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FREQUENCY ANALYSIS OF A GRAPHENE SHEET EMBEDDED IN AN ELASTIC MEDIUM WITH CONSIDERATION OF SMALL SCALE

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Abstract. The effect of length scale on the vibration response of a single-layer graphene sheet embedded in an elastic medium is studied using nonlocal Mindlin plate theory. The elastic medium is modeled using both Winkler-type and Pasternak-type elastic foundations. An explicit solution is derived for the natural frequencies of the graphene sheet. Through the analytical solution it is found that the vibration response of graphene sheet concerning the length scale effects considerably different from the results obtained by the classical theories. In comparison with the classical plate theory, the nonlocal model showed that the natural frequency of the graphene sheet decreases for smaller lengths of graphene sheet, higher aspect ratios, greater values of nonlocal parameter and stiffer elastic foundations.

Nomenclature

a	material constant depending on the internal length (nm),
b	graphene sheet width (nm),
e_0	calibrating constant suitable to each material,
h	graphene sheet thickness (nm),
l	graphene sheet length (nm),
k_w, k_s	Winkler modulus, shear modulus of the surrounding elastic medium,
m	half wave number,
n	half wave number,
p	force per unit area,
t	time (s),
u, v	displacement of the point (x, y, 0) of graphene sheet along x and y-axis (nm),
w	deflection of the graphene sheet at point (x, y) calculated (nm),
C_{ijkl}	classical stress tensor,
D	bending rigidity of graphene sheet,
E	Young's modulus (N/m^2),
G	shear modulus (N/m^2),
M_{xx}, M_{yy}	resultant moments (Nm),
M_{xy}	twisting moment (Nm),
N_{xx}	in-plane force (N),
Q_x, Q_y	transverse shear forces (N),
ε_{kl}, ε_{xx}, ε_{yy}, ε_{xy}, ε_{xz}, ε_{yz}, ε_{zz}	linear strain tensor,
κ^2	transverse shear correction coefficient,
ν	Poisson's ratio,
ρ	mass density (kg/m^3),
σ_{ij}, σ_{xx}, σ_{yy}, σ_{xy}, σ_{xz}, σ_{yz}	nonlocal stress tensor (N/m^2),
τ	nonlocal parameter,

ψ_x, ψ_y	rotational displacement (nm),
ψ_z	transverse displacement (nm),
ω_{nm}	related order natural frequency of the transverse vibration,
∇^2	Laplacian operator in 2D Cartesian coordinate system,

1. Introduction

Since the discovery of carbon nanostructures, extensive research studies have been conducted for characterizing the mechanical, chemical and electronical properties of such structures [1, 2]. A large number of research communities have reported properties of several allotropes of carbon such as diamond and graphite (3D), graphene sheets (2D), nanotubes (1D) and fullerenes (0D) [3, 4, 29-31]. Graphene is a two dimensional single layer of sp2 bonded carbon atoms densely packed in a honeycomb crystal structure. Graphene sheets have been recognized as attractive materials in nanoelectronics, nanosensors and micromechanical resonators due to the especial electronical and mechanical properties originating from their distinguished form and size [5, 6, 29-31]. In recent years, the investigation of mechanical behavior of graphene sheets has become an interesting subject and hence a few techniques and experimental methods have been used for investigating the properties of graphene sheets [7]. Accordingly, several studies have been reported in the literature using both molecular dynamics simulation and continuum mechanics models [8-10] for characterizing the viberational behavior of graphene sheets. For example, Sakhaee-pour *et al.* [8] modeled the vibrational behavior of defect-free single-layered graphene sheets at constant temperature using molecular structural mechanics. Using the concept of classical plate theory, Behfar and Naghdabadi [9] analyzed also the vibrational behavior of a multi-layered graphene sheet embedded in an elastic medium. However, the classical models have some drawbacks which restrict their generality and accuracy. For example, molecular dynamics simulations are purely computational and also the classical theories are not able to consider intrinsic size dependence in the elastic solutions of nanoscaled structures [11, 12]. Therefore, the modified form of the classical continuum theory (called the nonlocal elasticity theory) that takes into account the size effect of nanostructure materials, may provide a more accurate model to deal with small scale influences [13, 14]. Recently, bending, vibration, and buckling of graphene sheets have been investigated using nonlocal elasticity theory [15-17, 32-44]. For example, a nonlocal plate model was formulated by Lu et al. [16] to examine the length scale effect on bending and free vibration behavior of the Kirchhoff and the Mindlin plates. Murmu and Pradhan [18] investigated the vibration response of nanoplates under uniaxially pre-stressed conditions using nonlocal elasticity theory. In addition to the carbon nanostructures which are often used as embedded parts in elastic mediums [9, 19-21], graphene sheets can be also employed in manufacturing of the polymer composites to increase the strength of such materials. Pradhan and Phadikar [19] studied the vibration response of multilayered graphene sheets embedded in polymer matrix using nonlocal continuum plate theory and showed that the nonlocal effect is quite significant in the continuum model of graphene sheet.

