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STABILITY OF STRATIFIED VISCOELASTIC RIVLIN-ERICKSEN (MODEL) FLUID/PLASMA IN THE PRESENCE OF QUANTUM PHYSICS SATURATING A POROUS MEDIUM

Rajneesh Kumar[1*], Veena Sharma[2], Shaloo Devi[2]

[1]Department of Mathematics, Kurukshtra University, Kurukshtra, Haryana, 136119, India.

[2]Department of Mathematics& Statistics, H.P. University Shimla, 171005, India.

*e-mail:rajneesh_kuk@rediffmail.com

Abstract. The present investigation deals with the quantum effects on the Rayleigh –Taylor instability in an infinitely electrically conducting inhomogeneous stratified incompressible viscoelastic fluid/plasma through a porous medium. The linear growth rate is derived for the case where a plasma with exponential density, viscosity, viscoelasticity and quantum parameter distribution is confined between two rigid planes. The solution of the linearized equations of the system together with the appropriate boundary conditions leads to derive the dispersion relation (the relation between the normalized growth rate and square normalized wavenumber) using normal mode technique. The behavior of growth rate with respect to quantum effect and kinematic viscoelasticity are examined in the presence of porous medium, medium permeability and kinematic viscoelasticity. It is observed that the quantum effects bring more stability for a certain wave number band on the growth rate on the unstable configuration.

1. Introduction

Rayleigh-Taylor instability arises from the character of equilibrium of an incompressible heavy fluid of variable density (i.e. of a heterogeneous fluid). The simplest, nevertheless important, example demonstrating the Rayleigh-Taylor instability is when, we consider two fluids of different densities superposed one over the other (or accelerated towards each other); the instability of the plane interface between the two fluids, if it occurs, is known as Rayleigh-Taylor instability. Rayleigh (1900) [1] was the first to investigate the character of equilibrium of an inviscid, non- heat conducting as well as incompressible heavy fluid of variable density, which is continuously stratified in the vertical direction. The case of (i) two uniform fluids of different densities superposed one over the other and (ii) an exponentially varying density distribution, was also treated by him. The main result in all cases is that the configuration is stable or unstable with respect to infinitesimal small perturbations according as the higher density fluid underlies or overlies the lower density fluid. Taylor (1950) [2] carried out the theoretical investigation further and studied the instability of liquid surfaces when accelerated in a direction perpendicular to their planes. The experimental demonstration of the development of the Rayleigh –Taylor instability (in case of heavier fluid overlaying a lighter one, is accelerated towards it) is described by Lewis (1950) [3]. This instability has been further studied by many authors e.g. Kruskal and Schwarzschild (1954) [4], Hide (1955) [5], Chandrasekhar (1955) [6], Joseph (1976) [7], and Drazin and Reid (1981) [8] to include various parameters. Rayleigh-Taylor instability is mainly used to analyze the frequency of gravity waves in deep oceans, liquid vapour/globe, to extract oil from the earth to eliminate water drops, lazer and inertial confinement fusion etc.

Quantum plasma can be composed of electrons, ions, positrons, holes, and (or) grains, which plays an important role in ultra-small electronic devices which have been given by Dutta and McLennan (1990) [9], dense astrophysical plasmas system has been given by Madappa et al. (2001) [10], intense laser-matter experiments has been investigated by Remington (1999) [11], and non-linear quantum optics has been given by Brambilla et al. (1995) [12]. The pressure term in such plasmas is divided to two terms $p = p^C + p^Q$ (classical (p^C) and quantum (p^Q) pressure) and has been investigated by Gardner (1994) [13] for the quantum hydrodynamic model. In the momentum equation, the classical pressure rises in the form $(-\nabla p)$, while the quantum pressure rises in the form $Q = \frac{h^{\sim 2}}{2m_e m_i}\rho\nabla\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right)$, where $h^{\sim}$ is the Plank constant, m_e is the mass of electron and m_i is the mass of ion. The linear quantum growth rate of a finite layer plasma, in which the density is continuously stratified exponentially along the vertical, was studied by Goldston and Rutherford (1997) [14]. Nuclear fusion, which is plasma based, is one of the most promising candidates for the energy needs of the future when fossil fuels finally run out. It is well known that quantum effects become important in the behavior of charged plasma particles when the de Broglie wavelength of charge carriers become equal to or greater than the dimension of the quantum plasma system, which has been investigated by Manfredi and Haas (2001) [15]. Two models are used to study quantum plasmas systems. The first one is the Wigner-Poisson and the other is the Schrodinger-Poisson approaches (2001, 2005) [15-17] they have been widely used to describe the statistical and hydrodynamic behavior of the plasma particles at quantum scales in quantum plasma. The quantum hydrodynamic model was introduced in semiconductor physics to describe the transport of charge, momentum and energy in plasma (1994) [13].

