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# PLASTIC FORMING MODEL FOR AXISYMMETRIC SHELLS

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**Abstract.** The mathematical model of plastic forming dome-like shells is present. The forming is performed by pressure from a flat circular plate, clamped along the contour. Point force can be applied in the center. The semi-inverse method is used to resolve this physical and geometrical nonlinear problem.

**Keywords:** large deformations; nonlinear mathematical model; semi-inverse method; forming of dome-type shells.

## 1. Introduction

Mathematical modeling of plastic changes in the shape of shells of rotation is actual for improving the technologies of plastic forming dome-like shells having a predetermined critical load of buckling under the action of uniformly distributed load from the bulge. The manufacturing of such shells is performed by free stretching by the pressure of the flat round plates, rigidly clamped on a circular contour.

## 2. Equations

The model is based on the equations that allow large displacements and angles of rotation, changing of the middle surface metric and compression of the material normal. Power approximation of the diagrams of hardening material and physical deformation ratio with a logarithmic relative elongation are used [1].

It is proposed that relative elongations $e_k$, $\varepsilon_k=e_k|_{\zeta=0}$, $k$=1, 2, 3, is comparable to one; the compression of the material normal $e_3=\varepsilon_3$ is constant in thickness. Kinematic relations are of the form [1]:

$$e_1=(\varepsilon_1+\zeta\kappa_1),\ \ \kappa_1=\Phi_o'/\alpha_o-\delta_3K_1,\ \ \varepsilon_1=(w'\sin\Phi+u'\cos\Phi)/\alpha_o+\cos(\Phi-\Phi_o)-1;$$

$$e_2=(\varepsilon_2+\zeta\kappa_2),\ \ \varepsilon_2=u/r_o,\ \ \kappa_2=(\sin\Phi_o)/r_o-\delta_3K_2;\ \ K_1=\Phi'/\alpha_o,\ \ K_2=(\sin\Phi)/r_o.$$

$$\gamma=\gamma_o/\delta_1,\ \gamma_o=(w'\cos\Phi-u'\sin\Phi)/\alpha_o-\sin(\Phi-\Phi_o).$$

Here $\Phi_o$ and $\Phi$ are the angles of the incline of material normal to the rotation axis before and after deformation; $\kappa_1$ and $\kappa_2$ are the characteristics of change of the main curvatures; $\delta_k=1+\varepsilon_k$, $k$=1, 2, 3. For the plate in the initial state, the incline angle of the axis of symmetry $\Phi_o=0$. The angle of the transverse shear $\gamma$ is supposed small, and below is considered to be zero.

The differential system of equations for a plate, loaded with uniform pressure, has the dimensionless form:

$$d\overline{T}^o/d\xi=\alpha_o\xi\delta_1\delta_2 p\cos\Phi\,,\ \ d\overline{\Psi}^o/d\xi=\alpha_o\overline{N}_2^o-\alpha_o r_o\delta_1\delta_2 p\sin\Phi\,,$$

$$dM^o/d\xi=\alpha_o M_2^o\cos\Phi+\alpha_o r_o\delta_1\overline{Q}^o/\varepsilon_*,$$

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$$dw/d\xi = \alpha_o\delta_1 \sin\Phi\,,\ du/d\xi = \alpha_o(\delta_1\cos\Phi - 1)\,,\ d\Phi/d\xi = \alpha_o K_1, \quad (1)$$

where

$$\overline{T}^o = r_o\overline{V}^o,\ \ \Psi^o = r_o\overline{H}^o,\ \ M^o = r_o M_1^o;$$

$$\overline{V}^o = \overline{N}_1^o \sin\Phi + Q^o\cos\Phi\,,\ \ \overline{H}^o = \overline{N}_1^o\cos\Phi - Q^o\sin\Phi\,,$$

$$\overline{N}_1^o = \overline{V}^o\sin\Phi + \overline{H}^o\cos\Phi\,,\ \ \overline{Q}^o = \overline{V}^o\cos\Phi - \overline{H}^o\sin\Phi\,; \quad (2)$$

$$\overline{N}_j^o = N_j^o + \varepsilon_*(K_1M_1^o + K_2M_2^o)/\delta_j,\ j = 1, 2. \quad (3)$$

Here $\xi\in[0, 1]$ is the independent radial Lagrangian coordinate; $p$ is the intensity of hydrostatic pressure; $\varepsilon_*=h_*/R_*$ is the thin-wall parameter; $\overline{V}^o$ and $\overline{H}^o$ are the internal efforts in the direction of the axis of symmetry and radius of the cylindrical coordinate system; $M_1^o$ is the bending moment. The coefficient of Lame $\alpha_o$ is supposed equal to one.

