Materials Physics and Mechanics 22 (2015) 136-156 Received: December 1, 2014

NONLOCAL MECHANICS OF NONEQUILIBRIUM SHOCK-WAVE PROCESSES

Yu.I. Meshcheryakov[1*], T.A. Khantuleva[2]

[1]Institute of Problems of Mechanical Engineering RAS, Saint-Petersburg, Russia

[2]Saint-Petersburg State University, Russia

*e-mail: ym38@mail.ru

Abstract. Results of experimental research on shock loading of solid materials demonstrate that the revealed dependences of waveforms and threshold of the structure instability on strain-rate, target thickness and state of the material structure cannot be described in the framework of the conventional continuum mechanics. New concept of shock-wave processes in condensed matter is proposed on base of nonlocal theory of nonequilibrium transport which allowed a transition from the elastic medium reaction to the hydrodynamic one depending on the rate and duration of the loading. A new mathematical model of elastic-plastic wave is constructed to describe the elastic precursor relaxation and the plastic front formation taking into account the changing of material properties during the wave propagation. Analysis of experimental waveforms shows that for the shock-induced processes it is incorrect a priori to divide the components of stress and strain into elastic and plastic parts. The model allowed accounting for the inertial medium properties under short-duration loading and self-organization of new internal structures.

1. Introduction

In different regions of mechanics, the experimental researches of dynamic processes show a lot of common features which characterize an anomalous reaction of medium to an intense external action. Far from the thermodynamic equilibrium, transport processes are often accompanied by the formation of new multi-scale structures at mesoscopic scale level, such as shear bands, vortex structures, localized heterogeneities, velocity pulsations etc. The effects of self-organization observed in the target materials under study after shock loading [1-5] are determined not only by the material properties and its phase state but also by the loading and boundary conditions and by size and geometry of a system. The response to the high-rate external action begins to retard from the initial action and can be spread over space. The interaction between the structure elements causes the formed structures to evolve. The rate of the structure evolution and retarding responses to the actions could affect the relaxation characteristics of medium and lead to the system instability, fluctuations, to structure transformations, switching from one regime to another and to origin feedbacks. An influence of the individual factor among all the rest effects becomes indistinguishable because of the close-loops formed in the system. So, self-organization and self-regulation should be included into the mathematical model of an open system far from equilibrium to reveal indeterminacy effects.

At high-rate transport, the mean mass, impulse and energy densities are determined only in terms of the probability conception and don't entirely coincide with their definition near equilibrium in the framework of continuum mechanics. Unlike quasi-static processes well reproduced in experiments, reliability of experimental measurements for dynamic processes is

$f(U_{fs})$, corresponding to a threshold of structural transition equal to $U_{max} = U_{st}$= 323.8 m/s. Starting with the impact velocity of 376 m/s, the velocity defect begins to grow very fast (segment *B'C'* in Fig. 2b).

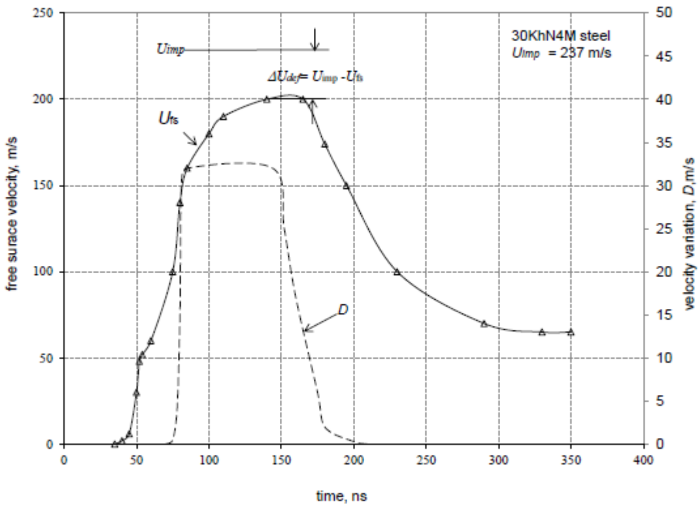
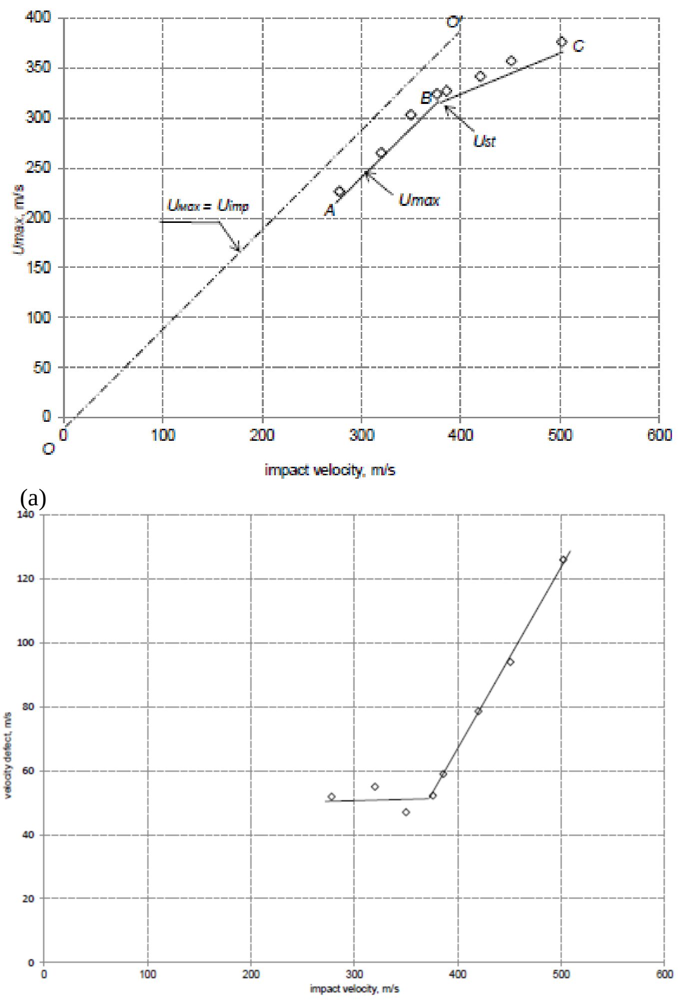


