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GENERALIZED THERMOELASTIC AXI-SYMMETRIC DEFORMATION PROBLEM IN A THICK CIRCULAR PLATE WITH DUAL PHASE LAGS AND TWO TEMPERATURES

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Abstract. In the present work, a thick circular plate with axisymmetric heat supply has been considered. It is assumed that lower and upper surfaces of the plate are traction free. The plate is subjected to an axisymmetric heat supply depending on the radial and axial directions of the cylindrical polar co-ordinate system with symmetry about z–axis. The initial temperature in the thick plate is given by a constant temperature T_0 and the heat flux is prescribed on the upper and lower boundary surfaces. Integral transform technique and its inversion is applied to determine displacement components, stresses, conductive temperature and temperature change. The results have been computed numerically and are illustrated graphically.

Keywords: thermoelastic problem; two-temperatures; dual phase lags; thermal stresses; thick circular plate; integral transform.

1. Introduction

The generalized theory of thermoelasticity is one of the modified versions of classical uncoupled and coupled theory of thermoelasticity and has been developed in order to remove the paradox of physical impossible phenomena of infinite velocity of thermal signals in the classical coupled thermoelasticity. Hetnarski and Ignaczak [1] examined five generalizations of the coupled theory of thermoelasticity.

Lord and Shulman [2] formulated the generalized thermoelasticity theory involving one thermal relaxation time. Green and Lindsay [3] developed a temperature rate-dependent thermoelasticity that includes two thermal relaxation times.

The third generalization of the coupled theory of thermoelasticity is developed by Hetnarski and Ignaczak [4] and is known as low temperature thermoelasticity. The fourth generalization to the coupled theory of thermoelasticity introduced by Green and Nagdhi [5]. The fifth generalization to the coupled theory of thermoelasticity is developed by Tzou [6] and Chandrasekhariah [7] and is referred to dual phase lag thermoelasticity.

Tzou [6] introduced two phase lags to both the heat flux vector and the temperature gradient and considered constitutive equations to describe the lagging behavior in the heat conduction in solids. Roychoudhuri [8] has recently introduced the three phase lag heat conduction equation in which the Fourier law of heat conduction is replaced by an approximation to a modification of the Fourier law with the introduction of three different phase lags for the heat flux vector, the temperature gradient and the thermal displacement gradient.

Chen and Gurtin [9], Chen et al. [10] and Chen et al. [11] have formulated a theory of heat conduction in deformable bodies which depends upon two distinct temperatures, the conductive temperature φ and the thermodynamical temperature T. For time independent situations, the difference between these two temperatures is proportional to the heat supply, and in absence of heat supply, the two temperatures are identical. For time dependent problems, the two temperatures are different, regardless of the presence of heat supply. The two temperatures T, φ and the strain are found to have representations in the form of a travelling wave plus a response, which occurs instantaneously throughout the body (Boley and Tolins [12]). Various investigators have studied different problems of thermoelastic dual and three phase lag models with two temperatures e.g. [13, 14, 15, 16, 17, 18, 19, 20]. Recently, Tripathi et al. [21] discussed generalized thermoelastic diffusion problem in a thick circular plate with axisymmetric heat supply.

Recently Kumar et. al [22] studied the effects of two temperatures and thermal phase lags in a thick circular plate. In this paper, we have extended the work of Kumar et. al. [22] and studied the axisymmetric deformation due to thermomechanical sources for dual phase lags in a thick circular plate with two temperatures.

Basic Equations. The constitutive relations, equations of motion, heat conduction in a homogeneous isotropic thermoelastic solid with dual phase lags and two temperatures in the absence of body forces, heat sources are:

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij}(\lambda_{ekk} - \beta_1 T) \tag{1}$$

$$(\lambda+\mu)\nabla(\nabla.u) + \mu\nabla^2 u - \beta_1\nabla = p\,\ddot{u} \tag{2}$$

$$\left(1+\tau_t\frac{\partial}{\partial t}\right)KT,\ \ _{ij} = \left(1+\tau_q\frac{\partial}{\partial t}+\tau_q^2\frac{\partial^2}{\partial t^2}\right)\left[\rho C_E\dot{T} + \beta_1 T_0\dot{e}_{kk}\right] \tag{3}$$

$$T = (1 - a\nabla^2)\varphi, \tag{4}$$

where λ, μ are Lame's constants, ρ is the density assumed to be independent of time, u_i are components of displacement vector u, K is the coefficient of thermal conductivity, C_E is the specific heat at constant strain, T is the absolute temperature of the medium, σ_{ij} and e_{ij} are the components of stress and strain respectively, e_{kk} is dilatation, S is the entropy per unit mass, $\beta_1 = (3\lambda + 2\mu)\alpha_t$, α_t is the coefficient of thermal linear expansion. τ_t, τ_q are respectively phase lag of temperature gradient, the phase lag of heat flux, 'a' is the two temperatures parameter. In the above equations, a comma followed by suffix denotes spatial derivative and a superposed dot denotes derivative with respect to time.

