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THERMAL STRESSES IN A SIMPLY SUPPORTED PLATE WITH THERMAL BENDING MOMENTS WITH HEAT SOURCES

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Abstract. In this paper, we consider a thin simply supported rectangular plate defined as $0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c$ and determined the temperature distribution function with heat generation. The thermal stress components $\sigma_{xx}, \sigma_{yy}, \sigma_{xy}$ are evaluated due to thermal bending moments. The results are obtained in the series forms in terms of trigonometric function. Also two special cases for point heat source and moving heat source are considered.

1. Introdution

Y. Tanigawa et al. [6] discussed thermal stress analysis of a rectangular plate and its thermal stress intensity factor for compressive stress field. M. Ishihara et al. [2] studied theoretical analysis of residual stresses removed by heat supply. Further V.M. Vihak et al. [5] investigated the solution of the plane thermoelastic problem for a rectangular domain. R.J. Adams et al. [1] determined thermoelastic vibration of a laminated rectangular plate subjected to a thermal shock. Gogulwar et al. [7] studied thermal stresses in a rectangular plate due to partially distributed heat supply. Kulkarni et al. [8] deals with the realistic problem of the quasi-static thermal stresses in a rectangular plate subjected to constant heat supply on the extreme edges $(x = a, y = b)$ whereas the initial edges $(x = 0, y = 0)$ are thermally insulated. Khandait et al. [9] determined the quasi-static thermal stresses in a finite thin rectangular plate. Also Deshmukh and Khandait [10] studied a quasi-static problem in a thermo-isotropic elasticity concerning on semi-infinite rectangular plate, when part of its boundary kept at insulated and the rectangular plate being subjected to a concentrated heat source located inside the plate. Alzaharnah [11] studied the thermal stresses in thick walled cylinders subjected to a periodic moving heat source.

Recently, we considered a simply supported rectangular plate and discussed the deflection with the help of resultant moment. Also, we evaluated the thermal stress component due to thermal bending and shearing stress function.

In this paper, we discuss the thermal stress components due the thermal bending and shearing stress function. Here we consider two different heat sources and discuss the thermoelasticity.

2. Formation of the problem

Consider a rectangular parallelepiped with its dimensions $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right) - \frac{1}{1-\nu} M_T \,, \tag{2.8}$$

$$M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right) - \frac{1}{1-\nu} M_T \,, \tag{2.9}$$

and

$$M_{xy} = (1-\nu) D \frac{\partial^2 w}{\partial x \partial y} \tag{2.10}$$

and the equilibrium equations of moments about x and y axes are

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{yx}}{\partial y} - Q_x = 0 \,, \tag{2.11}$$

$$\frac{\partial M_y}{\partial y} - \frac{\partial M_{xy}}{\partial x} - Q_y = 0 \tag{2.12}$$

where Q_x, Q_y are the shearing forces.

D is the bending rigidity of the plate and M_T is the thermally induced resultant moment of the plate respectively, which are defined by

$$D = \frac{Eh^3}{12(1-\nu^2)} \,, \tag{2.13}$$

where E be the Young's modulus and

$$M_T = \alpha E \int_0^C Tz dz \,. \tag{2.14}$$

The thermal stress components in terms of the resultant forces and resultant moments are given as [12]

$$\sigma_{xx} = \frac{1}{c} N_x + \frac{12z}{c^3} M_x + \frac{1}{(1-\nu)}\left(\frac{1}{c} N_T + \frac{12z}{c^3} M_T - \alpha E T\right), \tag{2.15}$$

$$\sigma_{yy} = \frac{1}{c} N_y + \frac{12z}{c^3} M_y + \frac{1}{(1-\nu)}\left(\frac{1}{c} N_T + \frac{12z}{c^3} M_T - \alpha E T\right), \tag{2.16}$$

and

$$\sigma_{xy} = \frac{1}{c} N_{xy} - \frac{12z}{c^3} M_{xy} \,, \tag{2.17}$$

where the resultant force is

$$N_T = \alpha \mathrm{E} \int_0^c \mathrm{Tdz} \,. \tag{2.18}$$

The deflection with $w = 0$ at $x = a, y = b$,

$$M_T = \alpha Ec\sqrt{\frac{2}{c}}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}\frac{\left\lfloor (-1)^{p+1}\right\rfloor}{\eta_p}K\left(\beta_m, x\right)K\left(\nu_n, y\right)\cdot e^{-\alpha(\beta_m^2+\nu_n^2+\eta_p^2)t}\,.$$

$$\left\lfloor \bar{F}(\beta_m,\nu_n,\eta_p)+\frac{\alpha}{k}\int_{t'=0}^{t}\bar{g}\left(\beta_n,\nu_n,\eta_p,t'\right)e^{\alpha\left(\beta_m^2+\nu_n^2+\eta_p^2\right)t'}dt'\right\rfloor. \qquad (3.7)$$

