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ELECTROMAGNETIC ELASTIC BALL UNDER NON-STATIONARY AXIALLY SYMMETRICAL WAVES

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Abstract. This paper studies propagation of non-stationary axially symmetrical kinematic or electromagnetic disturbances applied on the surface of a ball. To this end, linear equations of motion of an elastic ball together with Maxwell equations are used as well as linearized generalized Ohm law and Lorentz force equation. The required functions are expanded in series in terms of Legendre and Gegenbauer polynomials. Laplace integral time transformation and expansion of coefficients of series into power series in small parameter linking mechanical and electromagnetic properties of the medium enabled finding recurrent sequence of boundary value problems with respect to components of mechanical and electromagnetic fields. The solution of each problem is represented in the form of generalized convolution of functions corresponding to previous members of the recurrent sequence with Green functions.

Keywords: Green functions, electromagnetic disturbances, linear equations of motion of elastic ball, Ohm law

1. Introduction

Currently, the issues of considering coupled fields of mechanical and other nature, such as electromagnetic, are becoming increasingly important in various engineering problems. However, coupled problems of electromagnetic and mechanical fields inside conductors have not been enough studied at present. The relevance of this topic is beyond doubt, since the coupled fields are used in many areas of modern technology: electroacoustic, radio engineering, automatic systems. At the same time, mathematical modeling of the conjugate fields interaction is often simpler and more better visualized than physical experiment [1]. The only solutions are known is for piezoelectrics in the non-stationary formulation for a sphere. In this paper, we consider the propagation problem of electromagnetoelastic non-stationary two-dimensional waves inside isotropic conducting sphere under the influence of surface electric or mechanical disturbances.

Statements of non-stationary problems of electromagnetic elasticity are given in [2]. Solutions of corresponding uncoupled problems will be natural necessary components of this problem. The article [3] studies two-dimensional non-stationary electromagnetic fields induced by specific displacement field in a ball. The publication [4] investigates non-

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stationary process of axially symmetric deformation of elastic ball under volumetric forces. In [5] the coupled problem of electromagnetoelasticity for the spherical cavity located in the infinite isotropic conductor is solved. In contrast to the listed articles, in publications prior to those reviewed, as a rule, the connection of conjugate fields occurs through physical relations. Traditionally, solutions were built numerically and numeric-analytically. The numerical results of electroelasticity are known for canonical geometry bodies made of piezoelectric materials. In particular, the existence and uniqueness theorems for generalized solutions of coupled non-stationary two-dimensional electroelasticity problems was proved in [6] for some canonical geometry domains. In addition, the problem for a hollow piezoceramic finite length cylinder polarized along the radius was numerically solved in [7] and [8]. Moreover, in [8] more efficient numerical method for finding solutions was proposed. Analytical and numeric-analytical solutions of electroelasticity for piezoceramic cylinder are described in detail in [9, 10]. Similar problems of coupled electroelasticity for the thick-walled piezoelectric sphere were considered in [11, 12]. Moreover, in [12] the algorithm of finding an analytical solution is described. To find the analytical solutions for non-stationary problem, effective method is expansion in a series by the inverse parameter of the Laplace transform. It allows us to find solutions of the problem at small time intervals. Such approach in one-dimensional formulation was demonstrated in [13] by the example of electromagneto-thermoelastic spherical cavity. The present article offers further development of the results of recent three studies, now as applied to an electromagnetic elastic ball where mechanic and electromagnetic fields are coupled by Lorentz force acting as volumetric force in motion equation, and generalized Ohm law [14]. This problem has also practical applications of investigation of electromagnetic and mechanic fields, for instance in non-destructive inspections, as well as in designing electronic devices using conductor and conductive coating in harsh operational conditions.

2. Statement of problem

Consider a homogeneous isotropic conductive ball of radius r_1 with a center at point O at the boundary of which there are preset kinematic or electromagnetic conditions (r, θ, ϑ is spherical coordinate system where $r \geq 0,\ 0 \leq \theta \leq \pi,\ -\pi < \vartheta \leq \pi$):

$$u\big|_{r=r_1} = U_1(\tau, \theta),\ v\big|_{r=r_1} = V_1(\tau, \theta),\ E_\theta\big|_{r=r_1} = e_{01}(\tau, \theta). \tag{1}$$