2. Formulation and Solution of the Problem

Consider a thick circular plate of thickness $2b$ occupying the space D defined by $0 \le r \le \infty, -b \le z \le b$. Let the plate be subjected to an axisymmetric heat supply depending on the radial and axial directions of the cylindrical polar co-ordinate system (r, θ, z) with symmetry about z–axis. The initial temperature in the thick plate is given by a constant temperature T_0 and the heat flux $g_0 F(r, z)$ is prescribed on the upper and lower boundary surfaces. With these restrictions, thermomechanical quantities are required to be determined. As the considered problem is two dimensional axisymmetric problem, therefore we have

$$u = (u_r, 0, u_z) \tag{5}$$

Equations (2)-(3) with the aid of (5) takes the form

$$(\lambda+\mu)\frac{\partial e}{\partial r} + \mu\left(\nabla^2 - \frac{1}{r^2}\right)u_r - \beta_1\frac{\partial T}{\partial r} = \rho\frac{\partial^2 u_r}{\partial t^2} \tag{6}$$

$$(\lambda+\mu)\frac{\partial e}{\partial z} + \mu\nabla^2 u_z - \beta_1\frac{\partial T}{\partial z} = \rho\frac{\partial^2 u_z}{\partial t^2} \tag{7}$$

$$\left(1+\tau_t\frac{\partial}{\partial t}\right)K\nabla^2 T = \left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\left[\rho C_E\frac{\partial}{\partial t}(1-a\nabla^2)\varphi + \beta_1 T_0\frac{\partial}{\partial t}div\,u\right], \tag{8}$$

where

$$e=\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z}, e_{rr}=\frac{\partial u_r}{\partial r}, e_{\theta\theta}=\frac{u_r}{r}+\frac{\partial u\theta}{\partial \theta}, e_{zz}=\frac{\partial u_z}{\partial z}, e_{rz}=\frac{1}{2}\left(\frac{\partial u_r}{\partial z}+\frac{\partial u_z}{\partial r}\right) \tag{9}$$

To facilitate the solution, the following dimensionless quantities are introduced:

$$r'=\frac{\omega_1^*}{c_1}r, z'=\frac{\omega_1^*}{c_1}z, (u_r', u_z')=\frac{\omega_1^*}{c_1}(u_r, u_z), t'=\omega_1^* t, (T', \varphi')=\frac{\beta_1}{\rho c_1^2}(T, \varphi)$$

$$(\tau_q', \tau_t')=\omega_1^*(\tau_q, \tau_t), (\sigma_{rr}', \sigma_{\theta\theta}', \sigma_{zz}', \sigma_{rz}')=\frac{1}{\beta_1 T_0}(\sigma_{rr,}\sigma_{\theta\theta}, \sigma_{zz}, \sigma_{rz}), \tag{10}$$

where $\omega_1^*=\frac{\rho C_E c_1^2}{K}$ and $c_1^2=\frac{\lambda+2\mu}{\rho}$

In equations (6)-(8) and after that suppressing the primes and then applying the Laplace and Hankel transforms defined by:

$$\hat{f}(r,z,s)=\int_0^\infty f(r,z,t)e^{-st}dt \tag{11}$$

$$\bar{f}(\xi,z,s)=\int_0^\infty \hat{f}(r,z,s)rJ_n(r\xi)dr \tag{12}$$

On the resulting quantities and simplifying we obtain:

$$(D^2-\xi^2-k_i^2)\left(\overline{\varphi}_i^*, \overline{e}_i^*\right)=0 \tag{13}$$

The solution of (13) has the form:

$$\bar{e}^*=\sum_{i=1}^2 A_i(\xi,s)\cosh(q_i z) \tag{14}$$

$$\bar{\varphi}^*=\sum_{i=1}^2 d_i A_i(\xi,s)\cosh(q_i z), \tag{15}$$

where

$$q_i=\sqrt{\xi^2+k_i^2}, d_i=\frac{\tau_q^0\zeta_2}{\tau_q^0\zeta_1-\zeta_3 q_i^2}, \zeta_3=\tau_2^0\delta_1-\tau_t^0 k, \tau_q^0=1+s\tau_q+\frac{s^2\tau_q^2}{2}, \tau_t^0=1+s\tau_t$$

$$\zeta_1=\frac{\rho C_E c_1^2}{\omega_1^*}, \zeta_2=\frac{\beta_1^2 T_0}{\rho\omega_1^*}s, \delta_1=\frac{a\omega_1^{*2}}{c_1^2}$$

The inverse Hankel transform is given by the relation:

$$\bar{f}(r,z,s)=H^{-1}\hat{f}(\xi,z,s)=\int_0^\infty \hat{f}(\xi,z,s)\xi J_0(r\xi)d\xi$$

Applying inversion of Hankel transform on (14), and (15), we obtain:

$$\bar{e}=\int_0^\infty\left[\sum_{i=1}^2 A_i(\xi,s)\cosh(q_i z)\right]\xi J_0(\xi r)d\xi \tag{16}$$

$$\bar{\varphi}=\int_0^\infty\left[\sum_{i=1}^2 d_i A_i(\xi,s)\cosh(q_i z)\right]\xi J_o(\xi r)d\xi \tag{17}$$

Boundary Conditions. We consider a thermal source and normal force of unit magnitude along with vanishing of shear stress components at the stress free surface $z=\pm b$. Mathematically, these can be written as:

$$\frac{\partial\varphi}{\partial z}=\pm F(r,z) \tag{18}$$

$$\sigma_{zz}=H(t)f(r) \tag{19}$$

$$\sigma_{rz}=0, \tag{20}$$

where $\sigma_{zz}=2\mu e_{zz}+\lambda e-\beta_1(1-a\nabla^2)\varphi,\ \sigma_{rz}=\mu e_{rz}$ (21)

and $H(t)$ is the Heaviside unit step function.

