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DISTURBANCE DUE TO INCLINED LOAD IN TRANSVERSELY ISOTROPIC THERMOELASTIC MEDIUM WITH TWO TEMPERATURES AND WITHOUT ENERGY DISSIPATION

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Abstract. The present investigation is concerned with the two dimensional deformation in a homogeneous, transversely isotropic thermoelastic solids with two temperatures in context of Green-Naghdi theory of type-II as a result of an inclined load. The inclined load is assumed to be linear combination of normal load and tangential load. Laplace and Fourier transforms are employed to solve the problem. The components of displacements, stresses and conductive temperature distribution so obtained in the physical domain are computed numerically. Effect of two temperatures is depicted graphically on the resulting quantities.

1. Introduction

Thermoelasticity is the study of interaction between deformation and thermal fields. It deals with dynamical system whose interaction with surroundings is limited to mechanical work, external forces and heat exchange. It also comprises the heat conduction, stress and strain that arise due to flow of heat. Also, the change of body temperature is caused not only by external and internal heat sources but by a process of deformation itself. For this reason, thermoelasticity is to be regarded as a multi-field discipline, governed by the interaction of a temperature deformation field. It makes possible to determine the stresses produced by the temperature field and to calculate the temperature distribution due to an action of time dependent forces and heat sources.

Green and Naghdi [5] and [6] postulated a new concept in generalized thermoelasticity and proposed three models which are subsequently referred to as GN-I, II, and III models. The linearised version of model-I corresponds to classical Thermoelastic model. In model -II, the internal rate of production entropy is taken to be identically zero implying no dissipation of thermal energy. This model admits un-damped thermoelastic waves in a thermoelastic material and is best known as theory of thermoelasticity without energy dissipation. The principal feature of this theory is in contrast to classical thermoelasticity associated with Fourier's law of heat conduction, the heat flow does not involve energy dissipation. This theory permits the transmission of heat as thermal waves at finite speed. Model-III includes the previous two models as special cases and admits dissipation of energy in general. In context of Green and Naghdi model many applications have been found. Chandrasekharaiah and Srinath [1] discussed the thermoelastic waves without energy dissipation in an unbounded body with a spherical cavity.

Youssef [21] constructed a new theory of generalized thermoelasticity by taking into account two-temperature generalized thermoelasticity theory for a homogeneous and isotropic body without energy dissipation .Youssef [22] also obtained variational principle of two temperature thermoelasticity without energy dissipation. Chen and Gurtin [2], Chen et al. [3] and [4] have formulated a theory of heat conduction in deformable bodies which depends upon two distinct temperatures, the conductive temperature φ and the thermo dynamical temperature T. For time independent situations, the difference between these two temperatures is proportional to the heat supply, and in absence of heat supply, the two temperatures are identical. For time dependent problems, the two temperatures are different, regardless of the presence of heat supply. The two temperatures T, φ and the strain are found to have representations in the form of a travelling wave plus a response, which occurs instantaneously throughout the body.

Warren and Chen [17] investigated the wave propagation in the two temperature theory of thermoelasticity. Quintanilla [16] proved some theorems in thermoelasticity with two temperatures. Youssef AI-Lehaibi [19] and Youssef AI -Harby [20] investigated various problems on the basis of two temperature thermoelasticity. Kumar and Deswal [8] studied the surface wave propagation in a micropolar thermoelastic medium without energy dissipation. Kaushal, Kumar and Miglani [9] discussed response of frequency domain in generalized thermoelasticity with two temperatures. Sharma and Kumar [12-13] discussed elastodynamic response and interactions of generalized thermoelastic diffusion due to inclined load. Kumar and Kansal [10] discussed propagation of cylindrical Rayleigh waves in a transversely isotropic thermoelastic diffusive solid half-space. Kumar, Sharma and Garg [11] analyzed effect of two temperature on reflection coefficient in micropolar thermoelastic media with and without energy dissipation.

The deformation at any point of the medium is useful to analyze the deformation field around mining tremors and drilling into the crust of earth. It also contribute to the theoretical consideration of the seismic and volcanic sources since it can account for the deformation field in the entire volume surrounding the source region. The purpose of the present paper is to determine the expression for components of displacement, normal stress, tangential stress and conductive temperature due to inclined load by applying Integral transform techniques.

2. Basic equations

Following Youssef [21] the constitutive relations and field equations in absence of body forces and heat sources are:

$$t_{ij} = C_{ijkl}e_{kl} - \beta_{ij}T , \tag{1}$$

$$C_{ijkl}e_{kl,j} - \beta_{ij}T_{,j} = \rho\ddot{u}_i, \tag{2}$$

$$K_{ij}\varphi_{,ij} = \beta_{ij}T_0\ddot{e}_{ij} + \rho C_E\ddot{T}, \tag{3}$$

where

$$T = \varphi - a_{ij}\varphi_{,ij}, \tag{4}$$

$$\beta_{ij} = C_{ijkl}\alpha_{ij} \ , \tag{5}$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \quad i,j = 1,2,3. \tag{6}$$

Here C_{ijkl} $(C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk})$ are elastic parameters, β_{ij} is the thermal tensor, T is the temperature, T_0 is the reference temperature, t_{ij} are the components of stress tensor, e_{kl} are the components of strain tensor,u_i are the displacement components, ρ is the density, C_E is the specific heat, K_{ij} is the thermal conductivity, a_{ij} are the two temperature parameters, α_{ij} is the coefficient of linear thermal expansion.