Many metals and alloys at large plastic deformation behave as almost incompressible. Therefore, the condition $\delta_1\delta_2\delta_3=1$ is assumed to be satisfied.

The material properties are characterized by the diagram of loading, which is approximated by a power function in the area of hardening:

$\sigma=E\bar{e}$, $\bar{e}\le\bar{\varepsilon}_{0.2}$, $\sigma=C\bar{e}^{\,\eta}=E_S\bar{e}$, $\bar{e}\ge\bar{e}_{0.2}$, $\bar{e} = (2/\sqrt{3})\sqrt{\bar{e}_1^2 + \bar{e}_1\bar{e}_2 + \bar{e}_2^2}$,

where $\bar{e}$ is the logarithmic deformation intensity of an incompressible material; $\bar{e}_j = \ln(1+e_j)$; $C$, $\eta$ are material constants; $\sigma_{0.2}$, $\bar{\varepsilon}_{0.2}$ are the stress and strain of the conditional yield strength, $E$ is the Young's modulus, $E_s = C\bar{e}^{\,\eta-1}$ is the secant modulus.

We use the physical ratio of the Davis – Nadai for an incompressible material, coupling stress and the logarithmic strain in the principle axes:

$\sigma_1=(4/3)\Lambda(\bar{e}_1 + 0.5\bar{e}_2)$, $\sigma_2=(4/3)\Lambda(\bar{e}_1 + 0.5\bar{e}_2)$.

Here $\Lambda=E$ in the areas of elasticity $\bar{e}\le\bar{e}_{0.2}$ and $\Lambda(\bar{e})=E_c(\bar{e})$ in the plastic zones $\bar{e}_{0.2}<\bar{e}<\bar{e}_l$; $\tilde{\bar{e}} = \bar{e}/\bar{e}_l$, where, $\sigma_l$, $\bar{e}_l$ are the elastic limit stresses and strains.

Diagram of material properties is transferred to a single plane in the coordinates of the dimensionless stress $\tilde{\sigma}=\sigma/\sigma_l$ and the relative actual strains. These relations in dimensionless form take the view:

$\tilde{\sigma}=\sigma/\sigma_l$, $\bar{e} = \bar{e}_l\tilde{\bar{e}}$, $\tilde{\sigma}(\tilde{\bar{e}}) = \tilde{C}\bar{e}_l^{\eta}\tilde{\bar{e}}^{\eta} = \tilde{C}_l\tilde{\bar{e}}^{\eta}$, $\tilde{C} = \tilde{C}/\sigma_l, \tilde{C}_l = \tilde{C}\bar{e}_l^{\eta} = 1$,

$\tilde{E}_s(\tilde{\bar{e}}) = \tilde{C}\bar{e}_l^{(\eta-1)}\tilde{\bar{e}}^{(\eta-1)} = \tilde{C}_E\tilde{\bar{e}}^{(\eta-1)}, \tilde{C}_E = \tilde{C}\bar{e}_l^{(\eta-1)} = 1/\bar{e}_l$.

Further, to simplify the notation a tilde (~) is deleted. When constructing two-dimensional equations, $\Lambda(\bar{e})\approx\Lambda(\bar{\varepsilon})$ can be put. This is justified in the considered task of strong drawing. Then the formulae of the dependences of forces and moments on strain components in dimensionless form has the simpler form:

$$N_1^o = (k_\sigma/\varepsilon_*)B_1(\bar{\varepsilon}_1 + 0.5\bar{\varepsilon}_2)\,,\ N_2^o = (k_\sigma/\varepsilon_*)B_2(\bar{\varepsilon}_2 + 0.5\bar{\varepsilon}_1)\,, \quad (4)$$

$$M_1^o = -k_\sigma D_1\left(\overline{K}_1 + 0.5\overline{K}_2\right),\ M_2^o = -k_\sigma D_2\left(\overline{K}_2 + 0.5\overline{K}_1\right), \quad (5)$$

where

$\bar{\varepsilon}_j = \ln(1+\varepsilon_j) = \ln\delta_j$, $\overline{K}_j = K_j/\delta_j$, $j$=1, 2; $k_\sigma=\sigma_l/E_*$;

$\overline{B}_1 = \overline{B}/\delta_1$, $\overline{B}_2 = \overline{B}/\delta_2$, $\overline{D}_1 = \delta_3^3\delta_2\overline{D}$, $\overline{D}_2 = \delta_3^3\delta_1\overline{D}$,

$\overline{B} = (4/3)\Lambda(\bar{\varepsilon})h_o$, $\overline{D} = (1/9)\Lambda(\bar{\varepsilon})h_o^3$.