A magnetohydrodynamic model for semiconductor devices was investigated by Haas (2005) [16], which is an important model in astrophysics, space physics and dusty plasmas. The effect of quantum term on Rayleigh-Taylor instability in the presence of vertical and horizontal magnetic field, separately, has been studied by Hoshoudy (2009) [18, 19]. The Rayleigh-Taylor instability in a non-uniform dense quantum magneto-plasma has been studied by Ali et al. (2009) [20]. Hoshoudy (2010) [21] studied quantum effects on Rayleigh-Taylor instability of incompressible plasma in a vertical magnetic field. Rayleigh-Taylor instability in quantum magnetized viscous plasma has been studied by Hoshoudy (2011) [22]. External magnetic field effects on the Rayleigh-Taylor instability in an inhomogeneous rotating quantum plasma has been studied by Hoshoudy (2012) [23]. In all the above studies, the plasma/fluids have been considered to be Newtonian. With the growing importance of the non-Newtonian fluids in modern technology and industries, the investigations of such fluids are desirable. There are many elastico-viscous constitutive relation or Oldroyd constitutive relation. We are interested there in Rivlin-Ericksen Model. Rivlin-Ericksen Model (1955) [24] proposed a theoretical model for such elastic-viscous fluid. Molten plastics, petroleum oil additives and whipped cream are examples of incompressible viscoelastic fluids. Such types of polymers are used in agriculture, communication appliances and in bio-medical applications. Previous work on the effects of incompressible quantum plasma on Rayleigh-Taylor instability of Oldroyd model through a porous medium has been investigated by Hoshoudy (2011) [25], where the author has shown that both maximum k^*_{max} and critical k^*_c point for the instability are unchanged by the addition of the strain retardation and the stress relaxation. All growth rates are reduced in the presence of porosity of the medium, the medium permeability, the strain retardation time and the stress relaxation time. This paper aims at numerical analysis of the effect of the quantum mechanism on Rayleigh-Taylor instability for a finite thickness layer of incompressible viscoelastic plasma in a porous medium. Hoshoudy (2013) [26] has studied Quantum effects on Rayleigh-Taylor instability of a plasma-vacuum. Hoshoudy (2014) [27] studied Rayleigh-Taylor instability of Magnetized plasma through Darcy porous medium.

Sharma et al. (2014) [28] has investigated the Rayleigh-Taylor instability of two superposed compressible fluids in un- magnetized plasma. The present paper deals with quantum effects on the Rayleigh –Taylor instability in an infinitely electrically conducting inhomogeneous stratified incompressible, viscoelastic fluid/plasma through a porous medium. The solution of the linearized equations of the system together with the appropriate boundary conditions leads to the dispersion relation (the relation between the normalized growth rate and square normalized wavenumber). The behavior of growth rate with respect to quantum effect and kinematic viscoelasticity are examined in the presence of porous medium, medium permeability and kinematic viscoelasticity.

