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CORRESPONDENCE PRINCIPLE FOR SIMULATION HYDRAULIC FRACTURES BY USING PSEUDO 3D MODEL

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Abstract. The original pseudo three-dimensional (P3D) model is extended to an arbitrary stress contrast on the basis of the correspondence principle suggested. The principle employs the similarity between solutions to plain-strain elasticity problems for (i) the crack, corresponding to the central cross-sections of the P3D model, and (ii) the crack of the Khristianovich-Geertsma-de Klerk (KGD) model, when the sizes and average openings of the cracks are the same. This suggests using the propagation speeds of the KGD problem for assigning the speed of the height growth of the P3D model. This approach is applicable in all the cases when the KGD problem may be accurately solved; specifically, when accounting for an arbitrary stress contrast.

Keywords: hydraulic fracturing, pseudo three dimensional (P3D) model, stress contrast

1. Introduction

Hydraulic fracturing (HF) is the operation of injecting viscous fluid into the rock mass to create tensile cracks. The operation is widely used in petroleum industry for stimulation of oil and gas reservoirs [2]. Numerical simulators are developed for the design of this expensive operation to make it efficient. Commonly simulators are based on simplified mathematical models. Among those, the most popular is the pseudo-three dimensional (P3D) model [1, 5, 6, 10]. It combines physically clear prerequisites with computational efficiency (e.g. [1]). To assign the vertical speeds of fracture cross-sections, the authors introduced either actual [6, 9], or "apparent" [1] stress intensity factor (toughness) K_{IC} in the line of the classical fracture mechanics (e.g. [9]). Yet, being justified for high fracture toughness, the original model becomes irrelevant in practically important cases when the fluid viscosity dominates. Efforts to overcome the difficulty by assigning an apparent fracture toughness K_{IC} have actually succeeded merely for a particular case of a pay layer between half-spaces with the same (positive) stress contrast [1]. Authors obtained a solution, which in the particular case considered agrees with the solution to the truly 3D solution found by using the implicit level set algorithms (ILSA) suggested in [8]. Still, there has been no solution for the general case. The present work aims to consistently extend the P3D model to an arbitrary input of viscosity and to an arbitrary stress contrast.

2. Problem formulation

The original P3D model is based on three main assumptions:

1. The length of the fracture is greater than its height (Fig. 1). Then plane-strain conditions occur in central cross-sections parallel to the fracture front;

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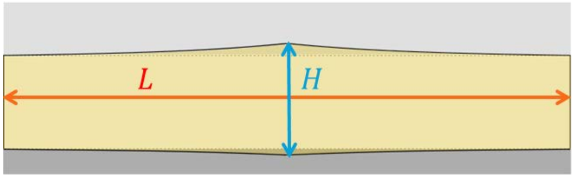

Fig. 1. Fracture footprint assumed in the P3D model

2. Elasticity modulus E and the Poisson ratio ν are the same in each layer of the elastic homogeneous isotropic media considered (Fig. 2);

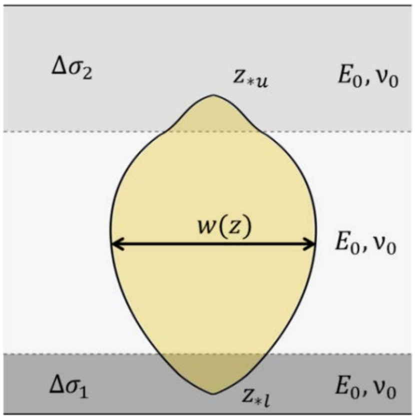

Fig. 2. Profile of the opening for the P3D model

3. The physical pressure p is the same along each cross-section, where plain-strain conditions are applicable (Fig. 3), while in-situ stress which acts and closes the fracture along the x-axis, commonly changes in the z-direction.

