Materials Physics and Mechanics 22 (2015) 107-117 Received: October 3, 2014

DISTURBANCE DUE TO INCLINED LOAD IN TRANSVERSELY ISOTROPIC THERMOELASTIC MEDIUM WITH TWO TEMPERATURES AND WITHOUT ENERGY DISSIPATION

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Abstract. The present investigation is concerned with the two dimensional deformation in a homogeneous, transversely isotropic thermoelastic solids with two temperatures in context of Green-Naghdi theory of type-II as a result of an inclined load. The inclined load is assumed to be linear combination of normal load and tangential load. Laplace and Fourier transforms are employed to solve the problem. The components of displacements, stresses and conductive temperature distribution so obtained in the physical domain are computed numerically. Effect of two temperatures is depicted graphically on the resulting quantities.

1. Introduction

Thermoelasticity is the study of interaction between deformation and thermal fields. It deals with dynamical system whose interaction with surroundings is limited to mechanical work, external forces and heat exchange. It also comprises the heat conduction, stress and strain that arise due to flow of heat. Also, the change of body temperature is caused not only by external and internal heat sources but by a process of deformation itself. For this reason, thermoelasticity is to be regarded as a multi-field discipline, governed by the interaction of a temperature deformation field. It makes possible to determine the stresses produced by the temperature field and to calculate the temperature distribution due to an action of time dependent forces and heat sources.

Green and Naghdi [5] and [6] postulated a new concept in generalized thermoelasticity and proposed three models which are subsequently referred to as GN-I, II, and III models. The linearised version of model-I corresponds to classical Thermoelastic model. In model -II, the internal rate of production entropy is taken to be identically zero implying no dissipation of thermal energy. This model admits un-damped thermoelastic waves in a thermoelastic material and is best known as theory of thermoelasticity without energy dissipation. The principal feature of this theory is in contrast to classical thermoelasticity associated with Fourier's law of heat conduction, the heat flow does not involve energy dissipation. This theory permits the transmission of heat as thermal waves at finite speed. Model-III includes the previous two models as special cases and admits dissipation of energy in general. In context of Green and Naghdi model many applications have been found. Chandrasekharaiah and Srinath [1] discussed the thermoelastic waves without energy dissipation in an unbounded body with a spherical cavity.

Youssef [21] constructed a new theory of generalized thermoelasticity by taking into

3. Formulation of the problem

We consider a homogeneous, transversely isotropic thermoelastic body initially at uniform temperature T_0. We take a rectangular Cartesian co-ordinate system (x_1, x_2, x_3) with x_3 axis pointing normally into the half space, which is thus represented by $x_3 \geq 0$. We consider the plane such that all particles on a line parallel to $x_2 -$ axis are equally displaced, so that the field component $u_2 = 0$ and u_1, u_3 and φ are independent of x_2 .we have used appropriate transformations following Slaughter [14] on the set of equations (1)-(3) to derive the equations for transversely isotropic thermoelastic solid with two temperature and without energy dissipation and we restrict our analysis to the two dimensional problem with

$$\vec{u} = (u_1, 0, u_3). \tag{7}$$

Equations (1) - (3) with the aid of (7) take the form

$$c_{11}\frac{\partial^2 u_1}{\partial x_1^2} + c_{44}\frac{\partial^2 u_1}{\partial x_3^2} + (c_{13}+c_{44})\frac{\partial^2 u_3}{\partial x_1 \partial x_3} - \beta_1 \frac{\partial}{\partial x_1}\left\{\varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x_1^2} + a_3 \frac{\partial^2 \varphi}{\partial x_3^2}\right)\right\} = \rho \frac{\partial^2 u_1}{\partial t^2}, \tag{8}$$