Fig. 1. Free surface velocity profile, U_{fs}(t), and variation velocity profile, *D*(t), for 5 mm 30KhN4M steel target loaded at the impact velocity of 237 m/s.



(a)

(b)

Fig. 2. Dependencies of maximum free surface velocity, U_{max}, (*a*) and velocity defect, U_{def}, (*b*) on the impact velocity for 30KHN4M steel.

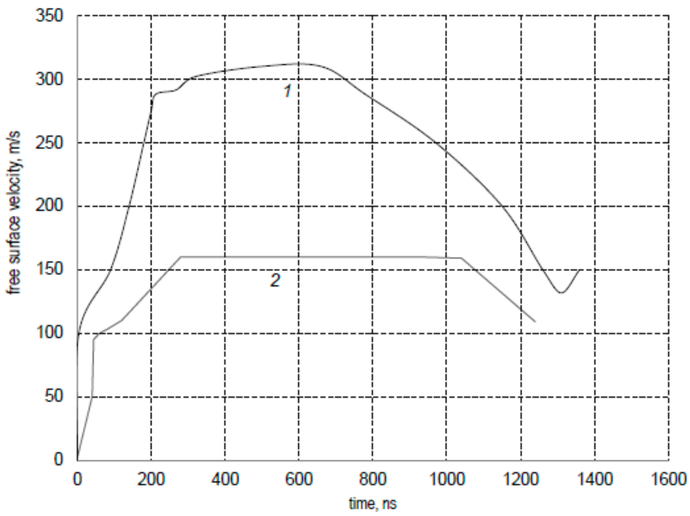


Fig. 4. Free surface velocity profiles, U_{fs}, for two kinds of 30KHN4M steel in different initial structural state before shock loading: 1) alloying at 250 ºC, 2) alloying at 400 ºC.

Below the critical strain-rate, the velocity defect depends on the strain-rate very weakly. Figure 5 shows the dependencies of the velocity defect on the impact velocity for two 5 mm targets of different kinds steel targets – 30KHN4M steel (1) and nitrogen steel (2). For the second kind of steel, the velocity defect remains invariable. Absence of the inflection point on the curve $U_{def} = f(U_{fs})$ for this material means that the material was not subjected to a structural instability transition all over the region of impact velocities under consideration.