3. Formulation of the problem

We consider a homogeneous, transversely isotropic thermoelastic body initially at uniform temperature T_0. We take a rectangular Cartesian co-ordinate system (x_1, x_2, x_3) with x_3 axis pointing normally into the half space, which is thus represented by $x_3 \geq 0$. We consider the plane such that all particles on a line parallel to $x_2 -$ axis are equally displaced, so that the field component $u_2 = 0$ and u_1, u_3 and φ are independent of x_2 .we have used appropriate transformations following Slaughter [14] on the set of equations (1)-(3) to derive the equations for transversely isotropic thermoelastic solid with two temperature and without energy dissipation and we restrict our analysis to the two dimensional problem with

$$\vec{u} = (u_1, 0, u_3). \tag{7}$$

Equations (1) - (3) with the aid of (7) take the form

$$c_{11}\frac{\partial^2 u_1}{\partial x_1^2} + c_{44}\frac{\partial^2 u_1}{\partial x_3^2} + (c_{13}+c_{44})\frac{\partial^2 u_3}{\partial x_1 \partial x_3} - \beta_1 \frac{\partial}{\partial x_1}\left\{\varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x_1^2} + a_3 \frac{\partial^2 \varphi}{\partial x_3^2}\right)\right\} = \rho \frac{\partial^2 u_1}{\partial t^2}, \tag{8}$$

$$(c_{13}+c_{44})\frac{\partial^2 u_1}{\partial x_1 \partial x_3} + c_{44}\frac{\partial^2 u_3}{\partial x_1^2} + c_{33}\frac{\partial^2 u_3}{\partial x_3^2} - \beta_3 \frac{\partial}{\partial x_3}\left\{\varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x_1^2} + a_3 \frac{\partial^2 \varphi}{\partial x_3^2}\right)\right\} = \rho \frac{\partial^2 u_3}{\partial t^2}, \tag{9}$$

$$k_1 \frac{\partial^2 \varphi}{\partial x_1^2} + k_3 \frac{\partial^2 \varphi}{\partial x_3^2} = T_0 \frac{\partial^2}{\partial t^2}\left(\beta_1 \frac{\partial u_1}{\partial x_1} + \beta_3 \frac{\partial u_3}{\partial x_3}\right) + \rho C_E \frac{\partial^2}{\partial t^2}\left\{\varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x_1^2} + a_3 \frac{\partial^2 \varphi}{\partial x_3^2}\right)\right\}, \tag{10}$$

$$t_{11} = c_{11}e_{11} + c_{13}e_{33} - \beta_1 T, \qquad t_{33} = c_{13}e_{11} + c_{33}e_{33} - \beta_3 T, \qquad t_{13} = 2c_{44}e_{13},$$

where

$$e_{11} = \frac{\partial u_1}{\partial x_1}, \ \ e_{33} = \frac{\partial u_3}{\partial x_3}, \qquad e_{13} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right), \ \ T = \varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x_1^2} + a_3 \frac{\partial^2 \varphi}{\partial x_3^2}\right),$$

$$\beta_1 = (c_{11}+c_{12})\alpha_1 + c_{13}\alpha_3, \qquad \beta_3 = 2c_{13}\alpha_1 + c_{33}\alpha_3 .$$

In the above equations we use the contracting subscript notations $(1 \to 11, 2 \to 22, 3 \to 33, 4 \to 23, 5 \to 31, 6 \to 12)$ to relate c_{ijkl} to c_{mn}.

The initial and regularity conditions are given by

$$u_1(x_1, x_3, 0) = 0 = \dot{u}_1(x_1, x_3, 0), \quad u_3(x_1, x_3, 0) = 0 = \dot{u}_3(x_1, x_3, 0),$$

$$\varphi(x_1, x_3, 0) = 0 = \dot{\varphi}(x_1, x_3, 0) \qquad \text{for } x_3 \geq 0, \quad -\infty < x_1 < \infty, \tag{11}$$

$$u_1(x_1, x_3, t) = u_3(x_1, x_3, t) = \varphi(x_1, x_3, t) = 0 \qquad for\ t > 0 \qquad \text{when} \quad x_3 \to \infty \tag{12}$$

To facilitate the solution, following dimensionless quantities are introduced:

$$x_1' = \frac{x_1}{L}, \ \ x_3' = \frac{x_3}{L}, \ \ u_1' = \frac{\rho c_1^2}{L\beta_1 T_0}u_1, \ \ u_3' = \frac{\rho c_1^2}{L\beta_1 T_0}u_3, \ \ T' = \frac{T}{T_0}, \ \ t' = \frac{c_1}{L}t, \ \ t_{11}' = \frac{t_{11}}{\beta_1 T_0},$$

$$t_{33}' = \frac{t_{33}}{\beta_1 T_0}, \ \ t_{31}' = \frac{t_{31}}{\beta_1 T_0}, \ \ \varphi' = \frac{\varphi}{T_0}, \ \ a_1' = \frac{a_1}{L}, \ \ a_3' = \frac{a_3}{L}, \tag{13}$$

where $c_1^2 = \frac{c_{11}}{\rho}$ and L is a constant of dimension of length.