$$(c_{13}+c_{44})\frac{\partial^2 u_1}{\partial x_1 \partial x_3} + c_{44}\frac{\partial^2 u_3}{\partial x_1^2} + c_{33}\frac{\partial^2 u_3}{\partial x_3^2} - \beta_3 \frac{\partial}{\partial x_3}\left\{\varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x_1^2} + a_3 \frac{\partial^2 \varphi}{\partial x_3^2}\right)\right\} = \rho \frac{\partial^2 u_3}{\partial t^2}, \tag{9}$$

$$k_1 \frac{\partial^2 \varphi}{\partial x_1^2} + k_3 \frac{\partial^2 \varphi}{\partial x_3^2} = T_0 \frac{\partial^2}{\partial t^2}\left(\beta_1 \frac{\partial u_1}{\partial x_1} + \beta_3 \frac{\partial u_3}{\partial x_3}\right) + \rho C_E \frac{\partial^2}{\partial t^2}\left\{\varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x_1^2} + a_3 \frac{\partial^2 \varphi}{\partial x_3^2}\right)\right\}, \tag{10}$$

$$t_{11} = c_{11}e_{11} + c_{13}e_{33} - \beta_1 T, \qquad t_{33} = c_{13}e_{11} + c_{33}e_{33} - \beta_3 T, \qquad t_{13} = 2c_{44}e_{13},$$

where

$$e_{11} = \frac{\partial u_1}{\partial x_1}, \quad e_{33} = \frac{\partial u_3}{\partial x_3}, \qquad e_{13} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right), \quad T = \varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x_1^2} + a_3 \frac{\partial^2 \varphi}{\partial x_3^2}\right),$$

$$\beta_1 = (c_{11}+c_{12})\alpha_1 + c_{13}\alpha_3, \qquad \beta_3 = 2c_{13}\alpha_1 + c_{33}\alpha_3 .$$

In the above equations we use the contracting subscript notations $(1 \to 11, 2 \to 22, 3 \to 33, 4 \to 23, 5 \to 31, 6 \to 12)$ to relate c_{ijkl} to c_{mn}.

The initial and regularity conditions are given by

$$u_1(x_1, x_3, 0) = 0 = \dot{u}_1(x_1, x_3, 0), \quad u_3(x_1, x_3, 0) = 0 = \dot{u}_3(x_1, x_3, 0),$$

$$\varphi(x_1, x_3, 0) = 0 = \dot{\varphi}(x_1, x_3, 0) \qquad \text{for } x_3 \geq 0, \quad -\infty < x_1 < \infty, \tag{11}$$

$$u_1(x_1, x_3, t) = u_3(x_1, x_3, t) = \varphi(x_1, x_3, t) = 0 \qquad for\ t > 0 \qquad \text{when} \quad x_3 \to \infty \tag{12}$$

To facilitate the solution, following dimensionless quantities are introduced:

$$x_1' = \frac{x_1}{L}, \quad x_3' = \frac{x_3}{L}, \quad u_1' = \frac{\rho c_1^2}{L\beta_1 T_0}u_1, \quad u_3' = \frac{\rho c_1^2}{L\beta_1 T_0}u_3, \quad T' = \frac{T}{T_0}, \quad t' = \frac{c_1}{L}t, \quad t_{11}' = \frac{t_{11}}{\beta_1 T_0},$$

$$t_{33}' = \frac{t_{33}}{\beta_1 T_0}, \quad t_{31}' = \frac{t_{31}}{\beta_1 T_0}, \quad \varphi' = \frac{\varphi}{T_0}, \quad a_1' = \frac{a_1}{L}, \quad a_3' = \frac{a_3}{L}, \tag{13}$$

where $c_1^2 = \frac{c_{11}}{\rho}$ and L is a constant of dimension of length.