Applying Laplace transform and Hankel transforms defined by (11)-(12) on the boundary conditions (18)-(20), we obtain:

$$\frac{d\bar{\varphi}}{dz}=\pm\bar{F}(\xi,z) \tag{22}$$

$$\bar{\sigma}_{zz}=\frac{1}{s}\bar{f}(\xi) \tag{23}$$

$$\bar{\sigma}_{rz}=0 \tag{24}$$

Substituting the values of $\bar{e}$ and $\bar{\varphi}$ from equations (16) and (17) in the boundary conditions (22)-(24), and using equations (6)-(10), we obtain the displacement components, stress components, conductive temperature and temperature change as:

$$\bar{u}_r(r,z,s) = \int_0^\infty \frac{1}{\Delta}\Bigg[\Delta_3\, cosh(qz)\xi J_0(\xi r) + \sum_{i=1}^2 \Delta_i \Bigg\{\Big(((-\eta_i+\mu_i)q_i^2\xi^2\, cosh(q_i z)\Big)J_1(\xi r) + \left\{\delta_1\mu_i\, cosh(q_i z)\left(\frac{\xi^3}{r}J_2 - J_1\left(\xi^4 - \frac{\xi^2}{r^2} + \xi^2 q_i^2\right) + \frac{\xi^3}{r}J_0\right)\right\}\Bigg\}\Bigg] d\xi \quad (25)$$

$$\bar{u}_z(r,z,s) = \int_0^\infty \frac{1}{\Delta}\Bigg[\Delta_3\, sinh(qz)\,\xi^3 J_0(\xi r) + \sum_{i=1}^2 \Delta_i \Bigg\{(-\eta_i+\mu_i)\, sinh(q_i z)\xi J_0(\xi r) - \delta_1\mu_i\, sinh(q_i z)\left(\frac{\xi^2}{r}J_2 - J_1\left(\xi^3 - \frac{\xi}{r} + \xi q_i^2 J_0\right)\right)\Bigg\}\Bigg] d\xi \quad (26)$$

$$\bar{\sigma}_{zz} = \frac{2\mu}{\Delta\beta_1 T_0}\int_0^\infty \Big[\xi J_o(\xi r)\{\xi^2\Delta_3\, cosh(qz) + (\sum_{i=1}^2(-\eta_i+\mu_i)q_i^2 - \zeta d_i + \lambda')\Delta_i\, cosh(q_i z)\} - \sum_{i=1}^2\Big\{\delta_1(\mu_i q_i - \zeta d_i)\Delta_i\, cosh(q_i z)\left(\xi^3 J_0(\xi r) - J_1(\xi r)\left(\frac{\xi-1}{r}\right) + \xi q_i^2 J_0(\xi r)\right)\Big\}\Big] d\xi \quad (27)$$

$$\bar{\sigma}_{rz} = \frac{\mu}{2\Delta\beta_1 T_0}\int_0^\infty \Bigg[\xi^2 J_1(\xi r)\left\{\left(\frac{q^2-\xi^2}{q}\right)\Delta_3 q\, sinh(qz) + 2\sum_{i=1}^2(\eta_i-\mu_i)q_i\Delta_i\, sinh(q_i z)\right\} + \sum_{i=1}^2\delta_1\mu_i\Delta_i\, sinh(q_i z)\Bigg\{\Bigg(q_i\left(\frac{\xi^3}{r}J_2(\xi r)\right) - J_1(\xi r)\left(\xi^4 + q_i^2\xi^2 + \frac{4}{\xi^2 r^2} + q_i^2\xi\right) + \left(\frac{2}{\xi r} - \xi^4 - \frac{\xi^2(\xi-1)}{r} + q_i^2\xi^2\right)J_0(\xi r)\Bigg)\Bigg\}\Bigg] d\xi \quad (28)$$

$$\bar{\varphi} = \int_0^\infty \frac{1}{\Delta}\{\sum_{i=1}^2 d_i\Delta_i(\xi,s)\, cosh(q_j z)\}\xi J_0(\xi r) d\xi \quad (29)$$

$$\bar{T} = \int_0^\infty \frac{1}{\Delta}\left\{\sum_{i=1}^2\left(d_i\Delta_i(\xi,s)\cosh(q_i z)\begin{bmatrix}\xi J_0(\xi r)(1+\delta_1 q_i^2 - \delta_1\xi) + \\ J_1(\xi r)\delta_1\xi\left(1-\frac{1}{r}\right)\end{bmatrix}\right)\right\} d\xi, \quad (30)$$

where

$$\Delta = -\frac{2\mu}{\beta_1 T_0}cosh(qb)(\Delta_{11}\Delta_{32} - \Delta_{12}\Delta_{31}) + sinh(qb)(\Delta_{11}\Delta_{22} - \Delta_{12}\Delta_{21})$$

$$\Delta_1 = g_0\bar{F}(\xi,z)(\Delta_{21}\Delta_{32} - \Delta_{22}\Delta_{31}) - \bar{f}(\xi)(\Delta_{11}\Delta_{32} - \Delta_{12}\Delta_{31})$$