Using the dimensionless quantities defined by (13) into (8)-(10) and after that suppressing the primes we obtain

$$\frac{\partial^2 u_1}{\partial x_1^2} + \delta_1 \frac{\partial^2 u_1}{\partial x_3^2} + \delta_2 \frac{\partial^2 u_3}{\partial x_1 \partial x_3} - \left[1 - \left(a_1 \frac{\partial^2}{\partial x_1^2} + a_3 \frac{\partial^2}{\partial x_3^2}\right)\right]\frac{\partial \varphi}{\partial x_1} = \frac{\partial^2 u_1}{\partial t^2}, \tag{14}$$

$$\delta_4 \frac{\partial^2 u_3}{\partial x_3^2} + \delta_1 \frac{\partial^2 u_3}{\partial x_1^2} + \delta_2 \frac{\partial^2 u_1}{\partial x_1 \partial x_3} - p_5\left[1 - \left(a_1 \frac{\partial^2}{\partial x_1^2} + a_3 \frac{\partial^2}{\partial x_3^2}\right)\right]\frac{\partial \varphi}{\partial x_3} = \frac{\partial^2 u_3}{\partial t^2}, \tag{15}$$

$$\frac{\partial^2 \varphi}{\partial x_1^2} + p_3 \frac{\partial^2 \varphi}{\partial x_3^2} - \zeta_1 \frac{\partial^2}{\partial t^2}\frac{\partial u_1}{\partial x_1} - \zeta_2 \frac{\partial^2}{\partial t^2}\frac{\partial u_3}{\partial x_3} = \zeta_3 \left[1 - \left(a_1 \frac{\partial^2}{\partial x_1^2} + a_3 \frac{\partial^2}{\partial x_3^2}\right)\right]\frac{\partial^2 \varphi}{\partial t^2}, \tag{16}$$

where

$\delta_1=\frac{c_{44}}{c_{11}}$, $\delta_2=\frac{c_{13}+c_{44}}{c_{11}}$, $\delta_4=\frac{c_{33}}{c_{11}}$, $p_5=\frac{\beta_3}{\beta_1}$, $p_3=\frac{k_3}{k_1}$, $\zeta_1=\frac{T_0\beta_1^2}{k_1\rho}$, $\zeta_2=\frac{T_0\beta_3\beta_1}{k_1\rho}$, $\zeta_3=\frac{C_E c_{11}}{k_1}$.

Apply Laplace and Fourier transforms defined by

$$\bar{f}(x_1,x_3,s)=\int_0^\infty f(x_1,x_3,t)e^{-st}dt, \tag{17}$$

$$\hat{f}(\xi,x_3,s)=\int_{-\infty}^\infty \bar{f}(x_1,x_3,s)e^{i\xi x_1}dx_1 \tag{18}$$

to equations (14)-(16), we obtain a system of three homogeneous equations. These resulting equations have non trivial solutions if the determinant of the coefficient $(\hat{u}_1,\hat{u}_3,\hat{\varphi})$ vanishes, which yield to the following characteristic equation

$$\left(P\frac{d^6}{dx_3^6}+Q\frac{d^4}{dx_3^4}+R\frac{d^2}{dx_3^2}+S\right)(\hat{u}_1,\hat{u}_3,\hat{\varphi})=0, \tag{19}$$

where

$P=\delta_1(\delta_4\zeta_3a_3s^2-\delta_4p_3-\zeta_2p_5a_3s^2)$,

$Q=(\zeta_3a_3s^2-p_3)\{(-\xi^2+s^2)\delta_4-\delta_1(b_1\xi^2+s^2)+\delta_2^2\xi^2\}+\delta_1\delta_4\{\xi^2-\zeta_3s^2-\xi^2\zeta_3s^2a_1\}+$
$\zeta_2s^2\{a_3p_5(\xi^2+s^2)+\delta_1p_5(a_1\xi^2+1)\}+\xi^2s^2\{-\delta_4a_3(p_5\zeta_1+\zeta_2-\zeta_1)\}$,

$R=(1+a_1\xi^2)\{-(\xi^2+s^2)\zeta_2p_5s^2+\xi^2s^2(p_5\zeta_1\delta_2+\zeta_2\delta_2-\zeta_1\delta_4)\}+$
$(\delta_1\xi^2+s^2)\{(\xi^2+s^2)(s^2\zeta_3a_3-p_3)-\delta_1(\xi^2-\zeta_3s^2-\zeta_3s^2a_1\xi^2)-\xi^2a_3\zeta_1s^2\}+$
$(\xi^2-\zeta_3s^2-\zeta_3s^2\xi^2a_1)\{-(\xi^2+s^2)\delta_4+\delta_2^2\xi^2\}$,

$S=(\delta_1\xi^2+s^2)\{(\xi^2+s^2)(\xi^2-\zeta_3s^2-\zeta_3s^2a_1\xi^2)+\xi^2(1+a_1\xi^2)\xi^2\zeta_1s^2\}$.

The roots of the equation (19) are $\pm\lambda_i$ ($i=1,2,3$), using the radiation condition that $\hat{u}_1,\hat{u}_3,\hat{\varphi}\to 0$ as $x_3\to\infty$ the solution of the equation (25) may be written as

$$\hat{u}_1=A_1e^{-\lambda_1x_3}+A_2e^{-\lambda_2x_3}+A_3e^{-\lambda_3x_3}, \tag{20}$$

$$\hat{u}_3=d_1A_1e^{-\lambda_1x_3}+d_2A_2e^{-\lambda_2x_3}+d_3A_3e^{-\lambda_3x_3}, \tag{21}$$

$$\hat{\varphi}=l_1A_1e^{-\lambda_1x_3}+l_2A_2e^{-\lambda_2x_3}+l_3A_3e^{-\lambda_3x_3}, \tag{22}$$

where

$$d_i=\frac{-\lambda_i^3P^*-\lambda_iQ^*}{\lambda_1^4R^*+\lambda_1^2S^*+T^*}\quad i=1,2,3 \tag{23}$$

$$l_i=\frac{\lambda_i^2P^{**}+Q^{**}}{\lambda_1^4R^*+\lambda_1^2S^*+T^*}\quad i=1,2,3 \tag{24}$$