Using equations (3.3) in (2.4), (2.5), (2.6), one obtains the thermal deflection as,

$$w(x,y) = \frac{\alpha Ec}{(1-\nu)D}\sqrt{\frac{2}{c}}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}\frac{\left\lfloor (-1)^{p+1}\right\rfloor}{\eta_p\left(\beta_m^2+\nu_n^2\right)}K\left(\beta_m, x\right)K\left(\nu_n, y\right)\cdot e^{-\alpha(\beta_m^2+\nu_n^2+\eta_p^2)t}\,.$$

$$\left\lfloor \bar{F}(\beta_m,\nu_n,\eta_p)+\frac{\alpha}{k}\int_{t'=0}^{t}\bar{g}\left(\beta_n,\nu_n,\eta_p,t'\right)e^{\alpha\left(\beta_m^2+\nu_n^2+\eta_p^2\right)t'}dt'\right\rfloor. \qquad (3.8)$$

Using (3.8) and (3.7) in (2.8),(2.9)and (2.10), one obtains the resultant moment as,

$$M_x = -\alpha Ec\sqrt{\frac{2}{c}}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}\frac{\left\lfloor (-1)^{p+1}\right\rfloor}{\eta_p}\left(\frac{\nu_n^2}{\beta_m^2+\nu_n^2}\right)K\left(\beta_m, x\right)K\left(\nu_n, y\right)\cdot e^{-\alpha(\beta_m^2+\nu_n^2+\eta_p^2)t}\,.$$

$$\left\lfloor \bar{F}(\beta_m,\nu_n,\eta_p)+\frac{\alpha}{k}\int_{t'=0}^{t}\bar{g}\left(\beta_n,\nu_n,\eta_p,t'\right)e^{\alpha\left(\beta_m^2+\nu_n^2+\eta_p^2\right)t'}dt'\right\rfloor. \qquad (3.9)$$

$$M_y = -\alpha Ec\sqrt{\frac{2}{c}}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}\frac{\left\lfloor (-1)^{p+1}\right\rfloor}{\eta_p}\left(\frac{\beta_m^2}{\beta_m^2+\nu_n^2}\right)K\left(\beta_m, x\right)K\left(\nu_n, y\right)\cdot e^{-\alpha(\beta_m^2+\nu_n^2+\eta_p^2)t}\,.$$

$$\left\lfloor \bar{F}(\beta_m,\nu_n,\eta_p)+\frac{\alpha}{k}\int_{t'=0}^{t}\bar{g}\left(\beta_n,\nu_n,\eta_p,t'\right)e^{\alpha\left(\beta_m^2+\nu_n^2+\eta_p^2\right)t'}dt'\right\rfloor. \qquad (3.10)$$

$$M_{xy} = \alpha Ec\sqrt{\frac{2}{a}}\sqrt{\frac{2}{b}}\sqrt{\frac{2}{c}}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}\frac{\left\lfloor (-1)^{p+1}\right\rfloor}{\eta_p}\left(\frac{\beta_m\nu_n}{\beta_m^2+\nu_n^2}\right)\cos\beta_m x\,\cos\nu_n y\cdot e^{-\alpha(\beta_m^2+\nu_n^2+\eta_p^2)t}\,.$$

$$\left\lfloor \bar{F}(\beta_m,\nu_n,\eta_p)+\frac{\alpha}{k}\int_{t'=0}^{t}\bar{g}\left(\beta_n,\nu_n,\eta_p,t'\right)e^{\alpha\left(\beta_m^2+\nu_n^2+\eta_p^2\right)t'}dt'\right\rfloor. \qquad (3.11)$$