## 3. Method

Let us consider the stretching by the pressure of a flat circular plate of thickness $h_p$, clamped along the contour of radius $r_p$, with ellipsoidal dome (segment) of a given height $w_a$. The meridian segment corresponds to an arc of an ellipse with semi-axes $a_e$ and $b_e$, the relationship $k_e = b_e/a_e$, and square of eccentricity $e_x^2=1-(k_e)^2$. This is true if $a_e > b_e$ (oblate ellipsoid). In the case of $a_e < b_e$ (elongated ellipsoid), $k_e = a_e/b_e$.

Under free stretching of the plate by hydrostatic pressure resulting dome passes through the stages of both a flattened and elongated ellipsoids of rotation. Previously, we have constructed a solution for only slightly flattened spheroids with the use of the smallness of the eccentricity, which was provided by additional force at the pole. It is necessary to develop the method for the purposes of the extension of control over the form, identification materials and evaluation of limit states.

Let the ellipsoid be formed by rotating the ellipse around $z$-axis. Consider two forms of the ellipse equation (the axial section of the ellipsoid). One of them is parametric:

$$r_1 = \varphi(\tau) = a_e \sin\tau,\ \ z = \psi(\tau) = z_e - b_e \cos\tau\,, \tag{6}$$

where $z_e$ is the coordinate of the ellipse centre. In this case, polar coordinate $r_1$ is connected with the Lagrangian $r$, counted on the plate, through the radial component of displacement $u$: $r_1 = r + u$. Since the considered ellipsoids are closed at the top, then $\tau_0 = 0$. We denote the values of the parameter $\tau$ on the segment contour by $\tau_c$. In the transition to dimensionless values, we propose: $h_*=h_p$, $R_*=r_p$, $E_*=\sigma_l$.

We define $\tau_c$ through the parameters of the ellipse and the plate. We have on the contour of the elliptical segment $r_1 = r_p,\ z = 0$. Then from (6) it follows:

$$\frac{r_p}{-z_e} = -\frac{a_e}{b_e}\tan\tau_c = -\frac{1}{k_e}\tan\tau_c\,,\ \ \tau_c = \arctan\left(\frac{k_e r_p}{z_e}\right).$$

The second form is the classical geometry in the axes $r_1$ and $z$ of cylindrical coordinate system:

$$(r_1/a_e)^2 + (z+z_c)^2/b_e^2 = 1\,. \tag{7}$$

Here the radial coordinate $r$ and $r_1$ change from $0$ to $r_p$ and dimensionless value $r_p=1$. Solving (7) with respect to z, we have:

$$z = f(r_1) = z_c - k_e\sqrt{a_e^2 - r_1^2}\,,\ \ k_e = b_e/a_e \tag{8}$$

Under a parametric specification of the surface in the form of (6), its square is calculated by the integral:

$$S = 2\pi\int_{\tau_0}^{\tau_c}\varphi(\tau)\sqrt{(\varphi_{,\tau})^2 + (\psi_{,\tau})^2}\,d\tau\,. \tag{9}$$

In the case of form (6) after substituting expressions for derivatives of functions $\varphi_{,\tau} = a_e\cos\tau,\ \psi_{,\tau} = b_e\sin\tau$, integral (9) hass the form:

$$S = 2\pi a_e\int_0^{\tau_c}F(\tau)d\tau\,, \tag{10}$$

where

$$F(\tau) = (\sin\tau)\sqrt{(a_e\cos\tau)^2 + (b_e\sin\tau)^2}\ .$$

To define the surface in the form (8), the square is given by the integral:

$$S = 2\pi\int_0^{r_p}r_1\sqrt{1+(f(r_1)_{,r_1})^2}\,dr_1\,. \tag{11}$$

Here, $f(r_1),_{r_1}$ is the derivative of the function. Explicitly

$$f(r_1),_{r_1} = k_e r_1 / \sqrt{(a_e^2 - r_1^2)} \,. \tag{12}$$

Let us denote:

$$\Pi(r_1) = r_1 \sqrt{1 + (f(r_1)_{r_1})^2} \,.$$

The volume of the shell with variable thickness $h(r_1)$, having a rotational ellipsoid as the middle surface, can be obtained, if the thickness is substituted in the integrals (10), (11):

$$V_o = 2\pi a_e \int_0^{\tau_c} h(\tau) F(\tau) d\tau \,, \quad V_o = 2\pi \int_0^{r_p} h(r_1) \Pi(r_1) dr_1 \,. \tag{13}$$