where $P^*=i\xi\{(-\zeta_1p_5a_3s^2+\delta_2(\zeta_3a_3s^2-p_3)\}$, $Q^*=\delta_2(\xi^2-\zeta_3s^2-\zeta_3s^2a_1\xi^2)+p_5\zeta_1(1+a_1\xi^2)s^2$, $R^*=-\zeta_2p_5a_3s^2+\delta_4(\zeta_3a_3s^2-p_3)$, $S^*=(\xi^2-\zeta_3s^2-\zeta_3s^2a_1\xi^2)\delta_4-(\delta_1\xi^2+s^2)(a_3\zeta_3s^2-p_3)+\zeta_2p_5s^2(1+a_1\xi^2)$, $T^*=-(\delta_1\xi^2+s^2)(\xi^2-\zeta_3s^2-\zeta_3s^2a_1\xi^2)$, $P^{**}=(\zeta_2\delta_2-\zeta_1\delta_4)s^2i\xi$, $Q^{**}=\zeta_1s^2(\delta_1\xi^2+s^2)$.

4. Boundary conditions

We consider a normal line load F_1 per unit length acting in the positive x_3 axis on the plane boundary $x_3=0$ along the x_2 axis and a tangential load F_2, per unit length, acting at the origin in the positive x_1 axis. The boundary conditions are

(1) $t_{33}(x_1,x_3,t)=-F_1\psi_1(x)H(t)$, (2) $t_{31}(x_1,x_3,t)=-F_2\psi_2(x)H(t)$,

(3) $\frac{\partial\varphi(x_1,x_3,t)}{\partial x_3}=0$, (25)

where F_1 and F_2 are the magnitudes of the forces applied, $\psi_1(x)$ and $\psi_2(x)$ specify the vertical and horizontal load distribution functions respectively along x_1 axis, $H(t)$ is the Heaviside unit step function (Fig. 2).

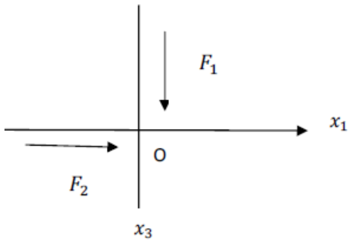

Fig. 1. Normal and tangential loadings.

Substituting the values of $\hat{u}_1$, $\hat{u}_3$ and $\hat{\varphi}$ from equations (20) – (22) in boundary conditions (25) and with the aid of (1), (4)-(6), (13), (17), and (18), we obtain the components of displacement, normal stress, tangential stress and conductive temperature as

$$\widehat{\widehat{u_1}} = \frac{F_1\widehat{\widehat{\psi_1}}(\xi)}{s\Delta}(-M_{11} + M_{12} - M_{13}) + \frac{F_2\widehat{\widehat{\psi_2}}(\xi)}{s\Delta}(M_{21} - M_{22} + M_{23}), \tag{26}$$

$$\widehat{\widehat{u_3}} = \frac{F_1\widehat{\widehat{\psi_1}}(\xi)}{s\Delta}(-d_1M_{11} + d_2M_{12} - d_3M_{13}) + \frac{F_2\widehat{\widehat{\psi_2}}(\xi)}{s\Delta}(d_1M_{21} - d_2M_{22} + d_3M_{23}), \tag{27}$$

$$\hat{\hat{\varphi}} = \frac{F_1\widehat{\widehat{\psi_1}}(\xi)}{s\Delta}(-l_1M_{11} + l_2M_{12} - l_3M_{13}) + \frac{F_2\widehat{\widehat{\psi_2}}(\xi)}{s\Delta}(l_1M_{21} - l_2M_{22} + l_3M_{23}), \tag{28}$$

$$\widehat{\widehat{t_{33}}} = \frac{F_1\widehat{\widehat{\psi_1}}(\xi)}{s\Delta}(-\Delta_{11}M_{11} + \Delta_{12}M_{12} - \Delta_{13}M_{13}) + \frac{F_2\widehat{\widehat{\psi_2}}(\xi)}{s\Delta}(\Delta_{11}M_{21} - \Delta_{12}M_{22} + \Delta_{13}M_{23}), \tag{29}$$

$$\widehat{\widehat{t_{31}}} = \frac{F_1\widehat{\widehat{\psi_1}}(\xi)}{s\Delta}(-\Delta_{21}M_{11} + \Delta_{22}M_{12} - \Delta_{23}M_{13}) + \frac{F_2\widehat{\widehat{\psi_2}}(\xi)}{s\Delta}(\Delta_{21}M_{21} - \Delta_{22}M_{22} + \Delta_{23}M_{23}), \tag{30}$$

where

$$M_{11} = \Delta_{22}\Delta_{33} - \Delta_{32}\Delta_{23},\ M_{12} = \Delta_{21}\Delta_{33} - \Delta_{23}\Delta_{31},\ M_{13} = \Delta_{21}\Delta_{32} - \Delta_{22}\Delta_{31},$$

$$M_{21} = \Delta_{12}\Delta_{33} - \Delta_{13}\Delta_{22},\ M_{22} = \Delta_{11}\Delta_{33} - \Delta_{13}\Delta_{31},\ M_{23} = \Delta_{11}\Delta_{32} - \Delta_{12}\Delta_{31}$$

$$\Delta_{1j} = \frac{c_{31}}{\rho c_1^2}i\xi - \frac{c_{33}}{\rho c_1^2}d_j\lambda_j - \frac{\beta_3}{\beta_1}l_j + \frac{\beta_3}{\beta_1 T_0}a_3l_j{\lambda_j}^2 - \frac{\beta_3}{\beta_1}l_ja_1\xi^2 \qquad j = 1,2,3,$$

$$\Delta_{2j} = -\frac{c_{44}}{\rho c_1^2}\lambda_j + \frac{c_{44}}{\rho c_1^2}i\xi d_j \quad j = 1,2,3,\ \Delta_{3j} = \lambda_jl_j \quad j = 1,2,3,\ \Delta = \Delta_{11}M_{11} - \Delta_{12}M_{12} + \Delta_{13}M_{13}.$$