2. Formulation of the problem and perturbation equations

We consider the initial stationary state whose stability is that of an incompressible, heterogeneous infinitely conducting viscoelastic Rivlin–Ericksen (Model) [24] fluid of thickness *h* bounded by the planes $z = 0$ and $z = d$. The variable density, kinematic viscosity, kinematic viscoelasticity and quantum pressure are arranged in horizontal strata electrons and immobile ions in a homogenous, saturated, isotropic porous medium with the Oberbeck–Boussinesq approximation for density variation are considered, so that the free surface behaves almost horizontal. The fluid is acted on by gravity force $= (0,0,-g)$.

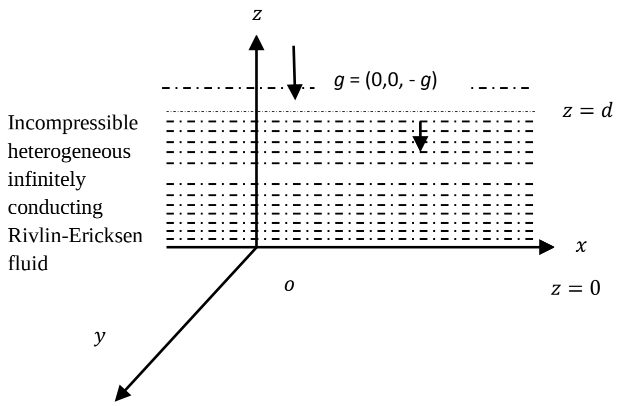

Fig. 1. Diagram of finite quantum plasma layer.

Following Hoshoudy (2009) [18, 19], the equations of motion, continuity (conservation of mass), incompressibility, Gauss divergence equation and Magnetic induction equations are taken as

$$\frac{\rho}{\varepsilon}\left[\frac{\partial}{\partial t}+\frac{1}{\varepsilon}(\mathrm{q}.\nabla)\right]\boldsymbol{q} = -\nabla p + \rho\boldsymbol{g} - \frac{1}{k_1}\left(\mu + \mu'\frac{\partial}{\partial t}\right)\boldsymbol{q} + \boldsymbol{Q}, \tag{1}$$

$$\nabla.\boldsymbol{q} = 0, \qquad \varepsilon\frac{\partial\rho}{\partial t} + (\boldsymbol{q}.\nabla)\rho = 0, \tag{2, 3}$$

where $\boldsymbol{q}, \rho, p, \mu, \mu', k_1, \varepsilon, \boldsymbol{Q}$ represent velocity, density, pressure, viscosity, viscoelasticity, medium permeability, medium porosity and Bohr vector potential, respectively. Equation (3) ensures that the density of a particle remains unchanged as we follow with its motion. Then equilibrium profiles are expressed in the form $\boldsymbol{u_0} = (0,0,0), \rho_0 = \rho_0(z), p = p_0(z)$ and $\boldsymbol{Q} = \boldsymbol{Q}_0(z)$.

To investigate the stability of hydromagnetic motion, it is necessary to see how the motion responds to a small fluctuation in the value of any flow of the variables. Let the infinitesimal perturbations in fluid velocity, density, pressure, magnetic field and quantum pressure be taken by

$$q = (u, v, w), \rho = \rho_0 + \delta\rho, p = p_0 + \delta p \text{ and } Q = Q_0 + Q_1(Q_x, Q_y, Q_z). \tag{4}$$

Using these perturbations and linear theory (neglecting the products of higher order perturbations because their contributions are infinitesimally very small), equations (1) - (3) in the linearized perturbation form become

$$\frac{\rho_0}{\varepsilon}\frac{\partial u}{\partial t} = -\nabla\delta p + g\delta\rho - \frac{1}{k_1}\left(\mu + \mu'\frac{\partial}{\partial t}\right)q + Q_1, \tag{5}$$

$$\nabla . q = 0,\ \varepsilon\frac{\partial}{\partial t}\delta\rho + w\frac{d\rho_0}{dz} = 0, \tag{6, 7}$$