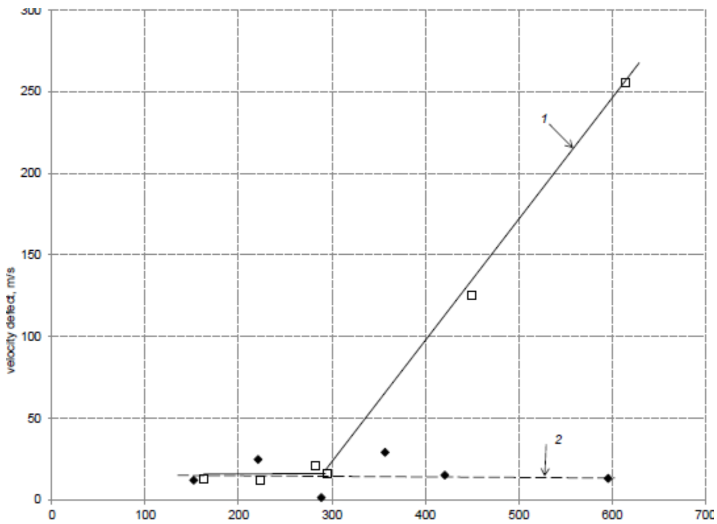
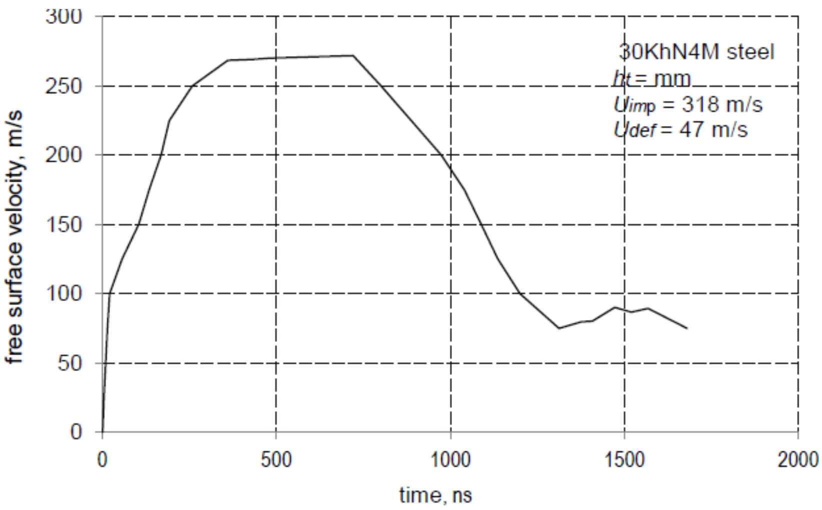
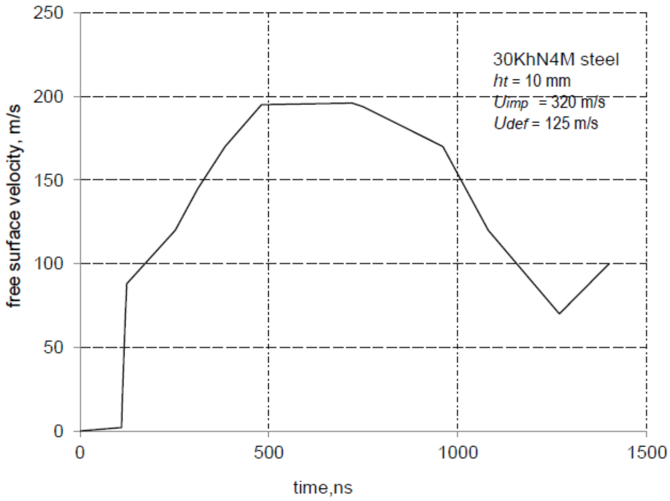


Fig. 5. Dependencies of the velocity defect, U_{def}, on the impact velocity for perlite steel target (1) and for nitrogen steel target (2).

Figure 6a,b shows a different situation observed for the free surface velocity profiles in AMg-6 aluminum alloy targets at the impact velocities of 178 m/c and 187 m/s. In Figure 6a maximal amplitude of compressive pulse equals to the impact velocity (double free surface velocity). Figure 6b shows that instead of the expected value of 187 m/s, the registered value of the free surface velocity equals to 116 m/s. It means that between the impact velocity 178 m/s and 187 m/s a transition from evolutional stage of dynamic straining to catastrophic stage (break of the interference signal is marked by symbol A) happens, which results in increase of the velocity defect up to 74 m/s. This result evidences that valuable part of impulse and energy is expended on the internal processes of structure formation.



(a)



(b)

Fig. 7. Free surface velocity profiles, U_{fs}, for 5 mm (*a*) and 10 mm (*b*) 30KHN4M steel targets shocked at the impact velocities of 318 m/s and 320 m/s, respectively.

Beyond the elastic limit, the mass transportation becomes irreversible, the waveform changes during propagation and the so-called two-wave structure of elastic-plastic wave is formed. Lastly, under long-time loading, any medium manifests a hydrodynamic behavior. The transport processes become irreversible because of the energy dissipation into heat. At the intermediate stage, a medium shows both elastic and hydrodynamic behavior. Elastic modules depend on strain-rate whereas viscosity and relaxation time are determined by size and geometry of the system. Under these conditions, the medium constants become functionals of transport processes whereas conventional models of elastic body and/or viscous liquid appear to be incorrect. Such transient processes are not described within continuum mechanics concepts. In transient processes, deformation cannot be correctly subdivided into elastic and plastic components as well as phase and group velocities cannot be properly defined for non-stationary waves [13, 14].

From the standpoint of continuum mechanics, the reaction of condensed matter to the dynamic load is anomalous as far as determined by the nonequilibrium transport processes beyond the validity of the continuum mechanics concept. These processes are accompanied by the effects of collective interaction of the medium elements, which result from inertia forces instead of the interaction potential. For the shock loading such characteristics as a rate of loading and duration of pulse play a principal role. As result, dynamic properties of medium depend on the loading regime and can essentially differ from the quasi-static ones

[32] are used to govern the evolution of the medium structure. Like in quantum mechanics, the boundary conditions for the model equations lead to discretization of scale spectrum of internal structure (structuration of system). By means of the structure parameters, the boundary problem formulation for the nonlocal equations becomes self-consistent: transport process depends on the structure of a system whereas the structure, in turn, is determined by the transport process itself. The close-loops between the structure evolution and the regime of loading are incorporated in the model. Only such multi-disciplinary approach at the crossroads of mechanics, physics and cybernetics leads to the closed self-consistent formulation of boundary problems for nonequilibrium transport processes in open systems. The approach allows a prediction of the dynamic medium properties under high-rate loading conditioned by the evolution of the inner medium structure. Successful description of the experimentally discovered effects within the nonlocal transport theory is shown to open a principally new opportunities to develop modern technologies in order to obtain materials with specified inner structure.