In this paper, the vibration of a single-layer graphene sheet embedded in an elastic medium and in the presence of length scale is studied based on the nonlocal Mindlin plate model. Analytical solutions for the graphene sheet with all the four edges simply supported are derived by solving the governing equations and considering the boundary conditions by Navier's approach. The influence of nonlocal parameter on the nondimensional frequency response of the graphene sheets with different parameters such as aspect ratio, vibrational mode and the length of graphene sheet are discussed in detail.

2. Nonlocal elasticity plate model

The basic concept of nonlocal elasticity, proposed by Eringen [14], is applied here for a rectangular plate model to predict the natural frequency of a single-layer graphene sheet embedded in an elastic medium. As shown in Fig. 1, the graphene sheet is modeled as a moderately thick rectangular plate resting on two-parameter elastic foundation with the elastic modulus E, Poisson's ratio ν, uniform thickness h, length l and width b. A coordinate system (x, y, z) is also defined in Fig. 1 with x, y and z axes along the length, width and thickness of the graphene sheet, respectively.

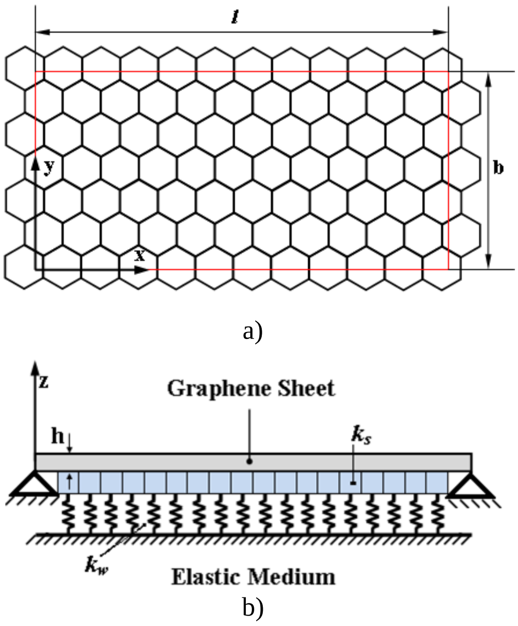

Fig. 1. Geometry of the considered graphene sheet embedded in elastic medium.

On the basis of the Mindlin plate theory, the governing differential equations of motion for the free vibration of the graphene sheet can be explained as follows [22]

$$M_{xx,x} + M_{xy,y} - Q_x = -\frac{1}{12}\rho h^3 \ddot{\psi}_x, \quad (1a)$$

$$M_{yy,y} + M_{xy,x} - Q_y = -\frac{1}{12}\rho h^3 \ddot{\psi}_y, \quad (1b)$$

$$Q_{x,x} + Q_{y,y} - p = \rho h \ddot{\psi}_z, \quad (1c)$$

where M_{xx} and M_{yy} are the resultant moments, M_{xy} is the twisting moment, and Q_x and Q_y are the transverse shear forces. All the aforementioned moments are per unit length. ψ_x and ψ_y are also the rotational displacements about y and x axes, respectively and ψ_z is the transverse displacement. In equation (1), t indicates also the time, ρ is the mass density and p is the force per unit area applied to the graphene sheet. It should be noted that in this paper, the symbol "," is used to show derivative operator. For example, $M_{xy,x}$ is equal to $\partial M_{xy}/\partial x$. Nonlocal elasticity theory states that the stress at each point in an elastic continuum medium depends not only on the strain of the same point but also on the strains at all other points in the domain [13, 14]. The nonlocal constitutive equations for a three-dimensional problem can be expressed as

$$(1 - \tau^2 l^2 \nabla^2)\sigma_{ij} = C_{ijkl}\varepsilon_{kl}, \quad (2)$$

where $\nabla^2, \sigma_{ij}, C_{ijkl}$ and ε_{kl} are the Laplacian operator, the stress tensor of the nonlocal elasticity, the classical stress tensor and the linear strain tensor, respectively. τ is also a material constant (called the nonlocal parameter) which depends on the internal length a [23] (such as the $C - C$ bond length, lattice parameter, granular size) and the external length l

(such as the graphene sheet length) of the system.