$$\boldsymbol{Q}_1 = \frac{h^2}{2m_e m_i}\begin{bmatrix} \frac{1}{2}\nabla(\nabla^2\delta\rho) - \frac{1}{2\rho_0}\nabla\delta\rho\nabla^2\rho_0 - \frac{1}{2\rho_0}\nabla\rho_0\nabla^2\delta\rho + \\ \frac{\delta\rho}{2\rho_0^2}\nabla\rho_0\nabla^2\rho_0 - \frac{1}{2\rho_0}\nabla(\nabla\rho_0\nabla\delta\rho) + \frac{\delta\rho}{4\rho_0^2}\nabla(\nabla\rho_0)^2 + \\ \frac{1}{2\rho_0^2}(\nabla\rho_0)^2\nabla\delta\rho + \frac{1}{\rho_0^2}(\nabla\rho_0\nabla\delta\rho)\nabla\rho_0 - \frac{\delta\rho}{\rho_0^3}(\nabla\rho_0)^3 \end{bmatrix}.$$

The Cartesian form of equations (5) - (7) yield

$$\frac{\rho_0}{\varepsilon}\frac{\partial u}{\partial t} = -\frac{\partial}{\partial x}\delta p - \frac{1}{k_1}\left(\mu + \mu'\frac{\partial}{\partial t}\right)u + Q_x, \tag{8}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial v}{\partial t} = -\frac{\partial}{\partial y}\delta p - \frac{1}{k_1}\left(\mu + \mu'\frac{\partial}{\partial t}\right)v + Q_y, \tag{9}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial w}{\partial t} = -\frac{\partial}{\partial z}\delta p - g\delta\rho - \frac{1}{k_1}\left(\mu + \mu'\frac{\partial}{\partial t}\right)w + Q_z, \tag{10}$$

$$\varepsilon\frac{\partial}{\partial t}\delta\rho = -w\frac{d\rho_0}{dz}, \quad \text{and} \tag{11}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{12}$$

where

$$Q_x = \frac{h^2}{2m_e m_i}\frac{\partial}{\partial x}\begin{bmatrix} \frac{1}{2}D^2\delta\rho - \frac{1}{2\rho_0}D\rho_0 D\delta\rho + \\ \left(\frac{1}{2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) - \frac{1}{2\rho_0}D^2\rho_0 + \frac{1}{2\rho_0^2}(D\rho_0)^2\right)\delta\rho \end{bmatrix}, \tag{13}$$

$$Q_y = \frac{h^2}{2m_e m_i}\frac{\partial}{\partial y}\begin{bmatrix} \frac{1}{2}D^2\delta\rho - \frac{1}{2\rho_0}D\rho_0 D\delta\rho + \\ \left(\frac{1}{2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) - \frac{1}{2\rho_0}D^2\rho_0 + \frac{1}{2\rho_0^2}(D\rho_0)^2\right)\delta\rho \end{bmatrix}, \tag{14}$$

$$Q_z = \frac{h^2}{2m_e m_i}\begin{bmatrix} \frac{1}{2}D^3\delta\rho - \frac{1}{\rho_0}D\rho_0 D^2\delta\rho + \\ \left(\frac{1}{2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) - \frac{1}{\rho_0}D^2\rho_0 + \frac{3}{2\rho_0^2}(D\rho_0)^3\right)D\delta\rho + \\ \left(-\frac{1}{2\rho_0}D\rho_0\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + \frac{1}{2\rho_0^2}D\rho_0 D^2\rho_0 - \frac{1}{\rho_0^3}(D\rho_0)^3\right)\delta\rho \end{bmatrix}. \tag{15}$$

Since the boundaries are assumed to be rigid. Therefore the boundary conditions appropriate to the problem are

$$w = 0,\ Dw = 0 \ \text{at}\ z = 0 \ \text{and}\ z = d, \text{on a rigid surface.} \tag{16}$$