5. Nonlocal model of the impulse transport

The problem on the plane elastic-plastic wave propagation in condensed medium is considered. The impulse transport is initiated by a plane shock on the medium surface at a velocity $V_0 << C$. C is the longitudinal sound velocity determined by relationship $\rho C^2 = K + \frac{4}{3}G$ in which ρ is the medium density, K, G are the elastic compression and shear modules respectively. Elastic waves propagate at the phase velocity C without mass transportation. Beyond the elastic limit the waveforms consist of two fronts: elastic precursor propagating at the phase velocity C and plastic front propagating at the group velocity $C^{pl} < C$. Inside the plastic part of the waveform, the mass transport at the mass velocity $v << C$ takes place. It is the irreversible mass transport that responds for the plastic deformation of material. Non-stationary waveforms can blur during propagation due to the medium dispersion, and therefore the velocity of uni-axial impulse transport cannot be a priori divided into two parts $u = dx/dt = C^{pl} + v$. At a distance from the shock surface, when pulses of moderate intensity propagate in solids at the constant group velocity C^{pl}, the mass velocity is determined correctly.

The medium state after such shock slightly deviates from the undisturbed one: $\rho = \rho_0 + \rho_1$, $\rho_0 = const$, $\rho_1/\rho_0 \sim v/C << 1$, ρ_1 is the density deviation. The longitudinal impulse flux component $J_{xx} = J_0 + J_1$, $J_0 = \rho_0 C^2 = \text{const}$ (it will be shown further) also contains a deviation J_1 related to the induced stress. In the linear approximation with respect to the parameter $v/C << 1$ equations of mass and impulse transport take a form

$$\frac{1}{\rho_0}\frac{\partial \rho_1}{\partial t} + \frac{\partial v}{\partial x} = 0\,, \qquad (2)$$

$$\rho_0 \frac{\partial v}{\partial t} + \frac{\partial J_1}{\partial x} = 0\,. \qquad (3)$$

If $J_1 = \rho_1 C^2$ the set (2)-(3) results the wave equation for the elastic wave propagation with stationary waveform

$$\frac{\partial^2 v}{\partial t^2} - C^2 \frac{\partial^2 v}{\partial x^2} = 0\,. \qquad (4)$$

However, beyond the elastic limit the two-front waveform evolves due to the relaxation of

6. Formulation of the problem on the shock-induced plane wave propagation
The wave propagation is characterized by three scale parameters:

1. Stress relaxation parameter $\tau = \frac{t_r}{t_R}$, defining the transport regime;
2. Retardation parameter $\theta = \frac{t_m}{t_R}$, defining the retardation of the maximal stress on the plastic front from the elastic precursor;
3. Nonlocality parameter (relaxation length) $\varepsilon = \frac{Ct_r}{L}$, defining the contribution of the waveform evolution during its propagation.

New reference connected to the elastic precursor running at the constant longitudinal sound velocity C is introduced $\zeta = \frac{1}{t_R}\left(t - \frac{x}{C}\right)$, $\xi = \frac{x}{L}$. In the reference the relationship between new coordinate derivatives $\tau \frac{\partial}{\partial \zeta} \gg \varepsilon \frac{\partial}{\partial \xi}$ is conditioned by an evaluation $\frac{\varepsilon}{\tau} << 1$, resulted from the experimental conditions $\frac{\varepsilon}{\tau} = \frac{Ct_R}{L} \approx \frac{6 \cdot 10^3 m/s \cdot 2 \cdot 10^{-9} s}{10^{-2} m} \approx 10^{-3}$. The typical scale division considers being necessary to describe the structure self-organization, and allows essential simplification of the nonlocal model for the impulse transport in the new variables.

$$\frac{\partial \rho_1 / \rho_0}{\partial \zeta} - \frac{1}{C}\frac{\partial v}{\partial \zeta} + \frac{\varepsilon}{\tau}\frac{\partial v}{\partial \xi} = 0 , \qquad (6)$$

$$\frac{\partial v}{\partial \zeta} - \frac{1}{C}\frac{\partial C\Pi}{\partial \zeta} + \frac{\varepsilon}{\tau}\frac{\partial \Pi}{\partial \xi} = 0,$$

$$\Pi = -\int_0^{Ct} d\xi' \int_o^{\omega} d\zeta' \Re(\zeta,\zeta';\tau)\delta(|\xi - \xi'|)\left[-\frac{\partial v}{\partial \zeta'} + \frac{\varepsilon}{\tau}\frac{\partial v}{\partial \xi}\right] = \qquad (7)$$

$$= \int_o^{\omega} d\zeta' \Re(\zeta,\zeta';\tau)\left[\frac{\partial v}{\partial \zeta'} - \frac{\varepsilon}{\tau}\frac{\partial v}{\partial \xi}\right], \quad \omega(\zeta) = \begin{cases} \zeta, \ \zeta < 1 \\ 1, \ \zeta \geq 1 \end{cases}.$$