Using equation (2), nonlocal stress constitutive relations can be written as:

$$\sigma_{xx} - (e_0 a)^2 \left(\frac{\partial^2 \sigma_{xx}}{\partial x^2} + \frac{\partial^2 \sigma_{xx}}{\partial y^2}\right) = \frac{E}{(1-\nu^2)}(\varepsilon_{xx} + \nu \varepsilon_{xx}), \tag{3a}$$

$$\sigma_{yy} - (e_0 a)^2 \left(\frac{\partial^2 \sigma_{yy}}{\partial x^2} + \frac{\partial^2 \sigma_{yy}}{\partial y^2}\right) = \frac{E}{(1-\nu^2)}(\varepsilon_{yy} + \nu \varepsilon_{xx}), \tag{3b}$$

$$\sigma_{xy} - (e_0 a)^2 \left(\frac{\partial^2 \sigma_{xy}}{\partial x^2} + \frac{\partial^2 \sigma_{xy}}{\partial y^2}\right) = 2G\varepsilon_{xy}, \tag{3c}$$

$$\sigma_{xz} - (e_0 a)^2 \left(\frac{\partial^2 \sigma_{xz}}{\partial x^2} + \frac{\partial^2 \sigma_{xz}}{\partial y^2}\right) = 2G\varepsilon_{xz}, \tag{3d}$$

$$\sigma_{yz} - (e_0 a)^2 \left(\frac{\partial^2 \sigma_{xz}}{\partial x^2} + \frac{\partial^2 \sigma_{xz}}{\partial y^2}\right) = 2G\varepsilon_{yz}. \tag{3e}$$

Here G and e_0 are shear modulus and calibrating constant suitable to each material, respectively. For the bending analysis of micro/nanoplates using nonlocal elasticity theory the value of scale coefficient or nonlocal parameter ($e_0 a$ or τ) has been considered in the range between 0 and 2 nm [17]. The general strain-displacement relations are expressed as

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + z\frac{\partial \psi_x}{\partial x}, \tag{4a}$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} + z\frac{\partial \psi_y}{\partial y}, \tag{4b}$$

$$\varepsilon_{zz} = 0, \tag{4c}$$

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + \frac{z}{2}\left(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x}\right), \tag{4d}$$

$$\varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + \psi_x\right), \tag{4e}$$

$$\varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial w}{\partial y} + \psi_y\right), \tag{4f}$$

where u, v, w are the displacements of a given point in the graphene sheet. The stress resultant-displacement relations have been also given in [24] as:

$$M_{ij} = \int_{-h/2}^{h/2} \sigma_{ij} z dz \ , \qquad i,j = x,y \tag{5a}$$

$$Q_k = \kappa^2 \int_{-h/2}^{h/2} \sigma_{f,z} dz \ , \qquad f = x,y \tag{5b}$$

where κ^2 is the transverse shear correction coefficient. Using equations (4a) to (4f) and (5a,b), the resultant bending moments, the twisting moment, and the transverse shear forces per unit length can be obtained from

$$M_{xx} - (e_0 a)^2 [M_{xx,xx} + M_{xx,yy}] = D(\psi_{x,x} + \nu \psi_{y,y}), \tag{6a}$$

$$M_{yy} - (e_0 a)^2 [M_{yy,yy} + M_{yy,xx}] = D(\psi_{y,y} + \nu \psi_{x,x}), \tag{6b}$$

$$M_{xy} - (e_0 a)^2 [M_{xy,xx} + M_{xy,yy}] = \frac{D}{2}(1-\nu)(\psi_{x,y} + \nu \psi_{y,x}), \tag{6c}$$

$$Q_x - (e_0 a)^2 [Q_{x,xx} + Q_{x,yy}] = \kappa^2 Gh\left(\frac{\partial w}{\partial x} + \psi_x\right), \tag{6d}$$