To investigate the stability of the system, we analyze an arbitrary perturbation into a complex set of normal modes individually. For the present problem, analysis is made in terms of two-dimensional periodic waves of assigned wavenumber. Thus to all quantities are ascribed describing the perturbation dependence on x, y and t of the forms

$$f_1(x, y, z, t) = f(z)exp i\left(k_x x + k_y y - nt\right), \tag{17}$$

where k_x and k_y are wavenumbers along x and y directions, $k = \left(k_x^2 + k_y^2\right)^{\frac{1}{2}}$ is the resultant wavenumber and n is the growth rate which is, in general a complex constant.
Using (17) in (8)-(11) and after some simplification, we obtain the characteristic equation:

$$\left[(-in) - \mathrm{A}\frac{(D\rho_0)^2}{{\rho_0}^2}\right]D^2\mathrm{w} + \left[\frac{(-in)(D\rho_0)}{\rho_0} - \mathrm{A}\frac{(D\rho_0)^3}{{\rho_0}^3} - 2\mathrm{A}\frac{(D\rho_0)(D^2\rho_0)}{{\rho_0}^2}\right]Dw + \left[-(-in)k^2 - \frac{gk^2}{\rho_0 in}(D\rho_0) - \frac{k^2\varepsilon}{\rho_0 k_1}\left(\mu + \mu'(-in)\right) + \mathrm{A}k^2\frac{(D\rho_0)^2}{{\rho_0}^2}\right]w = 0, \tag{18}$$

where $A = \frac{h^2k^2}{4(in)m_e m_i}$.
For the case of incompressible continuously stratified viscoelastic plasma layer considered in a porous medium, the density, viscosity, viscoelasticity and quantum pressure are taken as

$$\rho_0(z) = \rho_0(0)exp\left(\frac{z}{L_D}\right), \mu(z) = \mu_0 exp\left(\frac{z}{L_D}\right), \mu'(z) = \mu_0'(0)exp\left(\frac{z}{L_D}\right),$$

$$k_1(z) = k_{10}(0)exp\left(\frac{z}{L_D}\right), n_q(z) = n_{q_0}(0)exp\left(\frac{z}{L_D}\right), \varepsilon(z) = \varepsilon_0(0)exp\left(\frac{z}{L_D}\right), \tag{19}$$

where $\rho_0(0), \mu_0(0), \mu_0'(0), n_{q_0}(0), k_{10}(0), \varepsilon_0(0)$ and L_D are constants.
Making use of (19) in (18), yield

$$\left[(-in) - A\frac{1}{L_D^2}\right]D^2w + \left[\frac{(-in)}{L_D} - \frac{1}{L_D^3}\right]Dw + \left[-(-in)k^2 - \frac{gk^2}{L_D in} - \frac{k^2\varepsilon}{k_1}\left(\nu + \nu'(-in)\right) + \mathrm{A}\frac{k^2}{L_D^2}\right]w = 0, \tag{20}$$

and

$$\left[(-in) - \frac{n_q^2}{(in)}\right]D^2w + \left[\frac{(-in)}{L_D} - \frac{n_q^2}{(in)L_D}\right]Dw + \left[-(-in)k^2 - \frac{gk^2}{L_D} - \frac{k^2\varepsilon}{k_1}\left(\nu + \nu'(-in)\right) + \frac{k^2n_q^2}{(in)}\right]w = 0, \tag{21}$$

where $n_q^2 = \frac{h^2k^2}{4m_e m_i L_D^2}$ represents quantum effect.
In addition to the boundary conditions given by (16), we also have

$$D^2w = 0 \text{ at } z = 0 \text{ and } z = d. \tag{22}$$

Making use of (21) in (16) and (22) and assuming $w = sin(nz)exp(\lambda z)$, where $n = \frac{n_1\pi}{h}$, we obtain

$$(\lambda^2 - n^2)\left((-in) - \frac{n_q^2}{(in)}\right) + \lambda\left(\frac{(-in)}{L_D} - \frac{n_q^2}{(in)L_D}\right) + \left((in)k^2 - \frac{gk^2}{(in)L_D} - \frac{k^2\varepsilon}{k_1}\left(\nu + \nu'(-in)\right) + \frac{k^2n_q^2}{(in)}\right) = 0, \tag{23}$$

and

$$2\lambda\left(\frac{n_1\pi}{h}\right)\left((-in) - \frac{n_q^2}{(in)}\right) + \left(\frac{n_1\pi}{h}\right)\left(\frac{(-in)}{L_D} - \frac{n_q^2}{(in)L_D}\right) = 0. \tag{24}$$