4.1. Concentrated force. The solution due to concentrated normal force on the half space is obtained by setting

$$\psi_1(x) = \delta(x), \qquad (x) = \delta(x), \tag{31}$$

where $\delta(x)$ is dirac delta function. Applying Laplace and Fourier transform defined by (17)-(18) and (31) ,we obtain

$$\widehat{\psi_1(\xi)} = 1, \qquad \widehat{\psi_2(\xi)} = 1. \tag{32}$$

Using (32) in (26)- (30) , we obtain the components of displacement, stress and conductive temperature.

4.2. Uniformly distributed force. The solution due to uniformly distributed force applied on the half space is obtained by setting

$$\{\psi_1(x),\ \psi_2(x)\} = \begin{cases} 1 \ if\ \ |x| \le m \\ 0 \ if\ |x| > m \end{cases}, \tag{33}$$

The Laplace and Fourier transforms of $\psi_1(x)$ and $\psi_2(x)$ with respect to the pair (x,ξ) for the case of a uniform strip load of non-dimensional width 2 m applied at origin of co-ordinate system $x_1 = x_3 = 0$ is given by

$$\{ \widehat{\psi_1(\xi)}, \ \widehat{\psi_2(\xi)}\} = \left[2\sin(\xi m) \big/ \xi \right] \quad \xi \neq 0. \tag{34}$$

Using (34) in (26)-(30), yield components of displacement, stress and conductive temperature.

4.3. Linearly distributed force. The solution due to linearly distributed force applied on the half space is obtained by setting

$$\{\psi_1(x), \psi_2(x)\} = \begin{cases} 1 - \frac{|x|}{m} \ if \ |x| \leq m \\ 0 \ if \ |x| > m \end{cases}. \tag{35}$$

Here 2 m is the width of the strip load , using (13) and applying the transform defined by (17) and (18) on (35), we obtain

$$\{ \widehat{\psi_1(\xi)}, \ \widehat{\psi_2(\xi)}\} = \left[2\{1 - \cos(\xi m) \big/ \xi^2 m \right] \quad \xi \neq 0. \tag{36}$$

Using (36) in (26)-(30), we obtain components of displacement, stress and conductive temperature.

5. Applications

Inclined line load. Suppose an inclined load F_0, per unit length is acting on the x_2 axis and its inclination with x_3 axis is δ, we have (see Fig. 2)

$$F_1 = F_0 cos\delta, \qquad F_2 = F_0 sin\delta. \tag{37}$$

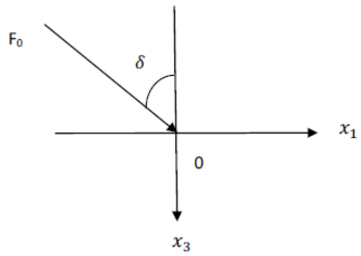

Fig. 2. Inclined load over a thermoelastic solid.

Using equation (37) in equations (26) –(30) and with aid of equations (31)-(36) we obtain the expressions for displacements, and stresses and conductive temperature for concentrated force, uniformly distributed force and linearly distributed force on the surface of anisotropic thermoelastic without energy dissipation.

6. Particular cases

(i) If $a_1 = a_3 = 0$, from equations (26)-(30) , we obtain the corresponding expressions for displacements, and stresses and conductive temperature for transversely isotropic thermoelastic solid without two temperature and without energy dissipation.

(ii) If we take $c_{11} = \lambda + 2\mu = c_{33}$, $c_{12} = c_{13} = \lambda$, $c_{44} = \mu$, $\beta_1 = \beta_3 = \beta$, $\alpha_1 = \alpha_3 = \alpha$, $K_1 = K_3 = K$ in equations (26)-(30) , we obtain the corresponding expressions for displacements, and stresses and conductive temperature in isotropic thermoelastic solid with two temperature and without energy dissipation.

7. Inversion of the Transformation

To obtain the solution of the problem in physical domain, we must invert the transforms in equations (26)-(30). Here the displacement components, normal and tangential stresses and conductive temperature are functions of x_3 , the parameters of Laplace and Fourier transforms s and ξ respectively and hence are of the form $f\,(\xi,\ x_3\ ,\ s)$. To obtain the function $f(x_1,x_3,t)$ in the physical domain, we first invert the Fourier transform using

$$\overline{f}(x_1,x_3,s)=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-i\xi x_1}\hat{f}(\xi,x_3,s)d\xi=\frac{1}{2\pi}\int_{-\infty}^{\infty}|\cos(\xi x_1)\,f_e-isin(\xi x_1)\mathrm{f_o}|\,d, \qquad (38)$$

where f_e and f_o are respectively the odd and even parts of $\hat{f}$ (ξ,x_3,s). Thus the expression (38) gives the Laplace transform $\overline{f}(x_1,x_3,s)$ of the function $f(x_1,x_3,t)$. Following Honig and Hirdes [7], the Laplace transform function $\overline{f}(x_1,x_3,s)$ can be inverted to $f(x_1,x_3,t)$.