$$\Delta_2 = -g_0\bar{F}(\xi,z)\left(\frac{2\mu}{\beta_1 T_0}cosh(qb)\Delta_{32} - \Delta_{22}\, sinh(qb)\right) + \frac{1}{s}\bar{f}(\xi)(-\Delta_{12}\, sinh(qb))$$

$$\Delta_3 = g_0\bar{F}(\xi,z)\left(\frac{2\mu}{\beta_1 T_0}cosh(qb)\Delta_{31} - \Delta_{21}\, sinh(qb)\right) + \frac{1}{s}\bar{f}(\xi)(\Delta_{11}\, sinh(qb))z \quad (31)$$

and

$$\Delta_{1i} = d_i q_i\, sinh(q_i b),\ \Delta_{2i} = \left((\mu_i - \eta_i)q_i^2 - \delta_1\mu_i q_i - \zeta d_i(1+\delta_1) + \lambda^0\right)cosh(q_i b)$$

$$\Delta_{3i} = (2(\eta_i - \mu_i)q_i + \delta_1\mu_i)\, sinh(q_i b),\ i = 1,2 \quad (32)$$

Applications. As an application of the problem, we take the source functions as:

$$F(r,z) = z^2 e^{-\omega r} \text{ and } f(r) = H\,(a-r) \quad (33)$$

Applying Laplace and Hankel Transforms defined by (11)-(12) on (33), we obtain:

$$\bar{F}(\xi,z) = \frac{z^2\omega}{(\xi^2+\omega^2)^{3/2}} \quad (34)$$

$$\bar{f}(\xi) = \frac{aJ_1(\xi a)}{\xi} \quad (35)$$

Making use of the values of $\bar{F}(\xi,z)$ and $\bar{f}(\xi)$ from (34) and (35) in (25)-(30), one can obtain the displacement components, stress components, conductive temperature and temperature change for thermomechanical sources.

Particular Cases. (1) In the limiting case $a \to 0$ in equation (25)-(30), yields the expressions for displacements and stresses, conductive temperature and temperature change for dual-phase-lag thermoelastic solid without two temperatures. These results are similar if we solve the problem directly.

(2) If $\tau_q = \tau_t = 0$ we obtain the corresponding expressions for thermoelasticity with two temperatures model .

(3) If $\tau_q = 0$ then the corresponding expressions for dual phase lag thermal model (DPLT) model reduce to single-phase-lag thermal model (SPLT) with two temperatures.

Numerical Inversion of Transforms. To obtain the solution of the problem in physical domain, we must invert the transforms in (25)-(30) for all the theories. Here the displacement components, normal and tangential stresses, temperature change, chemical potential and mass concentration are functions of z. We first invert the Hankel transform, which gives the Laplace transform expression $\bar{f}(r,z,s)$ of the function $f(r,z,t)$ as:

$$\bar{f}(r,z,s) = \int_0^\infty \eta \hat{f}(\eta,z,s) J_n(\eta r) d\eta \tag{36}$$

Now for fixed values of η, r and $z$the function $\bar{f}(r,z,s)$ in (36) can be considered as the Laplace transform $\bar{g}(s)$ of the same function $g(t)$. Following Honig and Hirdes [23], the Laplace transform function $\bar{g}(s)$ can be inverted as given below:

$$g(t) = \frac{1}{2\pi i}\int_{C-i\infty}^{C+i\infty} e^{st}\bar{g}(s)ds, \tag{37}$$

where C is an arbitrary real number greater than all the real parts of the singularities of $\bar{g}(s)$. Taking $s = C + iy$, we get:

$$g(t) = \frac{e^{Ct}}{2\pi}\int_{-\infty}^{\infty} e^{ity}\bar{g}(C+iy)dy, \tag{38}$$

Now, taking $e^{-Ct}g(t)$ as $h(t)$ and expanding it as Fourier series in $[0.2L]$ we obtain approximately the formula Honig and Hirdes [23]:

$$g(t) = g_\infty(t) + E_D \tag{39}$$

$$g_\infty(t) = (C_0/2) + \sum_{k=1}^{\infty} C_k, \text{ for } 0 \le t \le 2L \tag{40}$$

and

$$C_k = (e^{Ct}/L) Re\left[e^{ik\pi t/L}\bar{g}\left(C + (ik\pi/L)\right)\right] \tag{41}$$

E_D is the discretization error that can be made arbitrarily small by choosing a large enough C. The values of C and L are chosen according to the criteria outlined by Honig and Hirdes [23].

Since the infinite series in (40) can be summed up only to a finite number of N terms, the approximate value of $g(t)$ becomes:

$$g_N(t) = (C_0/2) + \sum_{k=1}^{N} C_k, \text{ for } 0 \le t \le 2L \tag{42}$$

We now introduce a truncation error E_T that must be added to the discretization error to produce the total approximate error in evaluating $g(t)$ using the above formula. Two methods are used to reduce the total error. The discretization error is reduced by using the ‘Korrecktur’ method Honig and Hirdes [23] and then the ‘ε-algorithm’ is used to reduce the truncation error and hence to accelerate the convergence. The Korrecktur method formula, to evaluate the function $g(t)$ is:

$$g(t) = g_\infty(t) - e^{-2CL}g_\infty(2L+t) + E'_D,$$

where $|E'_D| << |E_D|$ Honig and Hirdes [23].