$$Q_y - (e_0 a)^2 [Q_{y,xx} + Q_{y,yy}] = \kappa^2 Gh\left(\frac{\partial w}{\partial y} + \psi_y\right), \tag{6e}$$

where $D = Eh^3/12(1-\nu^2)$ is the bending rigidity of the graphene sheet. By ignoring the effect of nonlocal parameter $(e_0 a)$ in equations (6a) to (6e), the stress-resultant displacement relations will be identical to the relations obtained from the classical (or local) plate model. In addition, the in-plane edge load provides a force component for a deflected plate in transverse direction as:

$$p = N_{xx}\psi_{z,xx}, \tag{7}$$

where N_{xx} is the in-plane edge load per unit length. Consequently, by substituting equations (6a) to (6e) and (7) into equations (1a) to (1c), the governing differential equations of motion using nonlocal Mindlin plate theory are derived as follows:

$$\psi_{x,xx} + \psi_{x,yy} + \left(\frac{1+\nu}{1-\nu}\right)\left[\psi_{x,xx} + \psi_{y,xy}\right] - \left(\frac{2\kappa^2 Gh}{D(1-\nu)}\right)\left[\frac{\partial w}{\partial x} + \psi_x\right]$$

$$-\frac{1}{12}\rho h^3 \psi_{x,tt} + \frac{(e_0 a)^2}{12}\rho h^3\left[\psi_{x,xxtt} + \psi_{x,yytt}\right] = 0, \tag{8a}$$

$$\psi_{y,xx} + \psi_{y,yy} + \left(\frac{1+\nu}{1-\nu}\right)\left[\psi_{y,yy} + \psi_{x,xy}\right] - \left(\frac{2\kappa^2 Gh}{D(1-\nu)}\right)\left[\frac{\partial w}{\partial y} + \psi_y\right]$$

$$-\frac{1}{12}\rho h^3 \psi_{y,tt} + \frac{(e_0 a)^2}{12}\rho h^3\left[\psi_{y,xxtt} + \psi_{y,yytt}\right] = 0, \tag{8b}$$

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \left[\psi_{x,x} + \psi_{y,y}\right] + N_{xx}\frac{\partial^2 w}{\partial x^2} + k_s\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right]$$

$$- k_w w - \rho h\frac{\partial^2 w}{\partial t^2} + (e_0 a)^2 \rho h\left[\frac{\partial^4 w}{\partial x^2 \partial t^2} + \frac{\partial^4 w}{\partial y^2 \partial t^2}\right] - (e_0 a)^2\left[N_{xx}\frac{\partial^4 w}{\partial x^4} + N_{xx}\frac{\partial^4 w}{\partial x^2 \partial y^2} + \right.$$

$$\left. k_s\left[\frac{\partial^4 w}{\partial x^4} + \frac{\partial^4 w}{\partial y^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} - k_w\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right]\right]\right] = 0, \tag{8c}$$

where $k_s = K_s l^2/D$ and $k_w = K_w l^4/D$ are the dimensionless shear and Winkler foundation coefficients, respectively. It is worth mentioning that the aforementioned governing differential equations are reduced to those of the classical Mindlin plate model when the scale coefficient $(e_0 a)$ becomes equal to zero.

3. Solution using Navier's approach

In Navier's approach, the generalized displacement field is expanded in trigonometric series such that the boundary conditions of the problem are satisfied. In the following, the solution of governing differential equations obtained in previous section for the rectangular plates with simply supported boundary conditions is presented using Navier's method. The simply supported boundary conditions for the plate model are

$$w = 0,\ \psi_y = 0 \ \ and\ \ M_{xx} = 0 \quad at\ x = 0, l \tag{9a}$$

$$w = 0,\ \psi_x = 0 \ \ and\ \ M_{yy} = 0 \quad at\ y = 0, b \tag{9b}$$

The general form of expanded displacement components are assumed to be as

$$\psi_x = \textstyle\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \Psi_{nm}\cos(\zeta_m x)\sin(\eta_n y)\sin\omega_{nm}t, \tag{10a}$$

$$\psi_y = \textstyle\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \Phi_{nm}\sin(\zeta_m x)\cos(\eta_n y)\sin\omega_{nm}t, \tag{10b}$$

$$w = \textstyle\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{nm}\sin(\zeta_m x)\sin(\eta_n y)\sin\omega_{nm}t, \tag{10c}$$

where Ψ_{nm}, Φ_{nm} and W_{nm} are constant coefficients and ζ_m and η_n are defined as $\zeta_m =$