The last step is to calculate the integral in equation (38). The method for evaluating this integral is described in Press et al. [15]. It involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

8. Numerical results and discussion

Copper material is chosen for the purpose of numerical calculation which is transversely isotropic

$c_{11}=18.78\times10^{10}\ Kgm^{-1}s^{-2}$, $c_{12}=8.76\times10^{10}\ Kgm^{-1}s^{-2}$, $c_{13}=8.0\times10^{10}\ Kgm^{-1}s^{-2}$, $c_{33}=17.2\times10^{10}\ Kgm^{-1}s^{-2}$, $c_{44}=5.06\times10^{10}\ Kgm^{-1}s^{-2}$, $C_E=0.6331\times10^{3}JKg^{-1}K^{-1}$, $\alpha_1=2.98\times10^{-5}K^{-1}$, $\alpha_3=2.4\times10^{-5}K^{-1}$, $\rho=8.954\times10^{3}Kgm^{-3}$, $K_1=0.433\times10^{3}Wm^{-1}K^{-1}$, $K_3=0.450\times10^{3}Wm^{-1}K^{-1}$.

A comparison of values of normal displacement u_3, normal force stress t_{33}, tangential stress t_{31} and conductive temperature φ for a transversely isotropic thermoelastic solid with distance x has been made between with two temperature and without two temperature and is presented graphically for non-dimensional two temperature parameters a_1=0.03 and a_3=0.06 at $\delta=0°$ (initial angle), $\delta=45°$ (intermediate angle) and $\delta=90°$ (extreme angle) in Figs. 3-15.

1). The solid line, small dashed line, long dashed line respectively corresponds to a_1=0, a_3=0 with angle of inclination $\delta=0°$, $\delta=45°$ and $\delta=90°$ (without two temperature).

2). The solid line with centre symbol circle, the small dashed line with centre symbol diamond, the long dashed line with centre symbol cross respectively corresponds to a_1=0.03, a_3=0.06 and at angle of inclination $\delta=0°$, $\delta=45°$ and $\delta=90°$ (with two temperature).

Concentrated force. Near the loading surface, the values of normal displacement u_3 (Fig. 3) for a_1=0.03, a_3=0.06 is greater than a_1=0, a_3=0 for intermediate angle and extreme angle in the range 0 ≤ x ≤ 2 and 5 ≤ x ≤ 6. However in the remaining range, the behaviour is opposite. In Figure 4 for both intermediate and extreme angle, the values of normal stress t_{33}, for a_1=0, a_3=0 is greater than for a_1=0.03, a_3=0.06, except in the range 5 ≤ x ≤ 7. The values of tangential stress t_{31}, for a_1=0.03, a_3=0.06 at angle of inclination $\delta=45°$ and $\delta=90°$ are more than a_1=0.03, a_3=0.06 in the whole range. In Figure 6 the values of conductive temperature φ, have an oscillatory behaviour corresponding to both the cases at $\delta=0°$, $\delta=45°$ and $\delta=90°$. The behaviour of for a_1=0.03, a_3=0.06 and a_1=0, a_3=0 at initial angle is oscillatory as is depicted in Figs. 3-6.

Uniformly distributed force. It is depicted from Fig. 7, that the distribution curves for u_3 have same trend irrespective of angle of inclination i.e. for both intermediate as well as extreme , they move in the similar pattern. In the whole range, the values of u_3 for a_1 = 0 =

a_3 is greater than a_1=0.03, a_3=0.06 for $\delta = 45°$ and $\delta = 90°$. But at the initial angle, the variations are very small. In Figure 8 the values of normal stress t_{33} , at a_1 = 0 = a_3 are greater than for a_1=0.03, a_3=0.06 in the whole range for $\theta = 45°$ and $\theta = 90°$. Due to scale of graph the variations at θ =0^0 are not clear. The effect of temperature increases the magnitude of tangential stress t_{31} , i.e. the values for a_1=0.03, a_3=0.06 are greater than for a_1 = 0 = a_3 in the whole range as depicted in Fig. 9. At the initial angle, the pattern is oscillatory. In Figure 10 the values of the conductive temperature φ are increased in magnitude by taking a_1=0.03, a_3=0.06 at the initial angle as compared with a_1 = 0 = a_3. But at middle angle and at extreme angle behaviour is ascending oscillatory.

Linearly Distributed force. In Figure 11 the values of normal displacement u_3, for both the temperatures have oscillatory behaviour with same pattern for $\theta = 45°$ and $\theta = 90°$ in the range $0 \leq x \leq 6$ but it is opposite in the rest of the range. The values of normal stress t_{33}, for a_1=0= a_3 are greater than for the temperature parameter a_1=0.03, a_3=0.06 at angles of inclination $\theta = 90°$ and at $\theta = 45°$ as shown in Fig. 12. However at initial angle in the Figs. 9-12 small variations are depicted in values of normal displacement u_3, normal stress t_{33}, tangential stress t_{31} and conductive temperature φ. The values of tangential stress t_{31}, for a_1 = 0 = a_3 are smaller in magnitude than a_1=0.03, a_3=0.06 at both intermediate as well as extreme angle in the whole range, as is depicted in Fig. 13. The values of the conductive temperature corresponding to both the temperatures have oscillatory behaviour with change in magnitude except at initial angle.

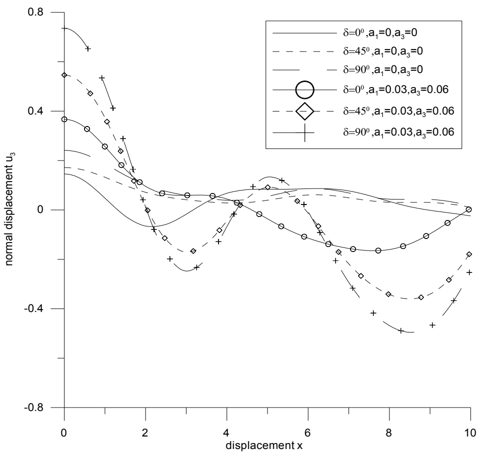

Fig. 3. Variation of normal displacement u_3 with displacement *x* (concentrated force).