Thus the approximate value of $g(t)$ becomes:

$$g_{N_k}(t) = g_N(t) - e^{-2CL}g_{N'}(2L+t), \tag{43}$$

where N' is an integer such that $N' < N$.

We shall now describe the ε-algorithm, which is used to accelerate the convergence of the series in (42). Let N be an odd natural number and $s_m = \sum_{k=1}^{m} C_k$ be the sequence of partial sums of (42). We define the ε-sequence by:

$$\varepsilon_{0,m} = 0, \varepsilon_{1,m} = s_m, \varepsilon_{n+1,m}\varepsilon_{n-1,m+1} + \frac{1}{\varepsilon_{n,m+1} - \varepsilon_{n,m}};\ n,m = 1,2,3 \tag{44}$$

It can be shown that Honig and Hirdes [23], the sequence $\varepsilon_{1,1}, \varepsilon_{3,1}, \dots, \varepsilon_{N,1}$ converges to $g(t) + E_D - (C_0/2)$ faster than the sequence of partial sums $s_{m,}\ m = 1,2,3$. The actual procedure to invert the Laplace transform consists of (43) together with the ε-algorithm.

The last step is to calculate the integral in equation (36). The method for calculating this integral is described by Press et al. [24]. It involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

3. Numerical Results and Discussion

The mathematical model is prepared with copper material for the purpose of numerical computation. The material constants for the problem are taken from Dhaliwal and Singh [25]

$\lambda = 7.76 \times 10^{10} Nm^{-2}, \mu = 3.86 \times 10^{10} Nm^{-2}, K = 386 JK^{-1}m^{-1}s^{-1}$

$\beta_1 = 5.518 \times 10^6 Nm^{-2}deg^{-1}, \rho = 8954 Kgm^{-3}, a = 1.2 \times 10^4 m^2s^{-2}k^{-1}$

$b = 0.9 \times 10^6 \dfrac{m^5}{kgs^2}, D = 0.88 \times 10^{-8}\, kgs/m^3, \beta_2 = 61.38 \times 10^6 Nm^{-2}deg^{-1}$

$T_0 = 293K, C_E = 383.1\, Jkg^{-1}k^{-1}$

The graphical representation has been given to study the effect of phase lags and two temperatures on the various quantities for the range $0 \le r \le 10$.

In all the figures, solid line with $(a = 0.08, t = 0.2)$ and without centre symbols $(a = 0, t = 0.2)$ corresponds to the dual-phase-lags thermoelastic with and without two temperatures. Small dash line with $(a = 0.08, t = 0.4)$ and without centre symbols $(a = 0, t = 0.4)$ corresponds to the dual-phase-lags thermoelastic with and without two temperatures.

Figure 1 exhibits variations of radial displacement u_r with distance r. Near the loading surface, there is a sharp decrease for the range $0 \le r \le 2$ and behaviour is oscillatory for the rest corresponding to all the cases with amplitudes of oscillations decreasing as r increases. Amplitude of oscillation is more with two temperatures than without two temperatures. For $a = 0$, small variations near zero are observed for $t = 0.2$ and $t = 0.4$.

Figure 2 shows variations of axial displacement u_z with distance r. Here behaviour is oscillatory for the whole range except for $0 \le r \le 1.5$, as for this range , there is a sharp decrease. Maximum variations are observed corresponding to $a = 0, t = 0.2$ for the range $0 \le r \le 4$. As r diverges from point of application of source, variations start decreasing.

Figure 3 shows variation of vertical stress σ_{zz} with distance r. .We find that there is a sharp increase for the range $0 \le r \le 3$ for all the cases and similar oscillatory trend is observed afterwards. Small variations near zero are noticed for the range $5 \le r \le 10$. Figure 4 gives variations of shear stress σ_{rz} with distance r. It is evident from this figure that the behaviour is descending oscillatory with a sharp decrease for the range $0 \le r \le 2$. For $a = 0$, more variations are noticed.

Figure 5 gives variations of conductive temperature φ with distance r. Here , we notice that either there are sudden increases and decreases or there are small variations. Here descents are observed at the points $r = 0.5$ and $r = 2.5$ and hikes are observed at the points $r = 6.5$ and $r = 9$ for all the cases. Figure 6 exhibits variations of temperature change T with displacement r. Here there is a descent at the point $r = 0.5$ and hikes at the points $r = 2.5$, $r = 4.5$, $r = 6.5$ and a small descent is observed at $r = 9$ and small variations are noticed for the remaining range except the small neighbourhoods of these points.