Using equations (2.7), (3.1), (3.6), (3.7), (3.9), (3.10), (3.11) in (2.15) (2.16) and (2.17), one obtains expressions for the thermal stresses as,

$$\sigma_{xx} = \left\{\frac{12z}{c^2}\left(-\alpha E\sqrt{\frac{2}{c}}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}\left\lfloor\frac{(-1)^{p+1}\nu_n^2}{\eta_p\left(\beta_m^2+\nu_n^2\right)}\right\rfloor K\left(\beta_m, x\right)K\left(\nu_n, y\right)\cdot e^{-\alpha(\beta_m^2+\nu_n^2+\eta_p^2)t}\right)\right.$$

$$+\frac{1}{c(1-\nu)}\left(\alpha E\sqrt{\frac{2}{c}}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}\left\lfloor\frac{(-1)^{p+1}+1}{\eta_p}\right\rfloor K\left(\beta_m, x\right)K\left(\nu_n, y\right).e^{-\alpha(\beta_m^2+\nu_n^2+\eta_p^2)t}\right)$$

$$w(x,y,t)=\frac{\alpha Ec}{(1-\nu)D}\sqrt{\frac{2}{c}}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}\frac{\left\lfloor(-1)^{p+1}\right\rfloor}{\eta_p\left(\beta_m^2+\nu_n^2\right)}K\left(\beta_m,x\right)K\left(\nu_n,y\right)K\left(\eta_p,z\right)\cdot e^{-\alpha(\beta_m^2+\nu_n^2+\eta_p^2)t}$$

$$\left\lfloor \bar{F}(\beta_m,\nu_n,\eta_p)+\frac{\alpha g_o}{k}K\left(\beta_m,x_o\right)K\left(\nu_n,y_o\right)K\left(\eta_p,z_o\right)e^{\alpha\left(\beta_m^2+\nu_n^2+\eta_p^2\right)\tau}\right\rfloor, \tag{3.17a}$$

$$\sigma_{xx}=\left\{\frac{12z}{c^2}\left(-\alpha E\sqrt{\frac{2}{c}}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}\left\lfloor\frac{(-1)^{p+1}\nu_n^2}{\eta_p\left(\beta_m^2+\nu_n^2\right)}\right\rfloor K\left(\beta_m,x\right)K\left(\nu_n,y\right)\cdot e^{-\alpha(\beta_m^2+\nu_n^2+\eta_p^2)t}\right)\right.$$

$$+\frac{1}{c(1-\nu)}\left(\alpha E\sqrt{\frac{2}{c}}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}\left\lfloor\frac{(-1)^{p+1}+1}{\eta_p}\right\rfloor K\left(\beta_m,x\right)K\left(\nu_n,y\right)\cdot e^{-\alpha(\beta_m^2+\nu_n^2+\eta_p^2)t}\right)$$

$$+\frac{12z}{c^2(1-\nu)}\left(\alpha E\sqrt{\frac{2}{c}}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}\left\lfloor\frac{(-1)^{p+1}}{\eta_p}\right\rfloor K\left(\beta_m,x\right)K\left(\nu_n,y\right)\cdot e^{-\alpha(\beta_m^2+\nu_n^2+\eta_p^2)t}\right)$$

$$\left.-\frac{1}{(1-\nu)}\left(\alpha E\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}K\left(\beta_m,x\right)K\left(\nu_n,y\right)K\left(\eta_p,z\right).e^{-\alpha(\beta_m^2+\nu_n^2+\eta_p^2)t}\right)\right\}$$

$$\left\lfloor \bar{F}(\beta_m,\nu_n,\eta_p)+\frac{\alpha g_o}{k}K\left(\beta_m,x_o\right)K\left(\nu_n,y_o\right)K\left(\eta_p,z_o\right)e^{\alpha\left(\beta_m^2+\nu_n^2+\eta_p^2\right)\tau}\right\rfloor, \tag{3.18a}$$

$$\sigma_{yy}=\left\{\frac{12z}{c^2}\left(-\alpha E\sqrt{\frac{2}{c}}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}\left\lfloor\frac{(-1)^{p+1}\beta_m^2}{\eta_p\left(\beta_m^2+\nu_n^2\right)}\right\rfloor K\left(\beta_m,x\right)K\left(\nu_n,y\right)\cdot e^{-\alpha(\beta_m^2+\nu_n^2+\eta_p^2)t}\right)\right.$$

$$+\frac{1}{c(1-\nu)}\left(\alpha E\sqrt{\frac{2}{c}}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}\left\lfloor\frac{(-1)^{p+1}+1}{\eta_p}\right\rfloor K\left(\beta_m,x\right)K\left(\nu_n,y\right)\cdot e^{-\alpha(\beta_m^2+\nu_n^2+\eta_p^2)t}\right)$$

$$+\frac{12z}{c^2(1-\nu)}\left(\alpha E\sqrt{\frac{2}{c}}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}\left\lfloor\frac{(-1)^{p+1}}{\eta_p}\right\rfloor K\left(\beta_m,x\right)K\left(\nu_n,y\right)\cdot e^{-\alpha(\beta_m^2+\nu_n^2+\eta_p^2)t}\right)$$