Here a new quantity is introduced $\Pi(\zeta,\xi;\tau,\theta) = J_1(x.t)/\rho_0 C$. The mass velocity is related to the shock velocity V_0. The coordinate ξ is counted from the back target surface which is at a distance L from the shock surface. The origin of the ζ-axis is an instance when the elastic precursor reaches the back target surface and is registered by the device. Due to the scale division the spatiotemporal correlations in the integral kernel in Eq. (7) are also divided. So, only memory effects remain in the kernel whereas the spatial nonlocality is neglected. The following model expression for the memory function is used [23, 24]:

$$\Re(\zeta,\zeta';\tau) = \exp\left\{-\frac{\pi\left(\zeta - \zeta' - \theta\right)^2}{\tau^2}\right\}. \qquad (8)$$

The parameters $\tau(\xi), \theta(\xi)$ depend on the distance traveled by the wave. In general case, the functions $\tau(\xi), \theta(\xi)$ are unknown due to the back influence of the wave propagation on the medium properties inside the wave.

7. Approximate solution for quasi-stationary wave propagation
The propagation of the shock-induced impulse of moderate intensity in condensed matter is

the impulse exchange essentially influenced on the short impulse transport in condensed medium. In order to introduce the rate of the initial impulse exchange, the initial waveform assumes to be trapeze-type which corresponds to the finite acceleration $\frac{\partial v}{\partial \zeta}$ induced by the constant force acting for a finite time interval t_R. For the velocity v and the variable ζ, normalized to the shock characteristics V_0, t_R, the acceleration during the impact $\frac{\partial v}{\partial \zeta} = 1$ equals to unit. Substitution of the initial acceleration into the Eq, (10) results an explicit automodel solution for the mass velocity wave front [23]:

$$v(\zeta;\tau,\theta) = \begin{cases} \frac{\tau}{2}\left(erf\,\frac{\sqrt{\pi}(\zeta-\theta)}{\tau} + erf\,\frac{\sqrt{\pi}\theta}{\tau} \right), & \zeta < 1, \\ \frac{\tau}{2}\left(erf\,\frac{\sqrt{\pi}(\zeta-\theta)}{\tau} + erf\,\frac{\sqrt{\pi}(1-\zeta+\theta)}{\tau} \right), & \zeta \geq 1. \end{cases} \tag{11}$$

The solution (11) relates the stress inside running wave to the initial strain-rate during the shock. During the loading below the elastic limit the solution (upper expression) describes the relaxation of the elastic precursor. After the loading (lower expression) the solution describes the shear relaxation in plastic front as aftereffect due to the medium inertia, but not due to a plastic flow. The non-stationary two-wave front is forming without previous division of stress into elastic and plastic parts. Further it will be demonstrated that the approximate solution (11) adequately describes all the experimentally observed effects related to the quasi-stationary elastic-plastic wave propagation. Figures 8, 9 show the two-wave front formation according to the solution (11) at different values of the model parameters.

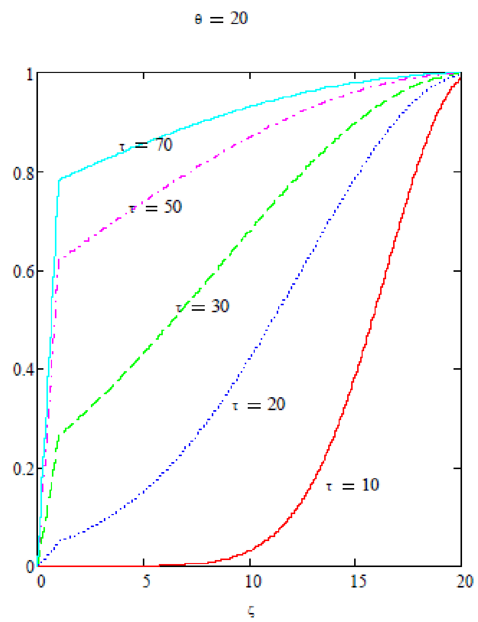


Fig. 8. Formation of the two-wave front at the fixed value of the retardation parameter θ with decrease of the relaxation parameter τ : (downwards) $\tau = 70, 50, 30, 20, 10.$

Depending on the parameters τ, θ the relaxation can be monotonous and nonmonotonous. Near the hydrodynamic limit with memory and retardation neglected, the stress after the loading begins to attenuate monotonously. At rather large values of parameters after the finished load beyond the elastic limit, the stress continues to grow due to inertia until it

near equilibrium) whereas in shock-induced processes aftereffects and inertia play the main role. On the plateau the acceleration is absent $\partial v / \partial \zeta = 0$, and therefore the force is not applied. It should be noticed that the doubling of the elastic precursor had never been observed in experiments on the shock compression of different materials.