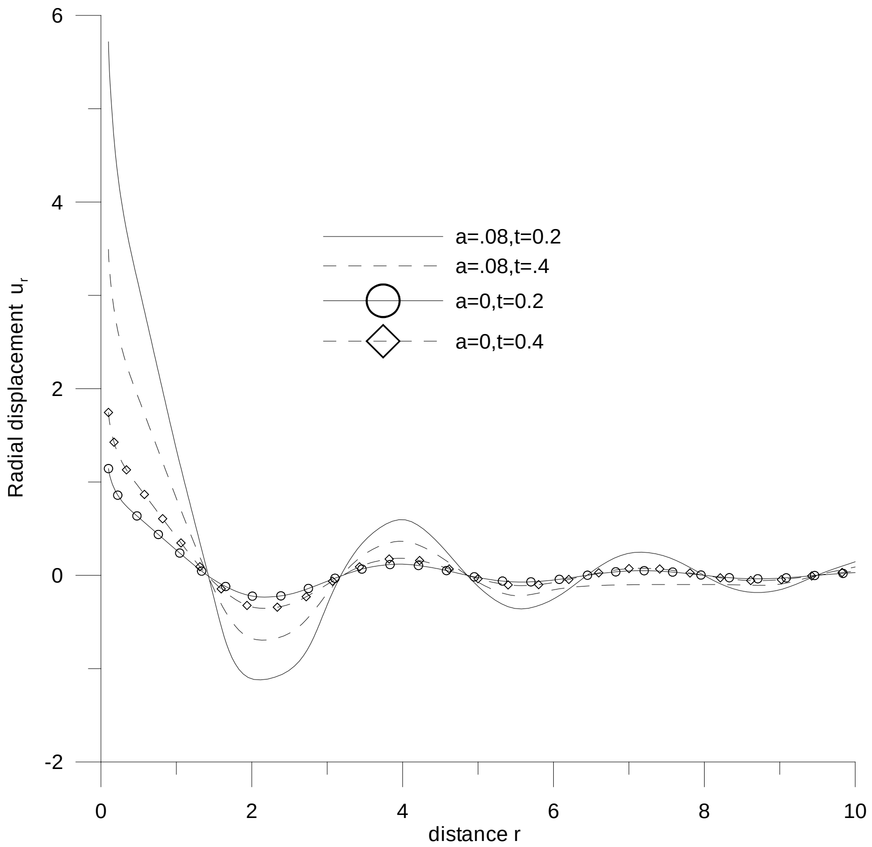

Fig. 1. Variations of radial displacement u_r with distance r.

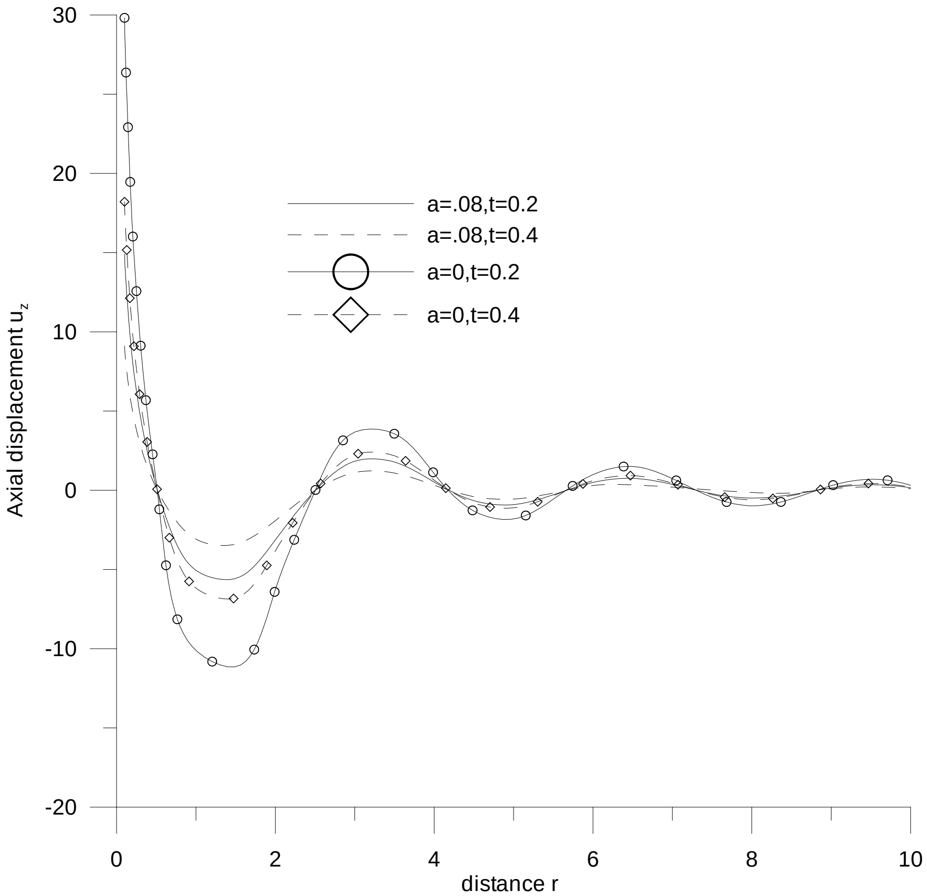

Fig. 2. Variations of axial displacement u_z with distance r.