Using the dimensionless quantities defined by (13) into (8)-(10) and after that suppressing the primes we obtain

$$\frac{\partial^2 u_1}{\partial x_1^2} + \delta_1 \frac{\partial^2 u_1}{\partial x_3^2} + \delta_2 \frac{\partial^2 u_3}{\partial x_1 \partial x_3} - \left[1 - \left(a_1 \frac{\partial^2}{\partial x_1^2} + a_3 \frac{\partial^2}{\partial x_3^2}\right)\right]\frac{\partial \varphi}{\partial x_1} = \frac{\partial^2 u_1}{\partial t^2}, \tag{14}$$

$$\delta_4 \frac{\partial^2 u_3}{\partial x_3^2} + \delta_1 \frac{\partial^2 u_3}{\partial x_1^2} + \delta_2 \frac{\partial^2 u_1}{\partial x_1 \partial x_3} - p_5 \left[1 - \left(a_1 \frac{\partial^2}{\partial x_1^2} + a_3 \frac{\partial^2}{\partial x_3^2}\right)\right]\frac{\partial \varphi}{\partial x_3} = \frac{\partial^2 u_3}{\partial t^2}, \tag{15}$$

$$\frac{\partial^2 \varphi}{\partial x_1^2} + p_3 \frac{\partial^2 \varphi}{\partial x_3^2} - \zeta_1 \frac{\partial^2}{\partial t^2}\frac{\partial u_1}{\partial x_1} - \zeta_2 \frac{\partial^2}{\partial t^2}\frac{\partial u_3}{\partial x_3} = \zeta_3 \left[1 - \left(a_1 \frac{\partial^2}{\partial x_1^2} + a_3 \frac{\partial^2}{\partial x_3^2}\right)\right]\frac{\partial^2 \varphi}{\partial t^2}, \tag{16}$$

where

where F_1 and F_2 are the magnitudes of the forces applied, $\psi_1(x)$ and $\psi_2(x)$ specify the vertical and horizontal load distribution functions respectively along x_1 axis, $H(t)$ is the Heaviside unit step function (Fig. 2).

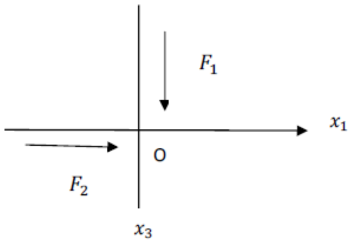

Fig. 1. Normal and tangential loadings.

Substituting the values of $\hat{u}_1$, $\hat{u}_3$ and $\hat{\varphi}$ from equations (20) – (22) in boundary conditions (25) and with the aid of (1), (4)-(6), (13), (17), and (18), we obtain the components of displacement, normal stress, tangential stress and conductive temperature as

$$\widehat{u_1} = \frac{F_1\widehat{\psi_1}(\xi)}{s\Delta}(-M_{11}+M_{12}-M_{13}) + \frac{F_2\widehat{\psi_2}(\xi)}{s\Delta}(M_{21}-M_{22}+M_{23}), \tag{26}$$

$$\widehat{u_3} = \frac{F_1\widehat{\psi_1}(\xi)}{s\Delta}(-d_1M_{11}+d_2M_{12}-d_3M_{13}) + \frac{F_2\widehat{\psi_2}(\xi)}{s\Delta}(d_1M_{21}-d_2M_{22}+d_3M_{23}), \tag{27}$$

$$\hat{\varphi} = \frac{F_1\widehat{\psi_1}(\xi)}{s\Delta}(-l_1M_{11}+l_2M_{12}-l_3M_{13}) + \frac{F_2\widehat{\psi_2}(\xi)}{s\Delta}(l_1M_{21}-l_2M_{22}+l_3M_{23}), \tag{28}$$

$$\widehat{t_{33}} = \frac{F_1\widehat{\psi_1}(\xi)}{s\Delta}(-\Delta_{11}M_{11}+\Delta_{12}M_{12}-\Delta_{13}M_{13}) + \frac{F_2\widehat{\psi_2}(\xi)}{s\Delta}(\Delta_{11}M_{21}-\Delta_{12}M_{22}+\Delta_{13}M_{23}), \tag{29}$$