$m\pi/l$ and $\eta_n = n\pi/b$. Also m and n are the half wave numbers. ω_{nm} is also the related natural frequency of the transverse vibration.
By substituting equations (10a) to (10c) into equations (8a) to (8c), one get

$$\begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix} \begin{Bmatrix} \Psi \\ \Phi \\ W \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{11}$$

where coefficients d_{11} through d_{33} are given in Appendix. By setting the determinant of the coefficient matrix $[d]$ equal to zero, the natural frequencies of the single-layer graphene sheet embedded in the elastic medium are obtained from the solution of characteristic equation for each combination of m and n. The lowest value of natural frequency corresponds to the mode where the transverse deflection is dominant, whereas the other two frequency responses are much higher and related to the shear modes [25].

4. Results and discussion

4.1. Comparative studies. Adopting the nonlocal Mindlin plate theory, as an example the vibration of moderately thick rectangular graphene sheet embedded in an elastic medium is investigated in detail for various geometries (i.e. different aspect ratios and nonlocal parameters). Consider a typical single layer graphene sheet with two opposite edges simply supported under uniformly distributed in-plane loads resting on Winkler/Pasternak elastic foundation. The length, width and thickness of graphene sheet are taken as $l = 15\,nm$, $b = 10\,nm$ and $h = 0.34\,nm$, respectively. The following values are also considered for the mechanical properties of the graphene: $E = 1GPa$, $\nu = 0.3$, $\rho = 2300\,kg/m^3$ and $\kappa^2 = 0.86667$. In order to examine the accuracy of the present solution, the frequency ratio defined as:

$$frequency\ ratio = \frac{natural\ frequency\ calculated\ using\ nonlocal\ Mindlin\ plate\ theory}{natural\ frequency\ calculated\ using\ classical\ Mindlin\ plate\ theory} \tag{12}$$

has been compared in Fig. 2 with the micro/nanoplates frequency results reported by Lu *et al*. [16] for the thin and moderately thick rectangular micro/nanoplates and for different values of *n*, *m* and $e_0 a$. *The good agreement that exists between the two sets of results demonstrates the accuracy of the developed nonlocal Mindlin theory for investigating the vibration response of the graphene sheets.*

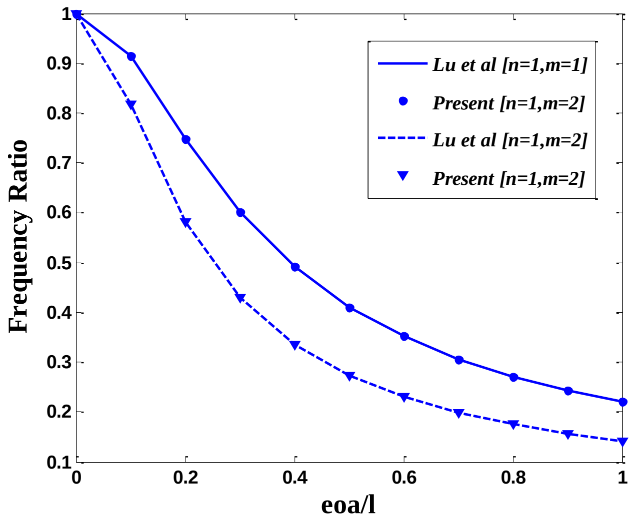

Fig. 2. Comparison between the results obtained in the present study and with those reported by Lu *et al* [20] for frequency ratio against different scale coefficient $(e_0 a/l)$.

4.2. Small scale effect on the frequency ratio of the graphene sheet. After verifying the accuracy of the present analytical solution, the following new results are presented for the graphene sheets under external in-plane loads and resting on elastic foundation. Figures 3 to 7 show the influence of scaling coefficient (e_0a), surrounding elastic medium (k_w and k_s), aspect ratio (l/b), length of graphene sheet (l) and the mode of vibration (m) on the frequency ratio of graphene sheet based on the nonlocal elasticity solutions. It can be demonstrated that by setting $e_0a = 0$ in the governing vibration equations of the nonlocal plate (i.e. equations (8a) to (8c)), the classical elastic plate vibration equations (presented by Hashemi *et al.* [24]) are obtained. In addition, by setting $D = EI$ and $b = 0$ in equations (8a) to (8c) the nonlocal solution for the free vibration of a beam is obtained that is consistent very well with the Reddy's results [26].