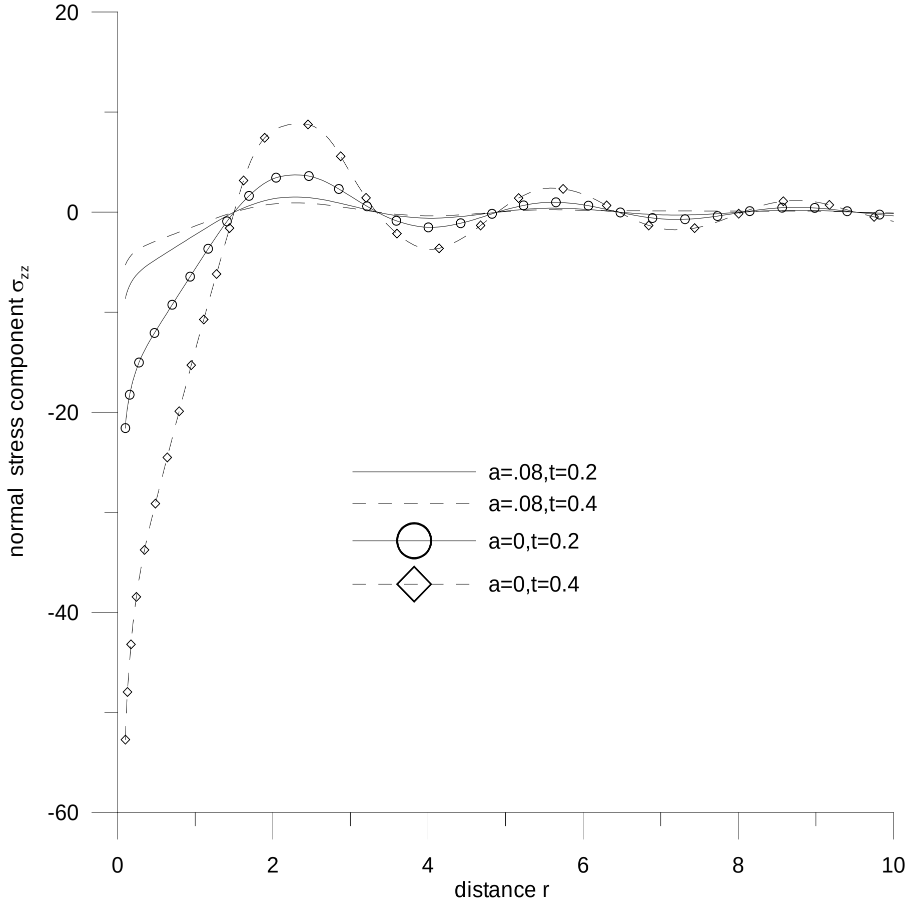

Fig. 3. Variation of normal stress component σ_{zz} with distance r.

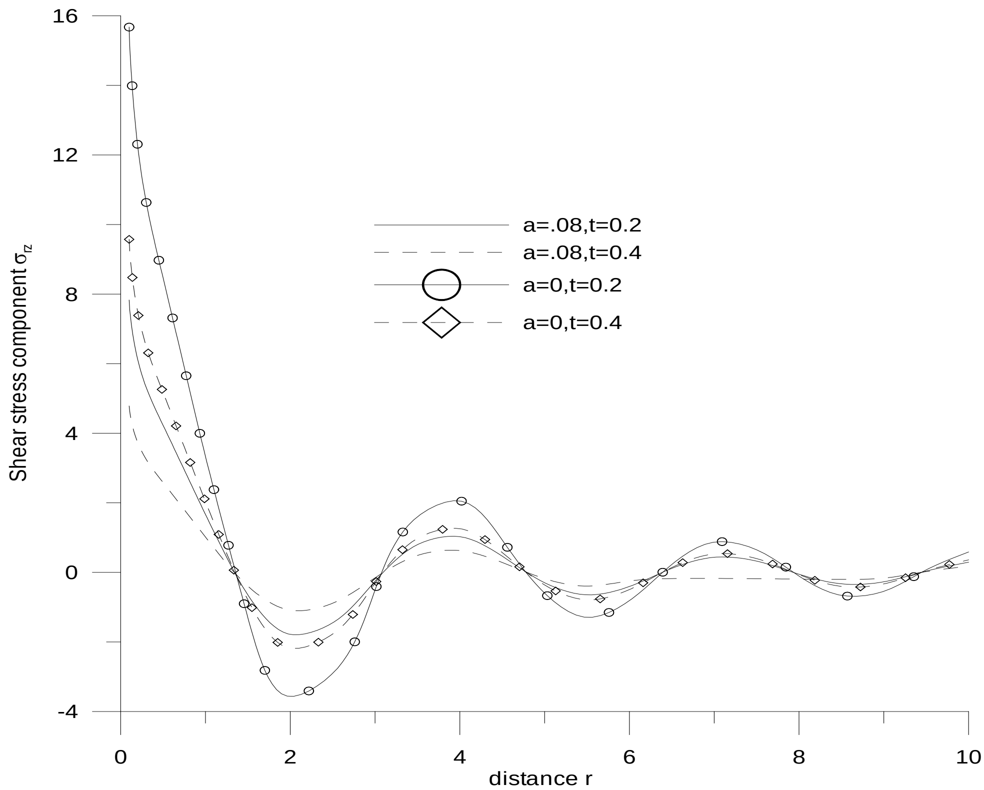

Fig. 4. Variations of shear stress component σ_{rz} with distance r.