The assumption about the form of the distribution function of the thickness is important. The analysis of experimental data and the performed measurements has shown that for shells with small eccentricity, the thickness can be approximated by a quadratic dependence:

$$h(x) = h_p \{1 - \delta[1 - (x/x_c)^2]\} \,. \tag{14}$$

However, when choosing an independent argument, different variants are possible here. The independent variables are the radii $r$ and $r_1$, the parameter $\tau$, the angle of inclination of the normal $\Phi$, the arc length of the curve $l$. For shells with a small $e_x$ the best results are given by the dependence on $\Phi$:

$$h(\Phi) = h_p \{1 - \delta[1 - (\Phi/\Phi_c)^2]\} \,. \tag{15}$$

The listed parameters are related one of others. From (6) it follows: $\tau = \arcsin(r_1 / a_e)$. It can be shown that $\tau$ and $\Phi$ are not identical and are related by the formulae:

$$\tau = \arctan\left( \frac{a_e}{b_e} \tan\Phi \right), \quad \Phi = \arctan\left( \frac{b_e}{a_e} \tan\tau \right). \tag{16}$$

These values can be identified only if $k_e = b_e/a_e$ is close to one.

Along with the angle of inclination of the normal $\Phi$, it is also logical to use the arc length of the meridian as an independent coordinate. For lengths of arcs of the curve, we have the formula:

$$L(\tau) = \int_0^{\tau} F_1(\tau) d\tau, \; L(r_1) = \int_0^{r_1} \Pi_1(r_1) d\tau \,, \tag{17}$$

where $F(\tau) = \sqrt{(a_e \cos\tau)^2 + (b_e \sin\tau)^2}$, $\Pi(r_1) = \sqrt{1 + (f(r_1)_{r_1})^2}$.

The full length of the meridian arc is $L_c = L(\tau_c) = L(r_p)$. Then, instead of formula (14), we can take the following:

$$h(r_1) = h_p \{1 - \delta[1 - (L(r_1)/L_c)^2]\} \,. \tag{18}$$

Variant (18) is more convenient for measurements in physical experiments.

For one-parameter approximation of the type (15), (18) coefficient $\delta$ can be determined directly, based on the condition of incompressibility. The volume of the original plate with radius $r_p$ and thickness $h_p$ will be $V_p = \pi r_p^2 h_p$. Since this is an incompressible material, so $V_o = V_p$. Applying the second of formulae (13), we get:

$$2\pi h_p \int_0^{r_p} \{1 - \delta[1 - (\Phi(r_1)/\Phi_c)^2]\} \Pi(r_1) dr_1 = \pi r_p^2 h_p \,. \tag{19}$$

The angle of inclination of the normal $\Phi$ is defined here as a function from $r_1$ using the formula $\tau = \arcsin(r_1 / a_e)$ and (16).

We consider (19) as an equation for determining the ratio $\delta$. It follows from:

$$\delta = [\int_0^{r_p} \Pi(r_1) dr_1 - r_p^2/2] \Big/ \int_0^{r_1} [1-(\Phi(r_1)/\Phi_c)^2] \Pi(r_1) dr_1 \,. \tag{20}$$

If the number of coefficients in the formula for the thickness is more than one ($n_h > 1$), then the condition of incompressibility will give the equation of connection between them. Then $n_h$–1 coefficients of these coefficients will be independent control parameters for the functional constructed below.

Further, we can construct a functional equation, from which with the help of the iterative process, the radial component of displacement is determined. With this aim, the condition of incompressibility is applicable to sites of the dome and plate, determined by the total Lagrangian coordinate *r*. We have the equation:

$$\int_0^{r_1(r)} [h(r_1)\Pi(r_1)] dr_1 = r^2/2, \tag{21}$$

where a variable upper limit $r_1(r) = r + u(r)$, and $r_1$ under the integral is the variable of integration. The independent variable in equation (18) is the Lagrangian coordinate *r*. Here, we will not specify the form of the thickness function.