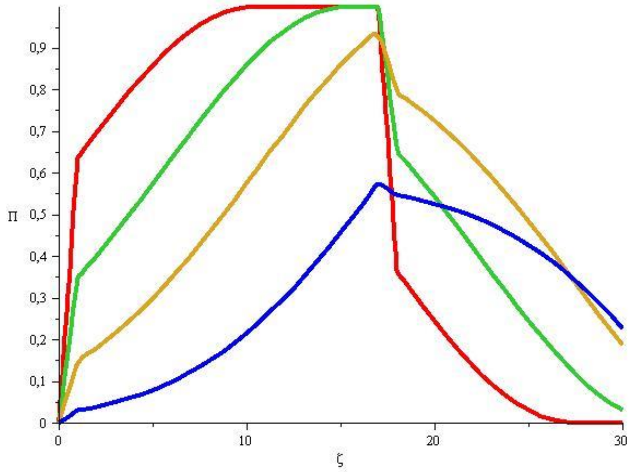


Fig. 10. Evolution of the finite duration waveform at the fixed value of the relaxation parameter τ with increase of the retardation parameter θ: (downwards) $\theta = 10$, $\theta = 15$, $\theta = 20$, $\theta = 25$.

Unlike the conventional approaches to the shock-induced processes [37-38], the plastic front is forming by the shear relaxation after the force of the shock proportional to the medium acceleration (or strain-rate) stops acting. The dependence of stress on the strain-rate is included into Eq. (10) and its solution (11) by means of the acceleration $\frac{\partial v}{\partial \zeta} = 1$. For quasi-static long-time loading at low strain-rates all aftereffects can be neglected as far as the medium reaction keeps up with the loading and the process history does not directly influence on it. The fundamental difference between quasi-static and shock loading can be seen on Fig. 11.

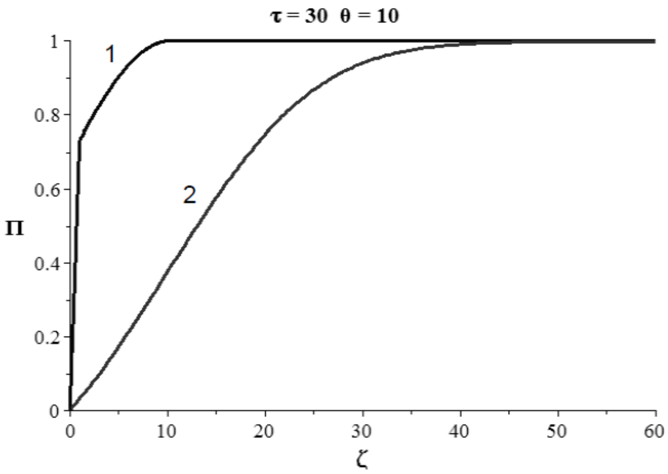


Fig. 11. Comparison of two waveforms of the same maximal amplitude induced by a shock (top) and quasi-static continuous loading (lower).

According to the solution (11), during the shock loading the elastic precursor reaches its maximal amplitude at the time $t = t_R$ and after that the shear relaxation forms the plastic front. During the slow continuous loading at the constant strain-rate no two-wave front forms, one

loading at the impact velocity in between the interval 200÷400 m/s and different target thicknesses for two materials: 30XH4M steel and Д16 aluminum alloy. It can be seen that each phase point moves along the trajectory away from the coordinate origin during the wave propagation. Both trajectories are straight lines under different angles to axis depending on the material elastic limit for the range of impact velocities. Both lines can be extrapolated to the origin though there is no experimental data close to the origin and the parameters loose their meaning at low values τ, θ.

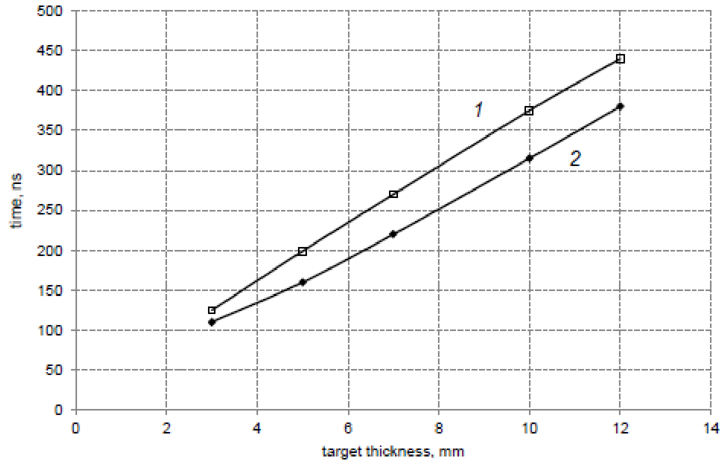


Fig. 12. Experimental dependencies of the relaxation parameter τ (1) and the retardation parameter θ (2) on the target thickness.