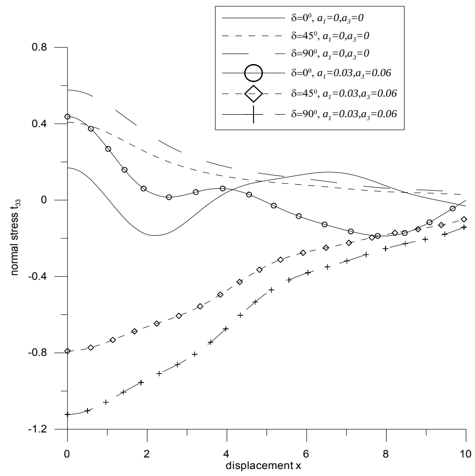

Fig. 4. Variation of normal stress t_{33} with displacement *x* (concentrated force).

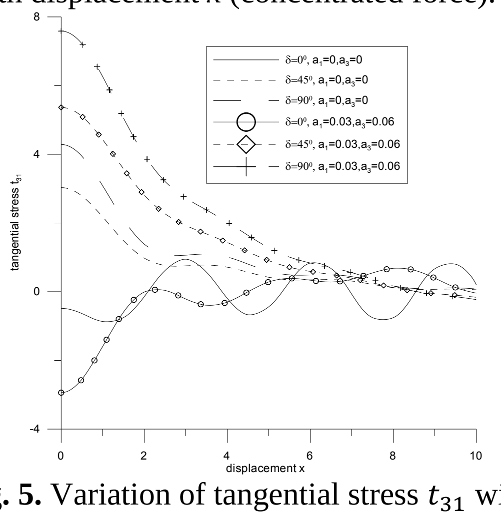

Fig. 5. Variation of tangential stress t_{31} with displacement *x* (concentrated force).

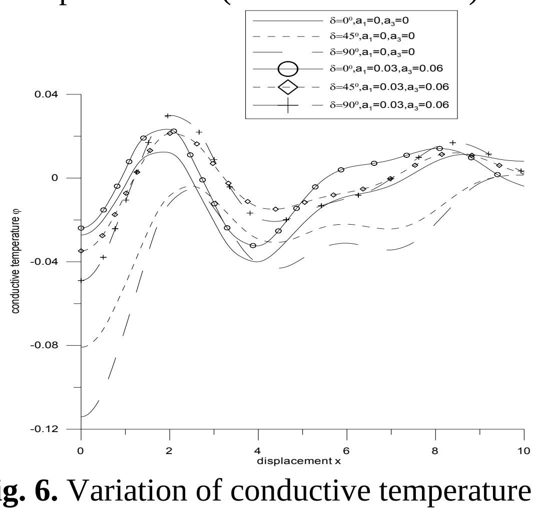

Fig. 6. Variation of conductive temperature φ with displacement *x* (concentrated force).

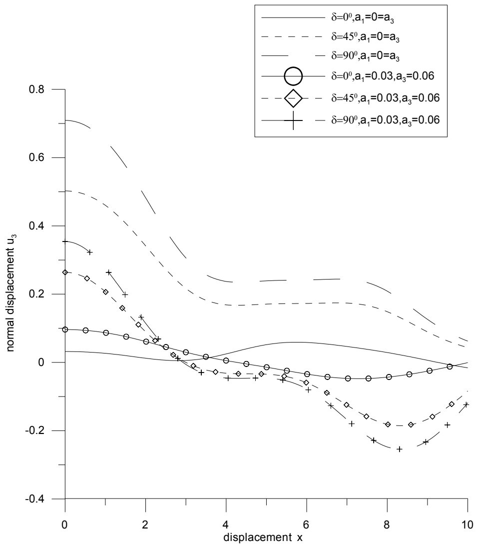

Fig. 7. Variation of normal displacement u_3 with displacement *x* (uniformly distributed force).

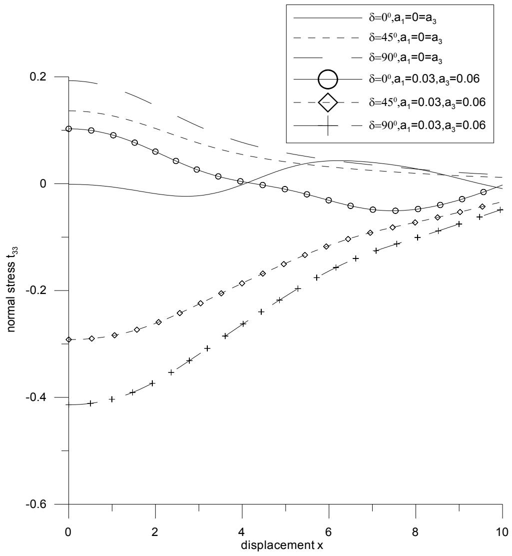

Fig. 8. Variation of normal stress t_{33} with displacement *x* (uniformly distributed force).

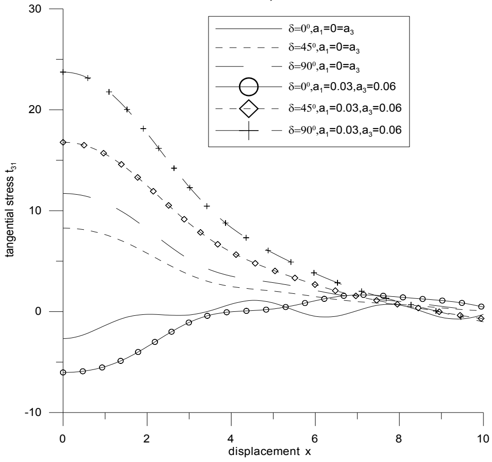

Fig. 9. Variation of tangential stress t_{31} with displacement *x* (uniformly distributed force).