In equation (24), implies that

$$\lambda = -\frac{1}{2L_D}. \tag{25}$$

Eq. no. (23) with the aid of (25) takes the form

$$\left(\frac{1}{4L_D^2}-n^2\right)\left((-in)-\frac{n_q^2}{(in)}\right)-\frac{1}{2L_D}\left(\frac{(-in)}{L_D}-\frac{n_q^2}{(in)L_D}\right)+\left((in)k^2-\frac{gk^2}{(in)L_D}-\frac{k^2\varepsilon}{k_1}\left(\nu+\nu'(-in)\right)+\frac{k^2n_q^2}{(in)}\right)=0. \quad (26)$$

To facilitate the problem, we introduce the non-dimensional quantities as $n^{*2}=\frac{n^2}{n_{pe}^2}, {n_q}^{*2}=\frac{n_q^2}{k^{*2}n_{pe}^2}, n_\varepsilon^*=\frac{\varepsilon}{n_{pe}}, n_\nu^*=\frac{\nu}{n_{pe}}, n_{\nu'}^*=\nu', n_{k_1}^*=\frac{k_1}{n_{pe}}, h^{*2}=\frac{h^2}{L_D^2},\ k^{*2}=k^2L_D^2,$ $g^*=\frac{g}{n_{pe}^2L_D}$, where $n_{pe}=\left(\frac{\rho e^2}{m_e^2\varepsilon_0}\right)^{\frac{1}{2}}$ is the plasma frequency, then using the differential equation given by (23) in (25) yield

$$\left(\frac{1}{4}-n^{*2}\right)\left(-in^*-\frac{{n_q}^{*2}\,k^{*2}}{in^*}\right)-\frac{1}{2}\left(-in^*-\frac{{n_q}^{*2}\,k^{*2}}{in^*}\right)+\left((in^*)\,k^{*2}-\frac{g^*\,k^{*2}}{(in^*)}-\frac{k^{*2}n_\varepsilon^*}{n_{k_1}^*}\left(n_\nu^*+n_{\nu'}^*(-in)\right)\right)=0 \quad (27)$$

Let $n^*=n_r^*+i\gamma$ and in the case of $n_r^*=0$ and $\gamma\neq 0$ (stable oscillations), the square normalized growth rate may be determined from equations (27) as

$$\left(\frac{1}{4}-n^{*2}\right)\left(\gamma+\frac{{n_q}^{*2}k^{*2}}{\gamma}\right)-\frac{1}{2}\left(\gamma+\frac{{n_q}^{*2}k^{*2}}{\gamma}\right)+\left(-\gamma k^{*2}+\frac{g^*k^{*2}}{\gamma}-\frac{k^{*2}n_\varepsilon^*}{n_{k_1}^*}\left(n_\nu^*+\gamma\,n_{\nu'}^*\right)\right)=0, \quad (28)$$

$$\gamma^2\left[\frac{1}{k^{*2}}\left(\frac{1}{4}+n^{*2}\right)+\left(1+\frac{n_\varepsilon^*n_{\nu'}^*}{n_{k_1}^*}\right)\right]+\gamma\left[\frac{n_\varepsilon^*n_\nu^*}{n_{k_1}^*}\right]+\left[\left(\frac{1}{4}+\frac{n_1^2\pi^2}{h^{*2}}\right){n_q}^{*2}-g^*\right]=0, \quad (29)$$