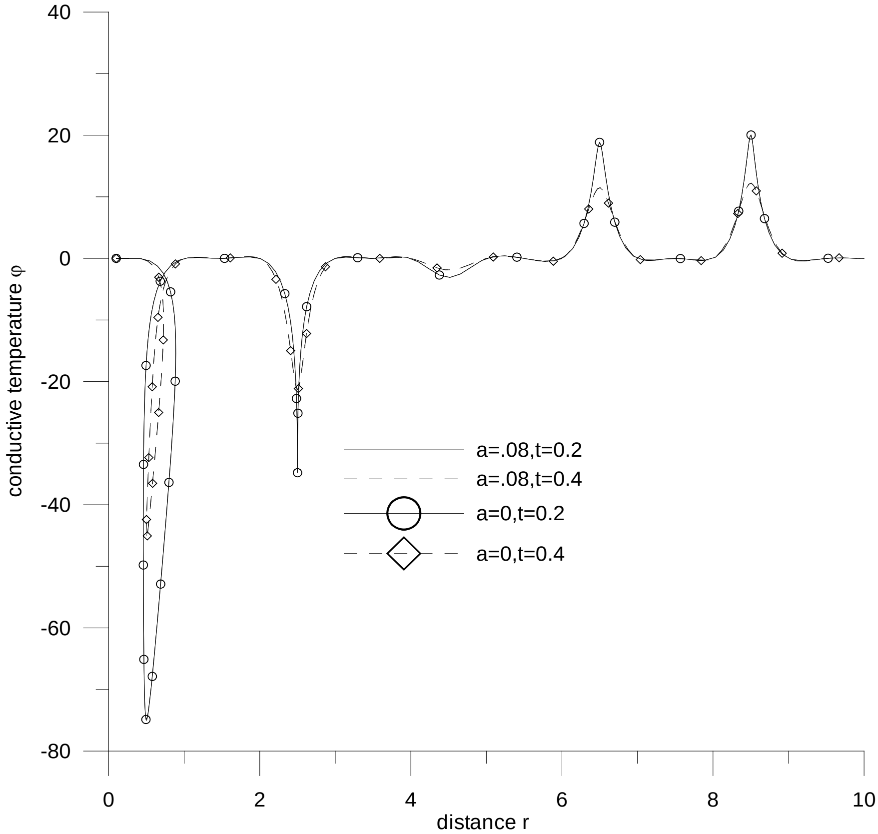

Fig. 5. Variation of conductive temperature φ with distance r.

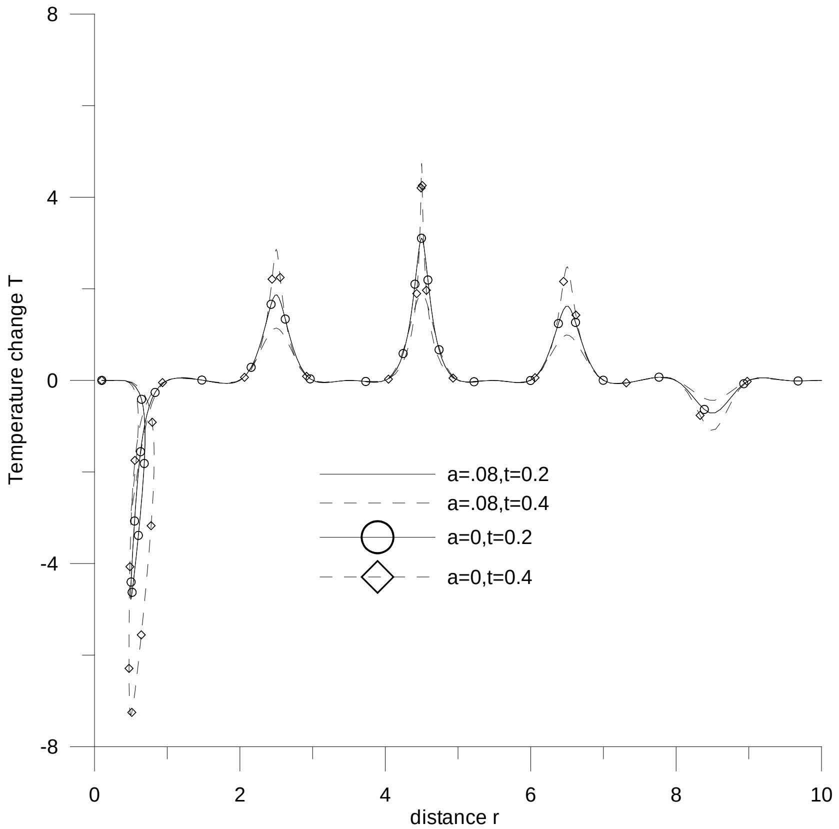

Fig. 6. Variations of temperature change T with distance r.

4. Conclusion

From the graphs, it is evident that there is a significant impact on behaviour of deformation of various components of stresses, components of displacement, conductive temperature and temperature change in the thick circular plate due to time variation with dual phase lags and two temperatures. Due to two temperatures and dual phase lags, instant hikes and descents are observed in temperature change, conductive temperature and radial stress σ_{rr}. Oscillatory trends are observed in components of displacement, shear stress and normal stress σ_{zz}. As r diverges from point of application of source, variations start decreasing and small variations near zero are observed. As disturbance travels through the constituents of the medium, it suffers sudden changes resulting in an inconsistent / non uniform pattern of graphs in case of conductive temperature, temperature change. The use of thermal phase-lags in the heat conduction equation gives a more realistic model of thermoelastic media as it allows a delayed response to the relative heat flux vector. The result of the problem is useful in the two dimensional problem of dynamic response due to various thermal and mechanical sources which has various geophysical and industrial applications.

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