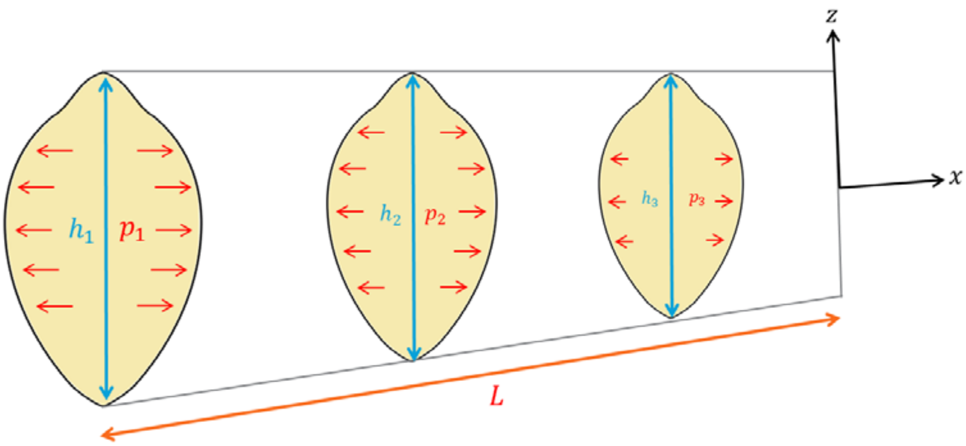

Fig. 3. Pressure in various cross-sections

These assumptions yield the system of equations, which is consists of 5 equation:

1. elasticity equation, obtained from the classical solution [7]

$$w_{av} = \frac{2z_*}{E'}\left[\frac{\pi p_{net}}{2} - \frac{1}{2}\int_{-1}^{1} \Delta\sigma(z(\varsigma))\sqrt{1-\varsigma^2}d\varsigma\right], \quad (1)$$

where w_{av} is the opening averaged over a cross-section, $E' = E/(1-\nu^2)$, $\varsigma = \left(z - (1/2(z_{*l} + z_{*u})\right)/z_*$, $z_*(x,t) = 1/2(z_{*u} - z_{*l})$ is the half-height, z_{*l} (z_{*u}) is the global coordinate of the lower (upper) tip; $p_{net} = p(x,t) - \sigma_p$ is the difference between the actual fluid pressure p and a fixed value σ_p of the confining rock pressure;

2. the continuity equation

$$\frac{\partial(2z_* w_{av})}{\partial t} = -\frac{\partial(2z_* w_{av} v_{av})}{\partial x} - Q_l + Q_0\delta(x), \quad (2)$$

where Q_l is the total leak-off through the surface of a cross-section, Q_0 is prescribed pumping rate at the source point;

3. the Poiseuille type equation

$$v_{av}(x,t) = F_v\left(-\frac{w_{av}^{n+1}}{\mu'}\frac{\partial p_{net}}{\partial x}\right)^{\frac{1}{n}}, \quad (3)$$

where $F_v(x,t) = \frac{\frac{1}{2z_*}\int_{z_{*l}}^{z_{*u}} w^{2+1/n} dz}{w_{av}^{2+1/n}}$, n is the fluid behavior index, $\mu' = 2[2(2n+1)/n]^n M$, M is the consistency index;

4. the speed equation for the fracture front

$$v_{*x}(t) = \frac{F_v(x_*,t)}{t_n}\left(\frac{1}{\pi z_*(n+2)}\right)^{1/n} C_w^{1+2/n}, \quad (4)$$

where $C_w = \frac{w_{av}(r_i)}{r_i^\alpha}$, $r = x_* - x$ is the distance from a point x behind the front to the front x_*, $\alpha = \frac{1}{n+2}$;

5. the speed equations for vertical growth of cross-sections

$$v_{zl}(x,t) = \frac{dz_{*l}}{dt} = f_{zl}(z_{*l}, z_{*u}, w_{av}); \; v_{zu}(x,t) = \frac{dz_{*u}}{dt} = f_{zu}(z_{*l}, z_{*u}, w_{av}), \quad (5)$$

where $f_{zl}(z_{*l}, z_{*u}, w_{av})$ and $f_{zu}(z_{*l}, z_{*u}, w_{av})$ are known functions.

The system (1)-(5) is complemented with the initial conditions (6) at an initial moment t_0:

$$x_*(t_0) = x_{*0}, \; w_{av}(x,t_0) = w_o(x), \; z_{*l}(t_0) = z_{*l0}, \; z_{*u}(t_0) = z_{*u0}. \quad (6)$$

To extend the original P3D model to an arbitrary regime of the fracture height growth we need to define $f_{zl}(z_{*l}, z_{*u}, w_{av})$ and $f_{zu}(z_{*l}, z_{*u}, w_{av})$. It can be done by using the correspondence principle formulated in the next section.

3. The correspondence principle

As a physically consistent approach, we suggest to use the next *correspondence principle*. Two plane-strain HF under the same conditions are assumed equivalent, as regards to the speeds of their height growth, when the HF have the same: (i) tip positions, and (ii) fluid volumes above and below the injection point. The "*same conditions*" mean that the rock structure, stress contrasts, leak-off parameters, and the properties of rock, fluid and proppant are the same for the both HF. Taking into account that Khristianovich-Geertsma-de Klerk (KGD) and P3D models employ the same elasticity equation (1) for a straight crack under plain-strain conditions, one may expect that profiles of the opening for both models are similar when items (i) and (ii) are met. In fact, the difference concerns with merely near-tip zones influenced by the asymptotics, while the openings of the central parts of fractures may be almost the same (Fig. 4).