In the integral (18), we select the term that can be easily integrated in an explicit form. For this, we will add and subtract 1:

$$\int_0^{r_1(r)} [1-1+h(r_1)\Pi(r_1)] dr_1 - r^2/2 = r + u(r) - \int_0^{r_1(r)} [1-h(r_1)\Pi(r_1)] dr_1 - r^2/2 \,. \tag{22}$$

From here

$$u(r) = \int_0^{r_1(r)} [1-h(r_1)\Pi(r_1)] dr_1 + r^2/2 - r \,. \tag{23}$$

For Equation (23) a simple iterative process can be organized on the base of the principle of contracting maps known in functional analysis [2]. By giving a certain initial approximation $u_0(r)$ and calculating the right-hand side in (20), using it, we obtain the first approximation $u_1(r)$. Further, the process is repeated and for (*k*+1)-th step we have:

$$u_{k+1}(r) = \int_0^{r_{1k}(r)} [1-h(r_1)\Pi(r_1)] dr_1 + r^2/2 - r \,, \tag{24}$$

where $r_{1k}(r) = r + u_k(r)$. As initial approximation, we can take the function:

$$u_0(r) = k_u r(1-r) \,. \tag{25}$$

The coefficient $k_u$ is determined by the application at the top of the dome of the incompressibility condition and proximity to the homogeneous strain state of the middle surface: $\varepsilon_1(0) \approx \varepsilon_2(0)$. It gives:

$$k_u = \frac{1}{\sqrt{1-\delta}} - 1 \,. \tag{26}$$

The process (24) converges quickly. For accuracy of few tenths of percent, less than five iterations are enough. After finishing the calculation process, the components of the deformation $\varepsilon_2(\tau)=u(\tau)/\xi$, $\delta_2(\tau)=1+\varepsilon_2(\tau)$, $\delta_1(\tau)=[\delta_2(\tau)\delta_3(\tau)]^{-1}$, $\varepsilon_1(\tau)=\delta_1(\tau)-1$, as well as internal forces and moments (2), (3) are determined.

Then the system (1) is integrated. From the first, second, fourth and fifth equations, the relations are defined as

$$\bar{T}^o(\tau) = pJ_{11}(\tau) + P_0/(2\pi) \,, \quad \bar{\Psi}^o(\tau) = J_{21}(\tau) - pJ_{22}(\tau) + r_p H_c \,; \tag{27}$$

$w(\xi)= -w_o+J_{41}(\xi),\ u(\xi)= J_{51}(\xi),$ (28)

where

$$J_{11}(\tau)=\int_0^\tau [\xi/\delta_3(\xi)]\cos\Phi(\xi)d\xi;\quad J_{22}(\tau)=\int_0^\tau [\xi/\delta_3(\xi)]\sin\Phi(\xi)d\xi,$$

$$J_{41}(\tau)=\int_0^\tau \delta_1(\xi)\sin\Phi(\xi)d\xi,\quad J_{51}(\tau)=\int_0^\tau \delta_1(\xi)\cos\Phi(\xi)d\xi,\quad J_{21}(\tau)=\int_0^\tau \overline{N}_2^o(\xi)d\xi. \tag{29}$$

Displacements (25) vanish on the boundary contour. Values $P_0$ and $H_c$ are constants of integration. Thus $P_0$ has the meaning concentrated in the top of the force affecting the shape of the shell, $H_c$ is the radial force, distributed along the contour.

From the third equation of the system (1), it is possible to determine the forming pressure dome of a given height $w_o$. This can be performed in two ways: differential and integral. Since the geometry of the resulting shell is parametrically known, it is possible to determine the moments and their derivatives through the shell curvatures and the obtained expressions for the relative elongations. This is the first approach, which gives the formula for the pressure $p$ :

$$p=\frac{[J_{21}(\tau)+r_pH_c]\sin\Phi(\tau)-[P_0/(2\pi)]\cos\Phi(\tau)+\varepsilon_* J(\tau)/\delta_1(\tau)}{J_{11}(\tau)\cos\Phi(\tau)+J_{22}(\tau)\sin\Phi(\tau)}\approx const, \tag{30}$$

where

$$J(\tau)=M_1^o(\tau)\{1-[\delta_1(\tau)/\delta_2(\tau)]\cos\Phi(\tau)\}+\tau dM_1^o(\tau)/d\tau.$$