$$\widehat{t_{31}} = \frac{F_1\widehat{\psi_1}(\xi)}{s\Delta}(-\Delta_{21}M_{11}+\Delta_{22}M_{12}-\Delta_{23}M_{13}) + \frac{F_2\widehat{\psi_2}(\xi)}{s\Delta}(\Delta_{21}M_{21}-\Delta_{22}M_{22}+\Delta_{23}M_{23}), \tag{30}$$

where

$$M_{11} = \Delta_{22}\Delta_{33} - \Delta_{32}\Delta_{23},\ M_{12} = \Delta_{21}\Delta_{33} - \Delta_{23}\Delta_{31},\ M_{13} = \Delta_{21}\Delta_{32} - \Delta_{22}\Delta_{31},$$

$$M_{21} = \Delta_{12}\Delta_{33} - \Delta_{13}\Delta_{22},\ M_{22} = \Delta_{11}\Delta_{33} - \Delta_{13}\Delta_{31},\ M_{23} = \Delta_{11}\Delta_{32} - \Delta_{12}\Delta_{31}$$

$$\Delta_{1j} = \frac{c_{31}}{\rho c_1^2}i\xi - \frac{c_{33}}{\rho c_1^2}d_j\lambda_j - \frac{\beta_3}{\beta_1}l_j + \frac{\beta_3}{\beta_1T_0}a_3l_j{\lambda_j}^2 - \frac{\beta_3}{\beta_1}l_ja_1\xi^2 \qquad j = 1,2,3,$$

$$\Delta_{2j} = -\frac{c_{44}}{\rho c_1^2}\lambda_j + \frac{c_{44}}{\rho c_1^2}i\xi d_j \quad j = 1,2,3,\ \Delta_{3j} = \lambda_j l_j \quad j = 1,2,3,\ \Delta = \Delta_{11}M_{11} - \Delta_{12}M_{12} + \Delta_{13}M_{13}.$$

4.1. Concentrated force. The solution due to concentrated normal force on the half space is obtained by setting

$$\psi_1(x) = \delta(x), \qquad (x) = \delta(x), \tag{31}$$

where $\delta(x)$ is dirac delta function. Applying Laplace and Fourier transform defined by (17)-(18) and (31) ,we obtain

$$\widehat{\psi_1(\xi)} = 1, \qquad \widehat{\psi_2(\xi)} = 1. \tag{32}$$

Using (32) in (26)- (30) , we obtain the components of displacement, stress and conductive temperature.

4.2. Uniformly distributed force. The solution due to uniformly distributed force applied on the half space is obtained by setting

$$\{\psi_1(x),\ \psi_2(x)\} = \begin{cases} 1 \ \ if \ \ |x| \le m \\ 0 \ if \ |x| > m \end{cases}, \tag{33}$$

7. Inversion of the Transformation

To obtain the solution of the problem in physical domain, we must invert the transforms in equations (26)-(30). Here the displacement components, normal and tangential stresses and conductive temperature are functions of x_3 , the parameters of Laplace and Fourier transforms s and ξ respectively and hence are of the form $f\,(\xi,\, x_3\,,\, s)$. To obtain the function $f(x_1, x_3, t)$ in the physical domain, we first invert the Fourier transform using

$$\overline{f}(x_1, x_3, s)=\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\xi x_1}\,\hat{f}(\xi, x_3, s)d\xi = \frac{1}{2\pi}\int_{-\infty}^{\infty}|\cos(\xi x_1)\, f_e - isin(\xi x_1)\mathrm{f_o}|\; d, \tag{38}$$

where f_e and f_o are respectively the odd and even parts of $\hat{f}$ (ξ, x_3, s). Thus the expression (38) gives the Laplace transform $\overline{f}(x_1, x_3, s)$ of the function $f(x_1, x_3, t)$. Following Honig and Hirdes [7], the Laplace transform function $\overline{f}(x_1, x_3, s)$ can be inverted to $f(x_1, x_3, t)$.