Figure 3 shows the variations of the frequency ratio of the graphene sheet (for the first natural mode) with plate length and for different values of the nonlocal parameter (e_0a). The nonlocal parameter was chosen between 0 to $2nm$ because according to Sudak [27], (e_0a) should be smaller than $2\,nm$ for single-walled carbon nanotubes. For plotting the curves of Fig. 3, Winkler and shear modulus parameters were considered to be constant and equal to $k_w = 250$ and $k_s = 0$, respectively and the modes of vibration were assumed as $m = 1$ and $n = 1$. As it is seen from Fig. 3, the length scale coefficient decreases the frequency ratio (i.e. the natural frequency of the graphene sheet). Moreover, according to this figure, while for the small lengths of the plate, the non-local parameter affects significantly the value of natural frequency, its influence becomes negligible for the larger plate lengths (typically $l \geq 20\,nm$). In addition when the nonlocal parameter increases, the natural frequencies of the nonlocal solutions become smaller than those of the classical solutions because of the negative sign of e_0a term in equations (8a) to (8c).

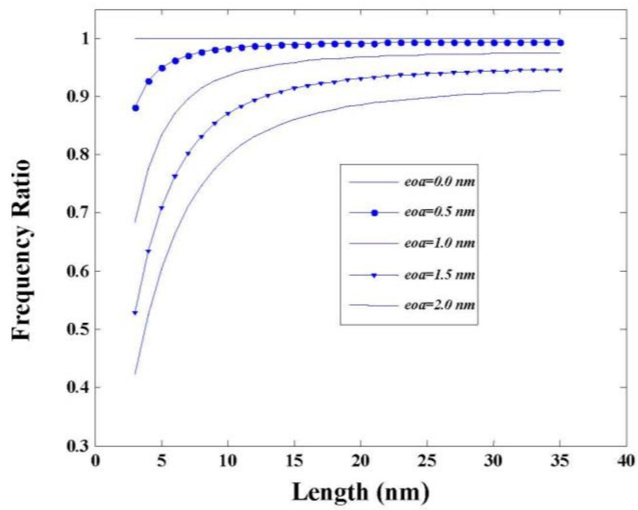

Fig. 3. Variations of frequency ratio with graphene sheet length (l) for different nonlocal parameters.

Figure 4 displays the variations of frequency ratio with the length of graphene sheet for different modes of vibration. It can be seen from Fig. 4 that for any given plate length, the corresponding value of natural frequency of the graphene sheet becomes smaller for the higher frequency mode numbers. The deference between the results of mode numbers increases by decreasing l showing the significant influence of nonlocal solution on the frequency response of graphene sheets having smaller plate lengths. Furthermore, at lower modes of vibration and for the higher lengths of plate all the results converge to the classical frequency of local plate [24] and the influences of small scale effects disappear.

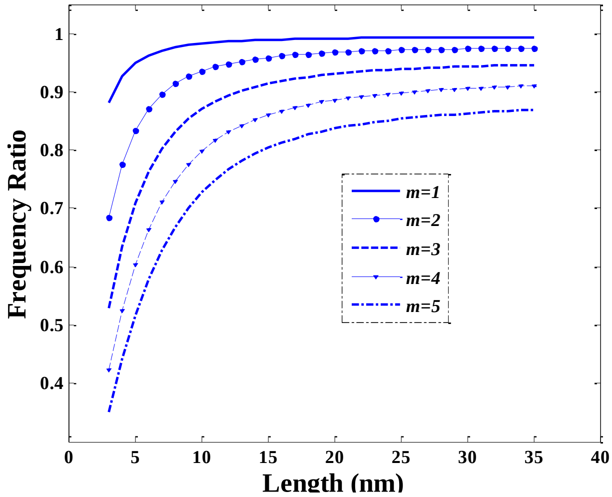

Fig. 4. Variation of frequency ratio with graphene sheet length (l) for different modes of vibration $m, (n = 1$ and $e_0a = 1.5\ nm)$.

