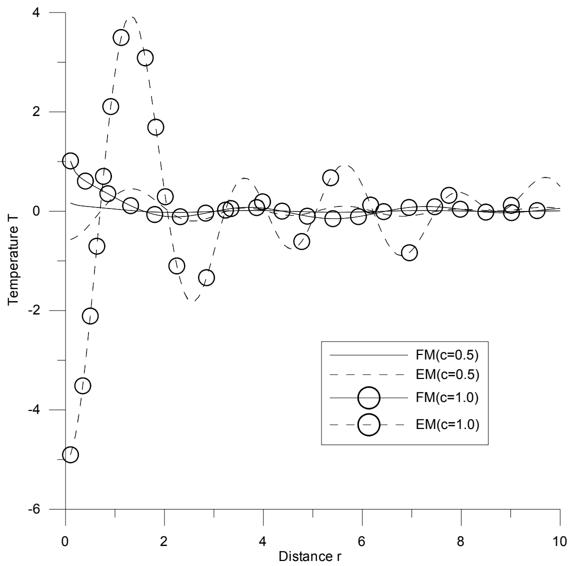

Fig. 9. Variation of temperature T w.r.t. distance r due to disc load.

Near the application of the load, the porosity effect increases the value of p and away from the source it become close to zero in oscillatory manner with the difference in their magnitude values.

Near the application of the load, the porosity effect decreases the value of T and away from the source it become close to zero in oscillatory manner for FM and for EM, near the application of the load, the porosity effect increases the value of T and away from the source these values oscillates with the difference in their magnitude values.

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