The last step is to calculate the integral in equation (38). The method for evaluating this integral is described in Press et al. [15]. It involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

8. Numerical results and discussion

Copper material is chosen for the purpose of numerical calculation which is transversely isotropic

$c_{11} = 18.78 \times 10^{10}\ Kgm^{-1}s^{-2}$, $c_{12} = 8.76 \times 10^{10}\ Kgm^{-1}s^{-2}$, $c_{13} = 8.0 \times 10^{10}\ Kgm^{-1}s^{-2}$,
$c_{33} = 17.2 \times 10^{10}\ Kgm^{-1}s^{-2}$, $c_{44} = 5.06 \times 10^{10}\ Kgm^{-1}s^{-2}$, $C_E = 0.6331 \times 10^3 JKg^{-1}K^{-1}$,
$\alpha_1 = 2.98 \times 10^{-5}K^{-1}$, $\alpha_3 = 2.4 \times 10^{-5}K^{-1}$, $\rho = 8.954 \times 10^3 Kgm^{-3}$,
$K_1 = 0.433 \times 10^3 Wm^{-1}K^{-1}$, $K_3 = 0.450 \times 10^3 Wm^{-1}K^{-1}$.

A comparison of values of normal displacement u_3, normal force stress t_{33}, tangential stress t_{31} and conductive temperature φ for a transversely isotropic thermoelastic solid with distance x has been made between with two temperature and without two temperature and is presented graphically for non-dimensional two temperature parameters a_1=0.03 and a_3=0.06 at $\delta = 0°$ (initial angle), $\delta = 45°$ (intermediate angle) and $\delta = 90°$ (extreme angle) in Figs. 3-15.

1). The solid line, small dashed line, long dashed line respectively corresponds to a_1=0, a_3=0 with angle of inclination $\delta = 0°$, $\delta = 45°$ and $\delta = 90°$ (without two temperature).

2). The solid line with centre symbol circle, the small dashed line with centre symbol diamond, the long dashed line with centre symbol cross respectively corresponds to a_1=0.03, a_3=0.06 and at angle of inclination $\delta = 0°$, $\delta = 45°$ and $\delta = 90°$ (with two temperature).

Concentrated force. Near the loading surface, the values of normal displacement u_3 (Fig. 3) for a_1=0.03, a_3=0.06 is greater than a_1=0, a_3=0 for intermediate angle and extreme angle in the range $0 \leq x \leq 2$ and $5 \leq x \leq 6$. However in the remaining range, the behaviour is opposite. In Figure 4 for both intermediate and extreme angle, the values of normal stress t_{33}, for a_1=0, a_3=0 is greater than for a_1=0.03, a_3=0.06, except in the range $5 \leq x \leq 7$. The values of tangential stress t_{31}, for a_1=0.03, a_3=0.06 at angle of inclination $\delta = 45°$ and $\delta = 90°$ are more than a_1=0.03, a_3=0.06 in the whole range. In Figure 6 the values of conductive temperature φ, have an oscillatory behaviour corresponding to both the cases at $\delta = 0°$, $\delta = 45°$ and $\delta = 90°$. The behaviour of for a_1=0.03, a_3=0.06 and a_1=0, a_3=0 at initial angle is oscillatory as is depicted in Figs. 3-6.

Uniformly distributed force. It is depicted from Fig. 7, that the distribution curves for u_3 have same trend irrespective of angle of inclination i.e. for both intermediate as well as extreme , they move in the similar pattern. In the whole range, the values of u_3 for a_1 = 0 =

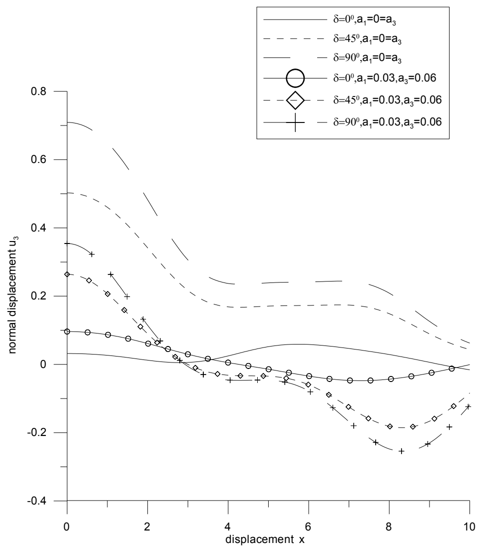

Fig. 7. Variation of normal displacement u_3 with displacement *x* (uniformly distributed force).