$$\left.-\frac{1}{(1-\nu)}\left(\alpha E\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}K\left(\beta_m,x\right)K\left(\nu_n,y\right)K\left(\eta_p,z\right)\cdot e^{-\alpha(\beta_m^2+\nu_n^2+\eta_p^2)t}\right)\right\}$$

$$\left\lfloor \bar{F}(\beta_m,\nu_n,\eta_p)+\frac{\alpha g_o}{k}K\left(\beta_m,x_o\right)K\left(\nu_n,y_o\right)K\left(\eta_p,z_o\right)e^{\alpha\left(\beta_m^2+\nu_n^2+\eta_p^2\right)\tau}\right\rfloor, \tag{3.19a}$$

$$\sigma_{xy}=\frac{-12z}{c^2}\left(\begin{array}{c}\alpha E\sqrt{\frac{2}{a}}\sqrt{\frac{2}{b}}\sqrt{\frac{2}{c}}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}\left\lfloor\frac{(-1)^{p+1}\beta_m\nu_n}{\eta_p\left(\beta_m^2+\nu_n^2\right)}\right\rfloor\cos\beta_m x.\cos\nu_n y\cdot e^{-\alpha(\beta_m^2+\nu_n^2+\eta_p^2)t}\\ \left[\bar{F}(\beta_m,\nu_n,\eta_p)+\frac{\alpha g_o}{k}K\left(\beta_m,x_o\right)K\left(\nu_n,y_o\right)K\left(\eta_p,z_o\right)e^{\alpha\left(\beta_m^2+\nu_n^2+\eta_p^2\right)\tau}\right]\end{array}\right). \tag{3.20a}$$

B) We consider the Heat source is a plane-surface heat source of strength $g_s(t)$. We

$$-\frac{1}{(1-\nu)}\left(\alpha E\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}K(\beta_m,x)K(\nu_n,y)K(\eta_p,z)\cdot e^{-\alpha(\beta_m^2+\nu_n^2+\eta_p^2)t}\right)\Bigg\}$$

$$\left[\bar{F}(\beta_m,\nu_n,\eta_p)+\frac{\alpha}{k}\left(K(\beta_m,x_o)K(\nu_n,y_o)K(\eta_p,z_o)\int_{t'=0}^{t}g_s(t)e^{\alpha(\beta_m^2+\nu_n^2+\eta_p^2)t'}dt'\right)\right], \qquad (3.19b)$$

$$\sigma_{xy}=\frac{-12z}{c^2}\left(\begin{array}{l}\alpha E\sqrt{\frac{2}{a}}\sqrt{\frac{2}{b}}\sqrt{\frac{2}{c}}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}\left[\frac{(-1)^{p+1}\beta_m\nu_n}{\eta_p(\beta_m^2+\nu_n^2)}\right]\cos\beta_m x.\cos\nu_n y.e^{-\alpha(\beta_m^2+\nu_n^2+\eta_p^2)t}\\ \left[\bar{F}(\beta_m,\nu_n,\eta_p)+\frac{\alpha g_o}{k}K(\beta_m,x_o)K(\nu_n,y_o)K(\eta_p,z_o)e^{\alpha(\beta_m^2+\nu_n^2+\eta_p^2)\tau}\right]\end{array}\right)$$

(3.20b)

5. Numerical calculations

Dimension

Length of rectangular plate $a=5m$;

Breadth of rectangular plate $b=4m$;

Height of rectangular plate $c=2\ m$.

Material properties

The numerical calculation has been carried out for a copper (pure) thin hollow disk with the material properties:

Thermal diffusivity $\alpha=112.34\times10^{-6}\quad m^2s^{-1}$;

Thermal conductivity $k=386\quad W\,m^{-1}K^{-1}$;

Density $\rho=8954\quad kg\,m^{-3}$;

Specific heat $c_p=383\quad J\,kg^{-1}K^{-1}$;

Poisson ratio $\nu=0.35$;

Coefficient of linear thermal expansion, $a_t=16.5\times10^{-6}\quad K^{-1}$;

Lamé constant $\mu=26.67$.