$$a_1\gamma^2+a_2\gamma+a_3=0, \quad (30)$$

where

$$a_1=1+\frac{\left(1+\frac{n_\varepsilon^*n_{\nu'}^*}{n_{k_1}^*}\right)}{\left(\frac{h^{*2}+n_1^2\pi^2}{4h^{*2}\,k^{*2}}\right)}, a_2=\frac{\left(\frac{n_\varepsilon^*n_\nu^*}{n_{k_1}^*}\right)}{\left(\frac{h^{*2}+n_1^2\pi^2}{4h^{*2}\,k^{*2}}\right)}, a_3=\left({n_q}^{*2}\,k^{*2}-\frac{4\,g^*h^{*2}\,k^{*2}}{h^{*2}+n_1^2\pi^2}\right). \quad (31)$$

Case (i). When $n_\varepsilon^*=0, n_\nu^*=0, n_{\nu'}^*=0, n_q^*=0$, in Eq. (29) we find that $a_1=1, a_2=0$ and $a_3=-\frac{4\,g^*h^{*2}\,k^{*2}}{h^{*2}+n_1^2\pi^2}$ and we obtain the classical normalized growth rate (γ_C) in the absence of quantum physics as

$$\gamma_C=\sqrt{\frac{4\,g^*h^{*2}\,k^{*2}}{h^{*2}+n_1^2\pi^2}}. \quad (32)$$

In the absence of viscoelastic parameter $n_{\nu'}^*=0$, in (29), we obtain the normal growth ratewhich is similar as given by Goldston and Rutherford (1997) [14].

Case (ii). When $n_\varepsilon^*=0, n_\nu^*=0, n_{\nu'}^*=0, n_q^*\neq 0$, we have $a_1=1, a_2=0$ while a_3 as in equation (31) and the quantum normalized growth rate is given by

$$\gamma_q=\sqrt{\frac{4\,g^*h^{*2}k^{*2}}{h^{*2}+n_1^2\pi^2}-{n_q}^{*2}k^{*2}}, \quad (33)$$

which is in good agreement with the earlier result obtained by Hoshoudy (2009) [18, 19]. It is clear from the comparison of expressions (31) and (33) that the quantum term stabilize the effect on Rayleigh-Taylor instability problem.

3. Results and discussion

We shall now analyze the effect of various parameters on the instability of the system under consideration. For this we solve equation (30) using the software Mathematica 5.2. For the role of porosity of the porous medium, the medium permeability, kinematic viscosity with quantum term one may be referred to (Hoshoudy 2009, [18, 19]). So, we shall confine our attention on numerical results to study the role of simultaneous presence of kinematic viscoelasticity and quantum effect. For numerical computation we taken following values of the relevant parameters $n_\varepsilon^* = 0.3$, $n_q^* = 0.6$, $n_{k_1}^* = 0.4$, $n = 1$, $h = 1, g^* = 10$, $n_\nu^* = 0.2$, $n_{\nu'}^* = 0.6$, respectively.

Figures 1 and 2 correspond to the variation of the square of the normalized growth rate γ^2 w.r.t the square normalized wave number k^{*2} for four different values ofkinematic viscoelasticity $n_{\nu'}^* = 0.1, 0.3, 0.5, 0.9$ and kinematic viscosity $n_\nu^* = 0.2, 0.4, 0.6, 0.8$, respectively. It is clear from the graphs that with the increase in kinematic viscosity and kinematic viscoelasticity, the growth rate of the unstable perturbation decreases; thereby stabilizing the system, however the critical wavenumber k_c^{*2} remains the same i.e. 1.6.

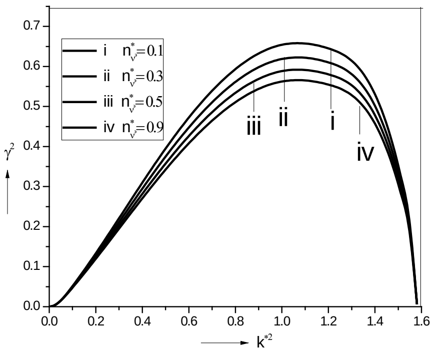

Fig. 1. Variation of γ^2 with k^{*2} for different values of kinematic viscoelasticity $n_{\nu'}^*$.