Then in any cross-section of the P3D model, the current tip positions and the current volumes above (and below) the injection point uniquely define the tip propagation speeds via the speeds in the corresponding (in the mentioned sense) KGD model (Fig. 5).

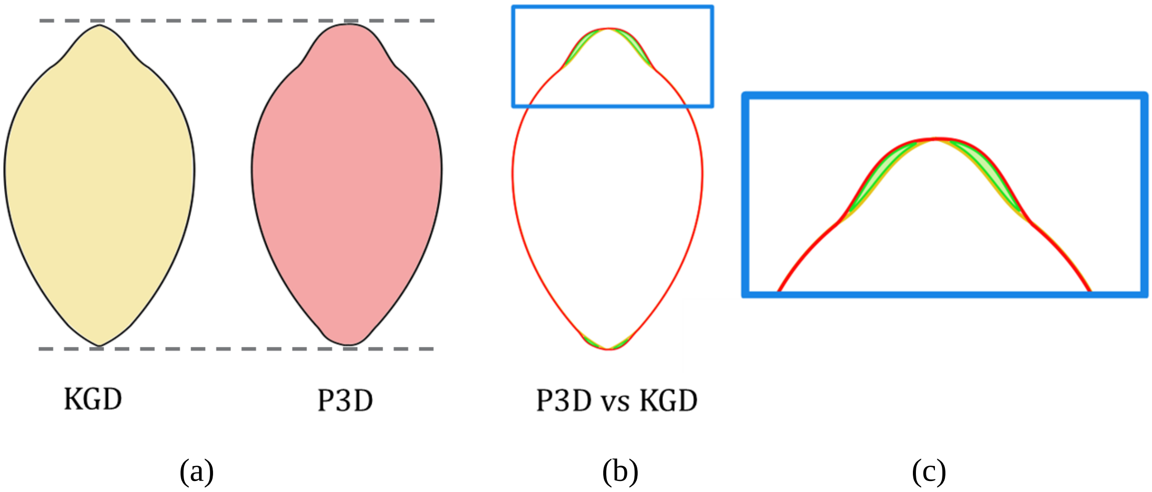

Fig. 4. Profiles of the opening for the KGD and P3D models: (a) full profiles; (b) comparison of the profiles; (c) magnified area with the difference between KGD and P3D openings

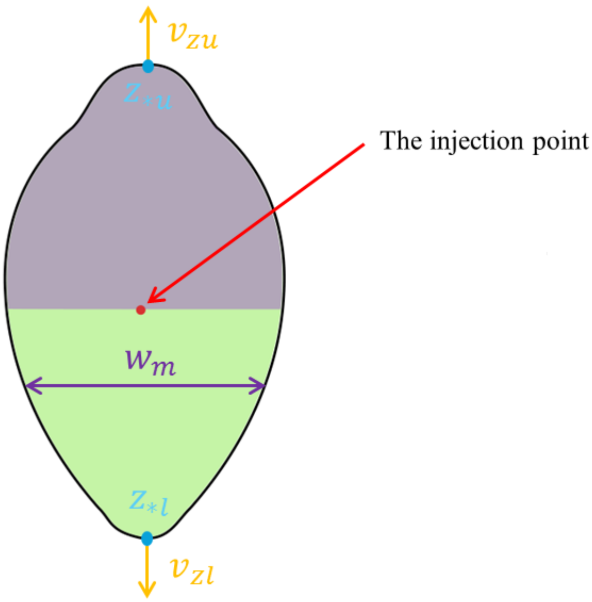

Fig. 5. Tip propagation speeds

The data to find the speeds via the tip positions and the volumes are prepared in advance by solving a set of extended KGD problems (e.g. [3]). For each cross-section of the P3D model, this gives the propagation speeds needed to update the tip positions on a time step. Clearly, the method may serve for an arbitrary propagation regime when the KGD problems are solved accounting for the universal asymptotics [4].

Numerical implementation of the method has confirmed that for the particular case studied in [3], the profiles of the opening found for P3D and KGD models are practically the same. The propagation speeds are close, as well, what follows from the perfect agreement of footprints. Since the latter were verified in [1] against the truly 3D bench-mark solutions, this implies that the method developed is quite accurate.