Roots of the transcendental equation

The $\beta_{m,}\ \nu_{n,}\ \eta_p$ are m^{th}, n^{th}, p^{th} roots of transcendental equations

$$\sin(\beta_m a)=0,\ \sin(\nu_n b)=0,\ \sin(\eta_p c)=0\ \text{ i.e. } \beta_m=\frac{m\pi}{a},\nu_n=\frac{n\pi}{b},\eta_p=\frac{p\pi}{c}.$$

The numerical calculation has been carried out with the help of computational mathematical software Mathcad-2000 and the graphs are plotted with the help of Excel (MS office-2000). For convenience setting

$$A=\frac{8T_0}{\pi^3},\ B=\frac{8\alpha Ec^2T_0}{\pi^4(1-\nu)D},\ C=\frac{8\alpha ET_0}{\pi^4(1-\nu)},\ D=\frac{8\alpha ET_0}{\pi^4}.$$

In order to examine the influence of constant heat supply on the extreme ends of plate one performed numerical calculations in *X* and *Y* direction. Considering

$$\lim_{n\to\infty}\beta_m=\lim_{m\to\infty}\upsilon_n=\lim_{m\to\infty}\eta_p=\infty,$$

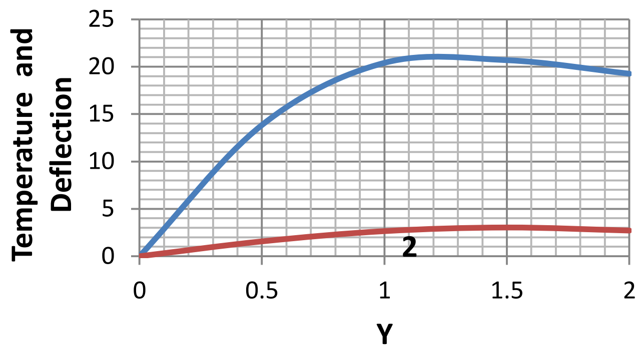

Fig. 4. Temperature distribution $\frac{T}{A}$ (1) and deflection $\frac{w}{B}$ (2) along Y axis.

From Fig. 5, it can be seen that, shear stresses develop tensile stresses whereas resultant stresses are compressive stresses in Y direction.

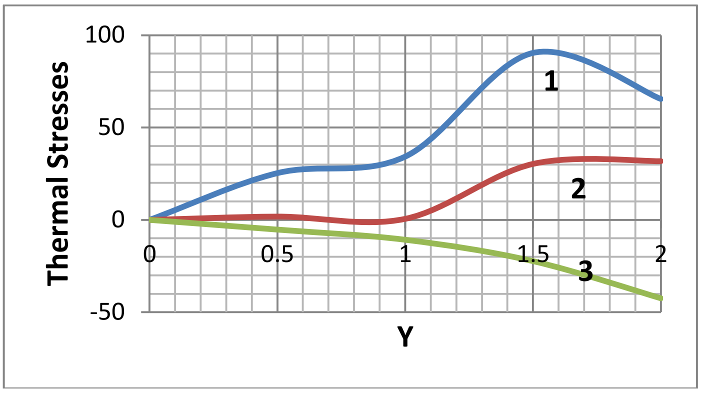

Fig. 5. Thermal stresses along Y axis: 1- $\frac{\sigma_{xx}}{C}$, 2- $\frac{\sigma_{yy}}{C}$, 3- $\frac{\sigma_{xy}}{D}$.

6. Conclusion

In discussing the thermal bending problem of a simply supported rectangular plate with thickness c, it will be assume that the deflection, which means a deformation in the out-of-plane direction of the plate is small. In order to analyze the thermo-elastic behavior of a simply supported rectangular plate we here introduce the concept of the resultant forces N_x, N_y, N_{xy} and the resultant moments M_x, M_y, M_{xy} per unit length of the plate by considering the equilibrium state in the in-plane-direction of x and y. Furthermore, the thermal stress components $\sigma_{xx}, \sigma_{yy}, \sigma_{xy}$ due to thermal bending moments are evaluated in which the in-plane resultant forces N_x, N_y, N_{xy} are omitted. As a special case the arbitrary initial heat supply $T(x, y, z, t) = T_o$ is considered and determined the expressions for the temperature distribution, thermal deflection and the stress functions when each boundary of a rectangular plate is of zero temperature.

From the figures, it can be observed that,

1) temperature and deflection take place at middle part of rectangular plate;
2) temperature and deflection are proportional to each other;
3) from Fig. 3 deflection occurs at extreme edge in Y direction;

4) shear stresses develop tensile stresses in both X and Y direction;
5) resultant stress develops compressive stresses in both X and Y direction.

It means we may find out that due to initial constant heat supply the stresses and deflection develops within rectangular plate. The both normal stress components and shear stress component change sharply from initial edges to extreme edges of rectangular plate. Also from the figures of deflection it can be observed that the deflection occur through middle part of rectangular plate towards Y the direction.

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