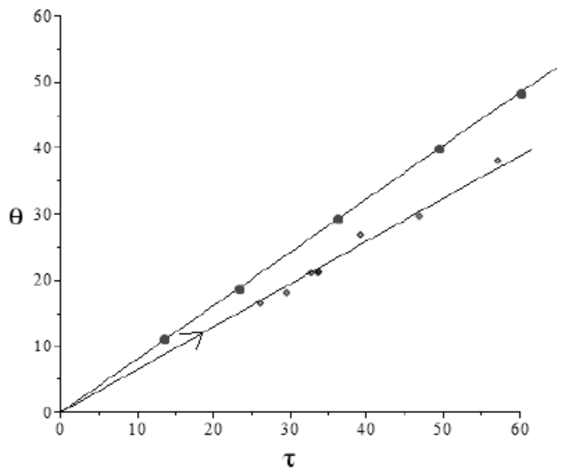


Fig. 13. Trajectories for two series of experiments on the shock loading at the impact velocity in the interval 200÷400 m/s and different target thicknesses for two materials: 30XH4M steel (lower) and Д16 aluminum alloy (top).

According to the parameters, the trajectories the waveforms sprawl in the propagation, and the plateau of the plastic front retards increasingly from the elastic precursor. The rate of the retardation can be evaluated from experimental data as follows: $C^{pl} = x/((x/C)+\theta) \approx 5000\,\mathrm{m/s} \approx C_0$. It means that all experimental data correspond to the quasi-stationary regime of wave propagation. In quasi-stationary regime the plastic front moves at the constant velocity and the amplitude of the elastic precursor has a constant value in its propagation. The phase trajectories for the quasi-stationary wave propagation are beams along which phase points run away from the origin. Since the plateau of the plastic front propagates at the volume sound velocity, the shear relaxation is already completed whereas the volume

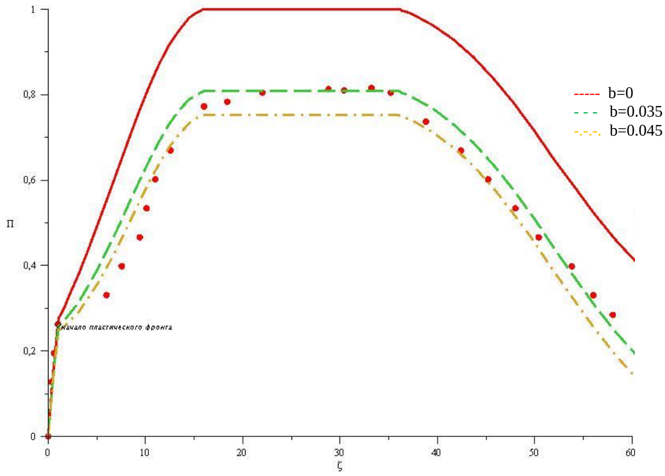


Fig. 14. Waveforms accounting the velocity defect (15) at the different exchange parameters b compared to the experimental waveform at the same impact velocity.

12. Discussion

Experimental research of the material response to the shock loading had revealed special features which distinguish dynamic processes from the quasi-static ones. From the standpoint of the conventional continuum mechanics the response seems to be anomalous due to the complex of accompanying relaxation and exchange processes which had never been included in conventionally used models. Aftereffects originated by relaxation and inertia make the response retarding. Impulse and energy exchange between different degrees of freedom induces a self-organization of new internal structures at the intermediate scale level between macro- and micro- scales. The internal structure, in turn, makes the response dispersed. The structure evolution leads to instabilities which cannot be predicted. All the features make the differential models in the framework of continuum mechanics inadequate for dynamic processes.

A need for a new theoretical approach able to describe the high-rate processes considering the whole complex of accompanying effects resulted in nonlocal theory of nonequilibrium transport. Application of the theory to the problem on the shock-induced wave propagation in solid materials allows an adequate explanation of the waveforms behavior during their propagation revealed in series of experiments on dynamic loading of different materials. It was found out that in dynamic processes stress and strain are related to different spatiotemporal points and cannot be correctly divided into elastic and plastic parts in advance. The so called plastic front appears to be the relaxation front and cannot be considered in the framework of hydrodynamic models. Only integral nonlocal models can describe all the special features inherent in the dynamic medium response to high-rate loading.

Until physicists use “rigid” differential models for high-rate nonequilibrium processes without involving of the internal structure evolution, the gap between fundamental science and practice would not be overcome.

Acknowledgement

This work was supported in part (for Yu.I.M.) by Saint-Petersburg State University research grant 6.37.671.2013.

References

[1] J.R. Asay, LC. Chhabildas, In: *Shock Waves and High-Strain-Rate Phenomena in Metals. Concepts and Applications*, ed. by M.A. Meyers and L.E. Murr (Plenum Publishing Corporation, N.Y., 1981).