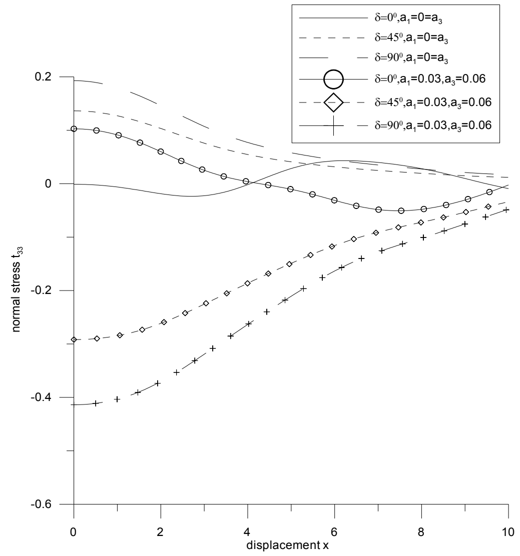

Fig. 8. Variation of normal stress t_{33} with displacement *x* (uniformly distributed force).

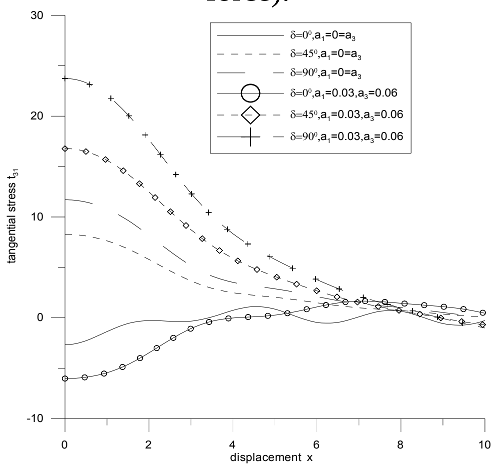

Fig. 9. Variation of tangential stress t_{31} with displacement *x* (uniformly distributed force).

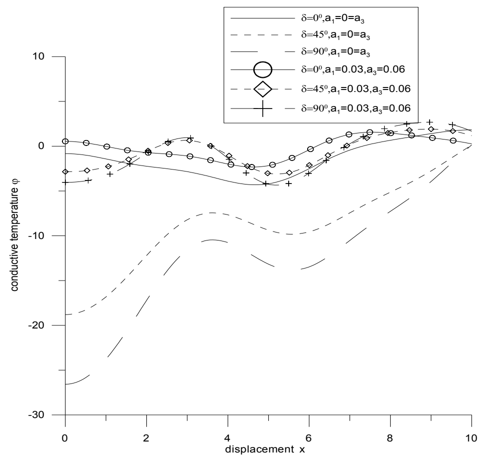

Fig. 10. Variation of conductive temperature φ with displacement *x* (uniformly distributed force).

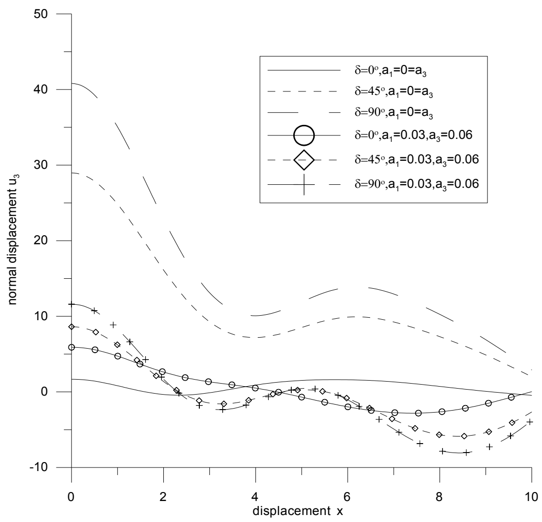

Fig. 11. Variation of normal displacement u_3 with displacement *x* (linearly distributed force).