4.3. Surrounding elastic medium effect on the frequency ratio of the graphene sheet. In this study, the single-layer graphene sheet embedded in an elastic medium was investigated using a Pasternak model with adding a shear layer to the Winkler model. Several studies have also been performed to predict the mechanical characteristics of carbon nanostructures (such as natural frequency and critical buckling loads) by employing a plate or beam on an elastic foundation [19, 20]. The graphene sheet embedded in an elastic medium can be simulated as a rectangular plate and the surrounding elastic medium with a Winkler-type elastic foundation. When the shear stiffness of the foundation is also taken into account, the two parameter foundation such as Pasternak foundation should be used. To study the effect of surrounding elastic medium on the frequency analysis of the graphene sheet, frequency ratio is analyzed here for different nonlocal parameters, Winkler and shear modulus. The variation of the frequency ratio $(\omega_{nonlocal}/\omega_{local})$ with nonlocal parameter has been displayed in Fig. 5 for different values of foundation stiffness(k_w). For this plot, the shear modulus parameter k_s was considered zero. According to Fig. 5, the effect of Winkler modulus parameter is more pronounced for larger values of nonlocal parameter (typically for $e_0a \geq 0.5\ nm$) such that the frequency ratio decreases dramatically for the stiffer foundations and greater nonlocal parameters.

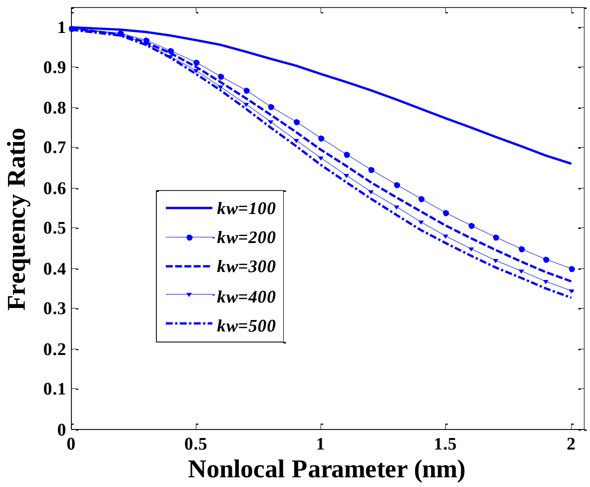

Fig. 5. Influence of small scale effects on the frequency ratio of graphene sheet for various Winkler modulus parameters k_w.

Similarly, Fig. 6 illustrates also the effect of shear modulus parameter k_s on the frequency ratio of the graphene sheet in the presence of nonlocal parameter. The elastic medium was modeled as a Pasternak type foundation model with $k_w = 250$ and the shear modulus parameter was varied as follows: $k_s = 2, 4, 6, 8, 10$. These values for the shear modulus parameter (k_s) were also used by Liew *et al.* [28]. It can be seen that by increasing the shear modulus parameter of elastic medium the frequency ratio of graphene sheet resting on a Pasternak type model foundation decreases for any given nonlocal parameters. Again, reduction in the frequency ratio becomes more noticeable for the greater nonlocal parameters.

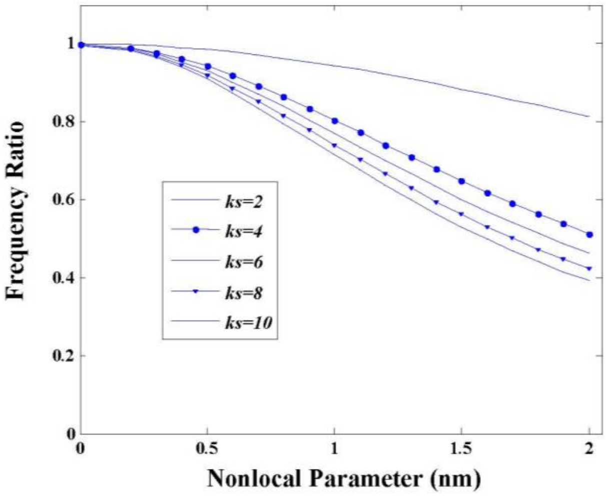

Fig. 6. Influence of small scale effects on the frequency ratio of graphene sheet for various shear modulus parameters k_s.