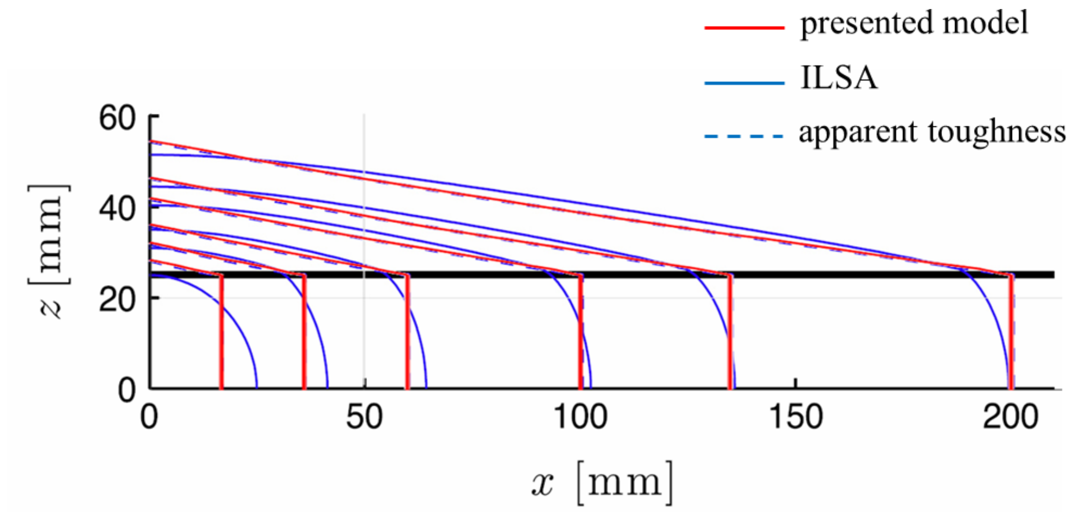

Fig. 6. Comparison of the footprints obtained by using the correspondence principle the benchmark solution given in [1]

4. Conclusions

The correspondence principle, based on similarities between KGD and P3D models, serves to extend the original P3D model to a general case. The proposed method allows one to obtain accurate results, which are comparable with those for the truly 3D model when a fracture grows in the viscosity-dominated regime. The comparison of fracture footprints, obtained by the method suggested, with published solutions to truly 3D benchmark problems, has shown good agreement. This confirms that the speeds of height growth are evaluated correctly when employing the correspondence principle. The method can be also used to determine the limits of applicability of the original P3D model.

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References

[1] Dontsov EV, Peirce AP. An enhanced pseudo-3D model for hydraulic fracturing accounting for viscous height growth, non-local elasticity, and lateral toughness. *Engineering Fracture Mechanics*. 2015;142: 116-139.

[2] Economides MJ, Nolte KG. *Reservoir stimulation*. 2nd ed. Houston, Texas, USA: Schlumberger Educational Services; 1989.

[3] Gladkov IO, Linkov AM. Khristianovich-Geertsma-de Klerk problem with stress contrast. To be published to *Appl. Math. Techn. Phys. Arxiv.* [Preprint] 2018. Available from: https://arxiv.org/abs/ 1703.05686 [Accessed 5th December 2018].

[4] Linkov AM. Particle velocity, speed equation and universal asymptotics for efficient modelling of hydraulic fractures. *J. Appl. Math. Mech.* 2015;79: 54–63.

[5] Linkov AM, Mishuris G. Modified formulation, ε-regularization and efficient solving of hydraulic fracture problem. In: Jeffrey R. (ed.) *Effective and Sustainable Hydraulic Fracturing: proceedings of the International Conference for Effective and Sustainable Hydraulic Fracturing, HF2013, 20-22 May 2013, Brisbane, Australia.* 2013. p.641–657.

[6] Mack MG, Warpinski NR. Mechanics of hydraulic fracturing. In: Economides MJ, Nolte KG. (eds.) *Reservoir Simulation*. 3-rd edn. Chichester, England: John Willey & Sons: 2000, p.6-1-6-48.
[7] Muskhelishvili NI. *Some Basic Problems of the Mathematical Theory of Elasticity*. Groningen, Noordhoff; 1975.
[8] Peirce A, Detournay E. An implicit level set method for modeling hydraulically driven fractures. *Comput. Methods Appl. Mech. Engng.* 2008;197: 2858–2885.
[9] Rice J. Mathematical analysis in the mechanics of fracture. In: Liebowitz H. (ed.) *Fracture, an Advanced Treatise.* New York: Academic Press; 1968;2. p.191–311.
[10] Settari A, Cleary MP. Development and testing of a pseudo-three-dimensional model of hydraulic fracture geometry. *SPE Prod. Eng.* 1986;1(6): 449–466.