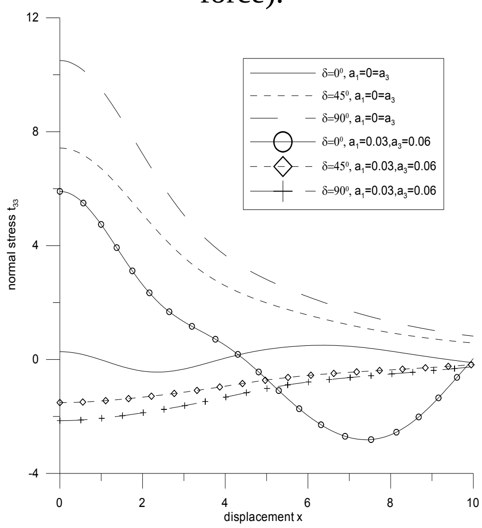

Fig. 12. Variation of normal stress t_{33} with displacement *x* (linearly distributed force).

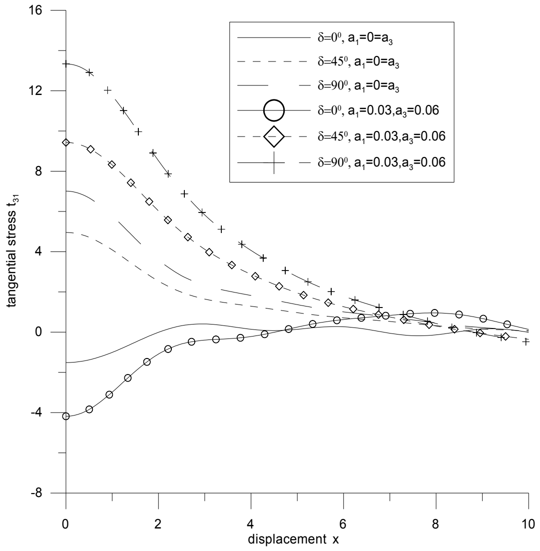

Fig. 13. Variation of tangential stress t_{31} with displacement *x* (linearly distributed force).

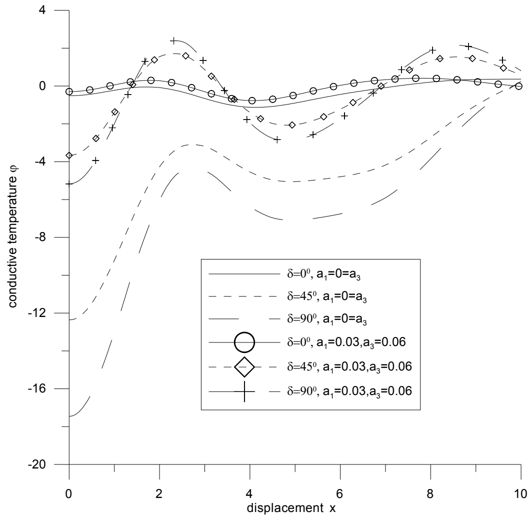

Fig. 14. Variation of conductive temperature φ with displacement *x* (linearly distributed force).

9. Conclusion

Effect of two temperature have significant impact on components of normal displacement, normal stress, tangential stress and conductive temperature. As disturbance travels through the constituents of the medium, it suffers sudden changes resulting in an inconsistent / non uniform pattern of graphs. The deformation in any part of the medium is useful to analyse the deformation field around mining tremors and drilling into the crust. It can also contribute to the theoretical consideration of the seismic and volcanic sources. Since it can account for the deformation fields in the entire volume surrounding the sources region.

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