4.4. Effect of aspect ratio on the frequency ratio of the graphene sheet. The variation of the frequency ratio ($\omega_{nonlocal}/\omega_{local}$) versus the aspect ratio (l/b) for the considered rectangular Mindlin plate (graphene sheet) has been presented in Fig. 7 for different values of e_0a. According to Fig. 7, the results obtained from nonlocal elastic solution are smaller than the corresponding results of the classical solution for the all aspect ratios. In addition the frequency ratio reduces by increasing l/b. It can be also seen that the length scale effects are more pronounced in vibration of rectangular graphene sheets than the strip-type graphene sheets (nanoribbons).

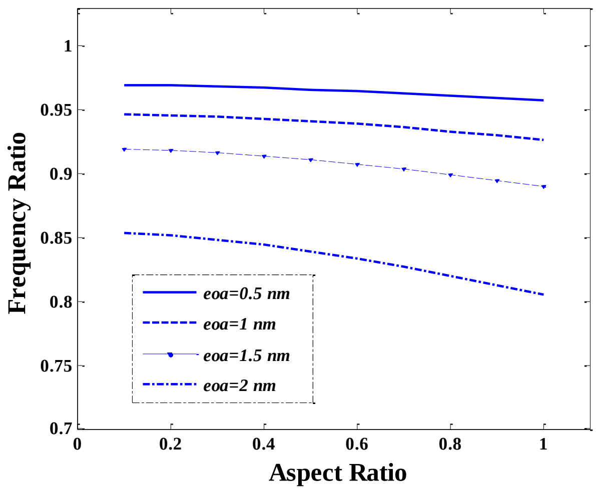

Fig. 7. Variations of frequency ratio with aspect ratio (l/b) for different nonlocal parameters,($m = 1, n = 1$).

5. Conclusions

- The vibration response of a single-layer graphene sheet embedded in an elastic medium was studied using nonlocal Mindlin plate theory.
- An explicit solution which takes into account the influence of length scale and surrounding elastic medium was derived for obtaining the natural frequencies of micro/nanoscaled Mindlin plates (such as graphene sheets).
- It was found that the effect of small scale is more pronounced for higher modes of vibration, greater Winkler and shear modulus parameters and larger length/width aspect ratios.
- The comprehensive model presented in this study provides useful results for vibration design aspects of NEMS devices such as graphene vibrators.

Appendix

Coefficients d_{11} through d_{33} are:

$$d_{11}=\zeta_m^2+\eta_n^2+\frac{2Ghk^2}{D(1-\upsilon)}+\frac{(1+\upsilon)}{(1-\upsilon)}\zeta_m^2-\frac{1}{12}\rho h^3\omega_{nm}^2-\frac{1}{12}\rho h^3\omega_{nm}^2(e_0a)^2(\zeta_m^2+\eta_n^2), \quad (A.1)$$

$$d_{12}=\zeta_m^2\eta_n^2\frac{(1+\upsilon)}{(1-\upsilon)}, \quad (A.2)$$

$$d_{13}=\frac{2Ghk^2}{D(1-\upsilon)}\zeta_m^2, \quad (A.3)$$

$$d_{21}=\zeta_m^2\eta_n^2\frac{(1+\upsilon)}{(1-\upsilon)}, \quad (A.4)$$

$$d_{22}=\zeta_m^2+\eta_n^2+\frac{2Ghk^2}{D(1-\upsilon)}+\frac{(1+\upsilon)}{(1-\upsilon)}\eta_n^2-\frac{1}{12}\rho h^3\omega_{nm}^2-\frac{1}{12}\rho h^3\omega_{nm}^2(e_0a)^2(\zeta_m^2+\eta_n^2), \quad (A.5)$$

$$d_{23}=\frac{2Ghk^2}{D(1-\upsilon)}\eta_n^2, \quad (A.6)$$

$$d_{31}=-\zeta_m^2, \quad (A.7)$$

$$d_{32}=-\eta_n^2, \quad (A.8)$$

$$d_{33}=-\zeta_m^2-\eta_n^2+\rho h\omega_{nm}^2+\rho h\omega_{nm}^2(e_0a)^2(\zeta_m^2+\eta_n^2)-k_s$$
$$-\zeta_m^2N_{xx}-(e_0a)^2[k_w(\zeta_m^4+\eta_n^4+\zeta_m^2\eta_n^2)-k_s(\zeta_m^2-\eta_n^2)+\zeta_m^4N_{xx}+\zeta_m^2\eta_n^2N_{xx}]. \quad (A.9)$$

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