If we integrate the third equation of the system, we get the formula:

$$p=\frac{\tau M_1^o(\tau)+r_pM_c-J_{31}(\tau)-J_{33}(\tau)+J_{34}(\tau)+J_{36}(\tau)}{J_{32}(\tau)+J_{35}(\tau)}\approx const, \tag{31}$$

where

$$J_{31}(\tau)=\int_0^\tau M_2^o\cos\Phi(\xi)d\xi;\quad J_{33}(\tau)=\frac{P_0}{2\pi\varepsilon_*}\int_0^\tau \delta_1(\xi)\cos\Phi(\xi)d\xi,$$

$$J_{34}(\tau)=\frac{1}{\varepsilon_*}\int_0^\tau \delta_1(\xi)J_{21}(\xi)\sin\Phi(\xi)d\xi,\quad J_{36}(\tau)=\frac{H_g}{\varepsilon_*}\int_0^\tau \delta_1(\xi)\sin\Phi(\xi)d\xi,$$

$$J_{32}(\tau)=\frac{1}{\varepsilon_*}\int_0^\tau \delta_1(\xi)J_{11}(\xi)\cos\Phi(\xi)d\tau,\quad J_{35}(\tau)=\frac{1}{\varepsilon_*}\int_0^\tau \delta_1(\xi)J_{22}(\xi)\sin\Phi(\xi)d\xi.$$

The output of the expression (30) by a constant is controlled by the parameters $P_0$, $H_c$, $e_x$.. In (31), these values are added to the constant $M_c$, which appears when integrating the third equation. A larger number of control parameters make it more advantageous to apply the formula (31), especially when forming high lift shells.

In constructing the solution for thickness, the formula (18) was mainly used. A generalized solution was tested by comparison with the more analytical version, built for a slightly flattened ellipsoid [1].

## 4. Numerical experiments

Along with the main formula of approximation of the thickness function (variant 1), we also considered other variants with additional terms to improve the accuracy of the output of the functional (31) to a stationary value. The variants 2 – 8 of the formulae for the thickness are present as

$h_2(r_1) = h_0[1-\delta(1-F_{lc}(r_1)^2) - g_h e_x^2 (1-F_{lc}(r_1))^2]$;

$h_4(r_1) = h_0[1-\delta(1-F_{lc}(r_1)^2) - g_k e_x^2 (1-F_{lc}(r_1))]$;

$h_3(r_1) = h_0[1-\delta(1-F_{lc}(r_1)^2) - g_g e_x^2 (1-F_{lc}(r_1)^4)]$;

$h_5(r_1) = h_0[1-\delta(1-F_{lc}(r_1)^2) - g_3 e_x^2 (1-F_{lc}(r_1)^3) - g_4 e_x^2 (1-F_{lc}(r_1)^4)]$;

$h_6(r_1) = h_0[1-\delta(1-F_{lc}(r_1)^2) - g_1 e_x^2 (1-F_{lc}(r_1)) - g_3 e_x^2 (1-F_{lc}(r_1)^3) - g_4 e_x^2 (1-F_{lc}(r_1)^4)]$;

$h_7(r_1) = h_0[1-\delta(1-F_{lc}(r_1)^2)(1-g_e e_x^2 F_{lc}(r_1))]$;

$h_8(r_1) = h_0[1-\delta(1-F_{lc}(r_1)^2)(1-g_e e_x^2 F_{lc}(r_1)) - g_e e_x^2 (1-F_{lc}(r_1))^2]$.

These variants allow to improve the accuracy of the output functional on the stationary value from 1 – 1.5% to 0.25 – 0.5%. As shown by numerical experiments, the fifth variant is the most preferable.

Let us consider the stretching of a flat circular plate of thickness $h_p$, clamped along the contour of radius $r_p$, by an ellipsoidal dome (segment) of a given height, $w_a$. When stretching is near the pinch circuit, a sharp bend appears, in the area of which the flat, clamped part of the plate passes into the dome. As experiments show, this zone is very narrow (about 0.5 of the thickness of the plate). Therefore, its size can be neglected and it could be assumed for the dome the radius of the reference contour is equal to the radius of the original plate. This area is conventionally cut off, and its force influence is replaced by reactions, namely vertical and horizontal linear forces and moment. These quantities appear naturally in the integration of the equilibrium equations.