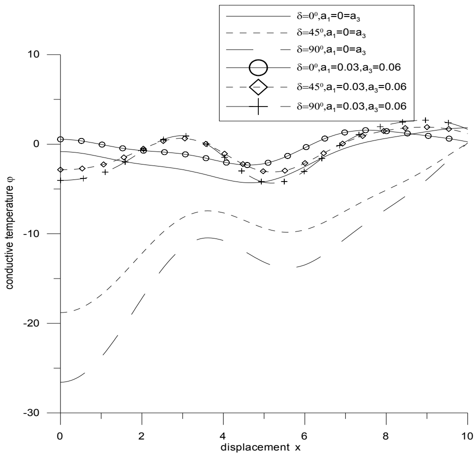

Fig. 10. Variation of conductive temperature φ with displacement *x* (uniformly distributed force).

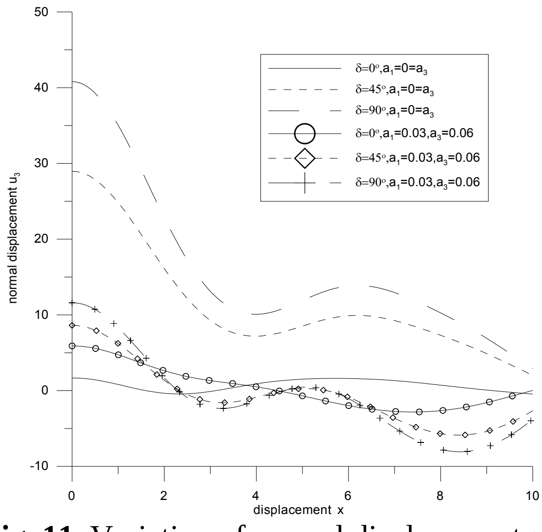

Fig. 11. Variation of normal displacement u_3 with displacement *x* (linearly distributed force).

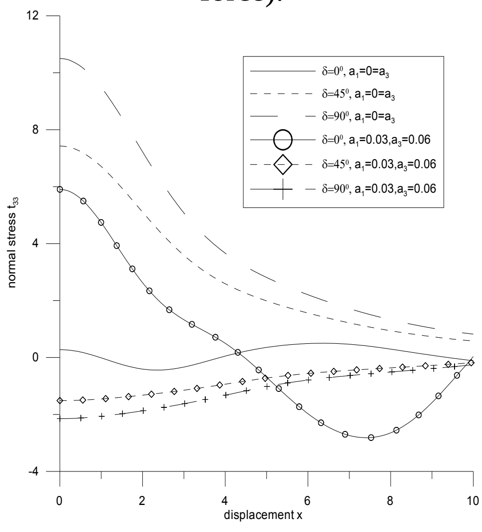

Fig. 12. Variation of normal stress t_{33} with displacement *x* (linearly distributed force).

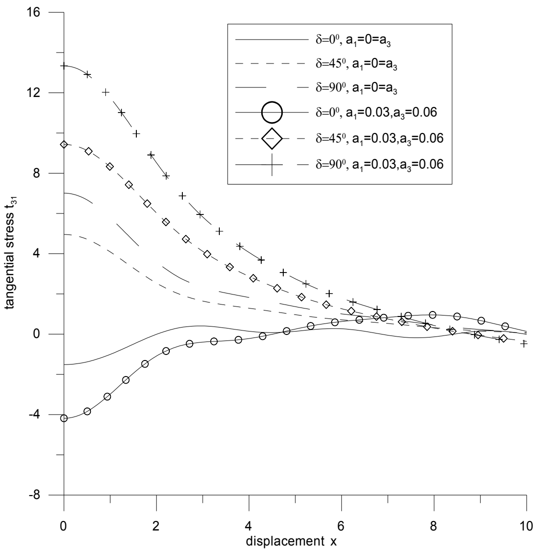

Fig. 13. Variation of tangential stress t_{31} with displacement *x* (linearly distributed force).

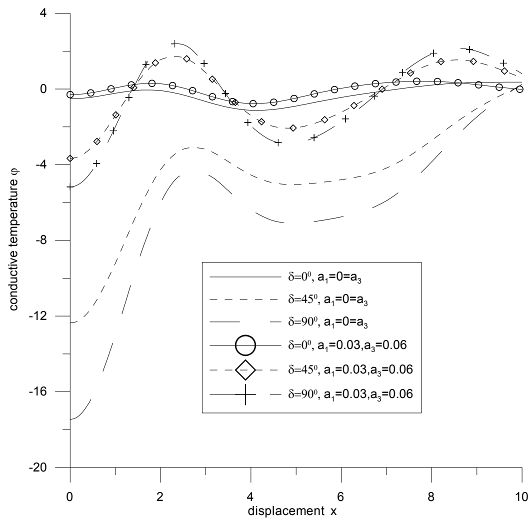

Fig. 14. Variation of conductive temperature φ with displacement *x* (linearly distributed force).

9. Conclusion

Effect of two temperature have significant impact on components of normal displacement, normal stress, tangential stress and conductive temperature. As disturbance travels through the constituents of the medium, it suffers sudden changes resulting in an inconsistent / non uniform pattern of graphs. The deformation in any part of the medium is useful to analyse the deformation field around mining tremors and drilling into the crust. It can also contribute to the theoretical consideration of the seismic and volcanic sources. Since it can account for the deformation fields in the entire volume surrounding the sources region.

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