[2] Yu.I. Meshcheryakov, S.A. Atroshenko // *International Journal of Solids and Structures* **29** (1992) 2761.
[3] Yu.I. Meshcheryakov, A.K. Divakov. N.I. Zhigacheva, I.P. Makarevich, B.K. Barakhtin // *Physical Review B.* **78** (2008) 64301.
[4] A.C. Koskelo, S.R. Greenfield, D.L. Raisley, K.J. McClellan, D.D. Byler, R.M. Dickerson, S.N. Luo, D.C. Swift, D.L. Tonk, P.D. Peralta, In: *Shock Compression of Condensed Matter - 2007. Proc. AIP-955* (N.Y., 2008), p. 557.
[5] M.Yu. Gutkin, I.A. Ovid'ko, Yu.I. Meshcheryakov // *Journal de Physique III France* **3** (1993) 1563.
[6] J.R. Asay, L.M. Barker // *Journal of Applied Phys*ics **45** (1974) 2540.
[7] Yu.I. Meshcheryakov, A.K. Divakov // *Dymat Journal* **1** (1994) 271.
[8] Yu.I. Meshcheryakov, A.K. Divakov, N.I. Zhigacheva // *International Journal of Solids and Structures* **41** (2004) 2349.
[9] J.R. Asay, In: *Shock Compression of Condensed Matter-2001*. *AIP Conference Proceedings-620*, ed. by M.D. Furnish, N.N. Thadhani, Y-Y. Horie (Melville, N.Y., 2002), p. 26.
[10] G. Ravichandran, A.J. Rosakis, J. Hodovany, P. Rosakis, In: *Shock Compression of Condensed Matter - 2001. AIP Conference Proceedings-620*, ed. by M.D. Furnish, N.N. Thadhani, Y-Y. Horie (Melville, N.Y., 2002) p. 557.
[11] Yu.I. Meshcheryakov, A.K. Divakov, S.A. Atroshenko, N.S. Naumova // *Technical Physics Letters* **35** (2009) 1125.
[12] Yu.I. Meshcheryakov, A.K. Divakov, N.I. Zhigacheva, B.K. Barakhtin // *International Journal of Impact Engineering* **57** (2013) 99.
[13] J.J. Gilman, In: *Shock Compression of Condensed Matter-2001*. *AIP Conference Proceedings-620*, ed. by M.D. Furnish, N.N. Thadhani, Y-Y. Horie (Melville, N.Y., 2002), p. 36.
[14] J.J. Gilman, In: *High Pressure Shock Compression of Solids VI. Old Paradigms and New Challenges*, ed. by Y-Y. Horie, L. Davison, N.N. Thadhani (Springer, 2002), p. 279.
[15] V.A. Morozov, In: *Models of continuum mechanics*. *Proceedings of 14 Inter. school on models of continuum mechanics* (Ghukovskii, 1992), Vol. 1, p. 324.
[16] O.D. Baizakov, V.A. Morozov, Yu.V. Sud'enkov, In: *Gas Dynamics and Thermo-exchange. Dynamics of uniform and nonuniform media* (Publ. Leningrad Univ., Leningrad, 1993), p. 9.
[17] T.A. Khantuleva, Yu.I. Mescheryakov // *International Journal of Solids and Structures* **36** (1999) 3105.
[18] T.A. Khantuleva, Yu.I. Mescheryakov // *Physical Mesomechanics* **2** (1999) 5.
[19] T.A. Khantuleva,. In: *Shock Compression of Condensed Matter - 1999. APS 1-56396-923-8/00*, ed. by M.D. Furnish, L.D. Chhabildas, and R.S. Hixon (2000), p. 371.
[20] T.A. Khantuleva // *Journal de Physique IV France* **10** (2000) 485.
[21] T.A. Khantuleva, In: *High-pressure compression of solids VI: old paradigms and new challenges*, ed. by Y. Horie, L. Davison, N.N. Thadhani (Springer, 2003), p. 215.
[22] T.A. Khantuleva // *Chemical Physics* **24** (2005) 36.
[23] T.A. Khantuleva. N.A. Serebryanskay // *Izvestiya VUZov, Fizika* **52** (2009) 165.
[24] T.A. Khantuleva, *Nonlocal theory of nonequilibrium transport processes* (Publ. Saint-Petersburg Univ., Saint-Petersburg, 2013).
[25] N.N. Bogolyubov, B.I. Sadovnikov, A.S. Shumovskii, *Mathematical methods of the mechanics of model systems* (Nauka, Moscow, 1989).
[26] J.M Richardson // *Journal of Mathematical Analysis and Applications* **1** (2013) 12.
[27] N. Ailavadi, A. Rahman, R. Zwanzig // *Physical Review A* **4** (1971) 1616.
[28] D.N. Zubarev, S.V. Tishchenko // *Physics* **59** (1972) 285.

[29] D.N. Zubarev // *Results of Science and Technics. Ser. Modern problems of mathematics* **15** (1980) 152.
[30] S.A. Vavilov // *Doklady AN SSSR* **316** (1991) 22.
[31] S.A. Vavilov // *Doklady AN SSSR* **323** (1992) 206.
[32] A.L. Fradkov, *Cibernetical physics* (Nauka, Saint-Petersburg, 2003).
[33] J.J. Gilman, In: *Shock Compression of Condensed Matter - 1991*, ed. by S.C. Schmidt, R.D. Dick, G.W. Forbes, D.G. Tasker (Elsevier Science Publishers B.V., 1992), p. 387.
[34] J.N. Johnson, L.M. Barker // *Journal of Applied Physics* **40** (1969) 4321.
[35] D. Wood // *Mechanics* 5(21) (1953) 150.
[36] T.A. Khantuleva, Yu.I. Mescheryakov // *Physical Mesomechanics* **17(5)** (2014) 21.