The equations of the model and the calculation process are fulfilled in dimensionless form. In particular, in the normalization of displacements and pressure, the following formulae are used:

$$\{u, w\}_n = \{u, w\}_d / R_*, \quad p_n = p_d/(E_*\varepsilon_*), \quad \varepsilon_* = h_*/R_* . \qquad (32)$$

Here $u$, $w$ are the horizontal and vertical movements; $h_*$, $R_*$ are typical small and large sizes; $\varepsilon_*$ is a thin-wall parameter; indices $n$ and $d$ point the dimensionless and dimensional quantities, respectively. As characteristic quantities, $h_* = h_p$, $R_* = r_p$, $E_* = E$ (Young's modulus) or $\sigma_v$ (tensile strength) were taken. The calculation results can be found both in dimensionless and dimensional forms.

We represent some results of calculations in the dimensional form, for the return to which formulae (32) are used. Let us consider a plate with a thickness $h_p$ = 0.38 mm and radius of the reference circuit $r_p$ = 100 mm, Young's modulus $E$=0.21·10$^6$ MPa, yield strength $\sigma_{02}$ = 360 MPa, tensile strength $\sigma_v$=720 MPa, ultimate strain intensity $\bar{\varepsilon}_v = 0.615$, yield strain $\varepsilon_{02}=\sigma_{02}/E$ = 0.001714, $\varepsilon_*$= 0.0038. The diagram of the plastic properties of the material corresponds to stainless steel 12X18H10T. Such plates are used for the manufacture of the destructible elements of the devices that protect technological equipment and tanks from destruction by excess pressure [3].

When the pressure is stretched freely with no force, applied at the apex, the shell passes through the stages of a segment of a slightly flattened ellipsoid, a spherical dome and an elongated ellipsoid.

The Table 1 shows the matching values of the height of the dome $w_a$, the forming pressure $p_f$ and the eccentricity $e_x$ of the ellipsoidal shell meridian. The nonlinear dependence of $p_f - w_a$ is also represented in Fig. 1 having the form that is observed in physical experiments.

Table 1. Values of the height of the dome $w_a$, the forming pressure $p_f$ and the eccentricity of the ellipsoids.

| $w_a$ (mm) | $p_f$ (MPa) | $e_x$ |
|---|---|---|
| 0.10 | 67 | 0.015 |
| 0.15 | 106 | 0.007 |
| 0.20 | 144 | 0.00 |
| 0.25 | 181 | 0.16 |
| 0.30 | 212 | 0.20 |
| 0.35 | 240 | 0.21 |
| 0.40 | 259 | 0.22 |
| 0.45 | 270 | 0.23 |
| 0.50 | 277 | 0.25 |

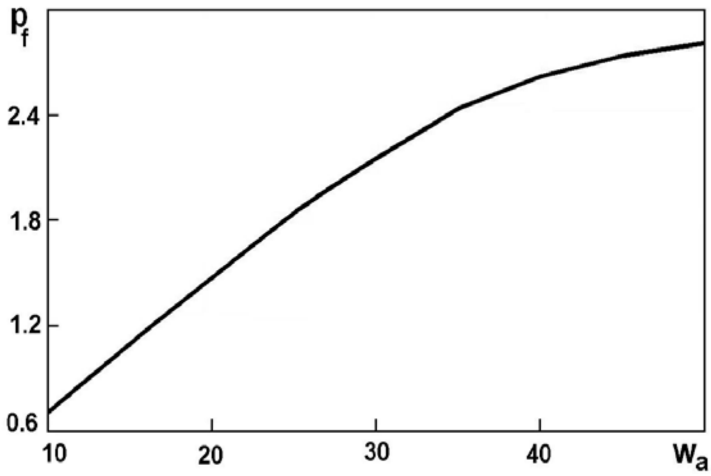


**Fig. 1.** Nonlinear dependence of pressure forming the dome height.

## 5. Conclusions

A more general form of the constructed algorithm makes it possible to consider variants for ellipsoids with fairly large eccentricities. The solution, where the power series expansion by eccentricity is used, follows from the generalized one as a special case. Although for more efficient calculations, this technique can be used for small eccentricities, a small amount of eccentricity is not assumed in the generalized approach presented here. This makes it possible to expand the possibilities of applying this semi-return method in the case of deformations of plastic materials close to the limiting ones.

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