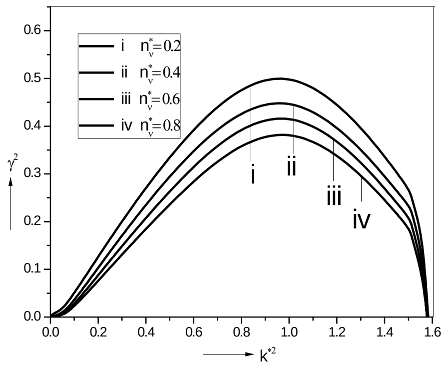

Fig. 2. Variation of γ^2 with k^{*2} for different values of kinematic viscosity n_ν^*.

Figures 3 and 4 correspond to the variation of the square of the normalized growth rate γ^2 w.r.t the square normalized wave number k^{*2} for three different values of medium porosity $n_\varepsilon^* = 0.1, 0.3, 0.7$ and quantum plasma $n_q^* = 0.0, 0.4, 0.6, 0.9$, respectively. It is clear from the graphs that in the presence of medium porosity n_ε^* has a slight stabilizing effect, whereas the critical wavenumber remains the same. i.e. 1.6. It is clear from the figure that in the presence of quantum plasma n_q^* square of the normalized growth rate γ^2 increases with the increasing k^{*2} until arrives at the maximum instability, then decrease with the increasing k^{*2} until arrives at the complete stability, where the maximum instability appears at k_{max}^{*2}=0.7 and the complete stability appears at k_c^{*2}=1.1. This graph shows that quantum effect play a major role in securing a complete stability.

4. Conclusions

The effect of quantum term on the Rayleigh-Taylor instability of stratified viscoelastic Rivlin –Ericksen (Model) fluid /plasma saturating a porous media has been studied. The principal conclusions of the present analysis are as follows:

1. The kinematic viscoelasticity stabilizing effect on the system and the critical wavenumber is k_c^{*2}=1.6.
2. The kinematic viscosity has a slight stabilizing effect on the system.
3. The medium porosity has a large stabilizing effect on the system.
4. Quantum plasma plays a major role in approaching a complete stability implying thereby the large enough stabilizing effect on the system.

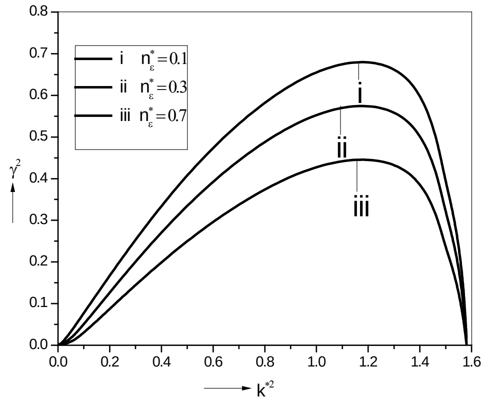

Fig. 3. Variation of γ^2 with k^{*2} for different values of medium porosity n_ε^*.

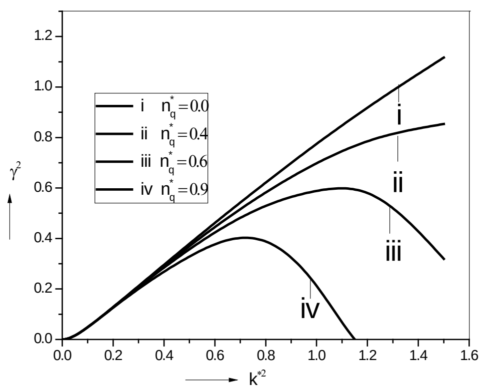

Fig. 4. Variation of γ^2 with k^{*2} for different values of quantum plasma n_q^*.

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