They are complemented by the boundary condition of the medium stress-strain components and the electromagnetic field. The initial electromagnetic field is assumed to be stationary, radial and satisfies the conditions $E_{0r} = E_0(r),\ E_{0\theta} \equiv 0,\ H_0 \equiv 0$ (hereinafter zero indices indicate the initial condition). At the initial moment of time the ball is in undisturbed state (dots label the time derivatives):

$$u\big|_{\tau=0} = \dot{u}\big|_{\tau=0} = v\big|_{\tau=0} = \dot{v}\big|_{\tau=0} = E_r\big|_{\tau=0} = \dot{E}_r\big|_{\tau=0} = E_\theta\big|_{\tau=0} = \dot{E}_\theta\big|_{\tau=0} = H\big|_{\tau=0} = \dot{H}\big|_{\tau=0} = 0,$$

where u and v are radial and tangential displacements, E_r and E_θ - components of electric field vectors; H is a non-zero component of magnetic field vector.

Resulting from the motion equations, Maxwell equations and generalized Ohm law, closed coupled equation system with assumed axially symmetric motion will have the form [2]. The obvious type of the corresponding system of the equations is given in [5]. To non-dimension sizes entered in work [4], in the system of the equations for a cavity for the considered sphere the ratio is added: $r = \dfrac{r_1'}{L}$. At the same time, in designations which are used in [5] and further: t is time; j_r and j_θ are radial and tangential densities of current; ρ_e is the density of surface charges; F_k are non-zero radial and tangential components of Lorentz forces; c, c_1 and c_2 are speed of light and speed of propagation of strain-stress and shear waves; λ and μ are elastic Lame constants; ε and μ_e are coefficients of dielectric and magnet permittivity; σ is coefficient of electric conductivity; L and E_* are some specific linear size and electric field intensity.

3. Expansion in series over angle

Let the required functions, similar to work [5] be represented in form of series in terms of polynomials of Legendre $P_n(x)$ and Gegenbauer $C_{n-1}^{3/2}(x)$ [15]. Taking the functions of displacement and magnetic field strength as primary unknown variables, after Laplace time transform τ (when s is its parameter; index L designates a transform domain) and using the fact that as shown in [16], even in one-dimensional case the solution of corresponding boundary value problem has Bessels functions with indices depending on the parameter of the Laplace transformation. Obviously, for this reason it is impossible to find a solution analytically. To this reason, we represent the required functions in power series in small parameter $\alpha = \dfrac{\varepsilon E_*^2}{4\pi(\lambda+2\mu)}$. Index n treats decomposition coefficients on Legendre and Gegenbauer polynomials, m - to decomposition coefficients in the following series:

$$\begin{gathered} u_n = \sum_{m=0}^{\infty} u_{nm}(r,\tau)\alpha^m,\ v_n = \sum_{m=0}^{\infty} v_{nm}(r,\tau)\alpha^m,\ H_n = \sum_{m=0}^{\infty} H_{nm}(r,\tau)\alpha^m, \\ \rho_n = \sum_{m=0}^{\infty} \rho_{nm}(r,\tau)\alpha^m, E_{rn} = \sum_{m=0}^{\infty} E_{rnm}(r,\tau)\alpha^m,\ E_{\theta n} = \sum_{m=0}^{\infty} E_{\theta nm}(r,\tau)\alpha^m. \end{gathered} \tag{2}$$

Substitution of these series to the system of the equations on coefficients of series of decomposition of required functions on Legendre and Gegenbauer polynomials [5] will lead to a recurrent sequence of system of equations with respect to limited functions

$$s^2 u_{00}^L = l_{110}\left(u_{00}^L\right); \tag{3}$$

$$s^2 u_{0m}^L = l_{110}\left(u_{0m}^L\right) + g_u\left(E_{r0,m-1}^L, \rho_{0,m-1}^L\right),\ (n=0,\ m\ge 1);$$

$$s_e^2 E_{r0m}^L = -s^2 \rho_{e0} u_{0m}^L\ ; \qquad (s+\gamma)\rho_{0m}^L = -\frac{s}{r^2}\frac{\partial\left(r^2\rho_{e0}u_{0m}^L\right)}{\partial r},\ (n=0,\ m\ge 0);$$

$$s_e^2\eta_e^2 H_{nm}^L = \Delta_n H_{nm}^L + \eta_e^2 s l_H\left(u_{nm}^L, v_{nm}^L\right); \tag{4}$$

$$s^2 u_{n0}^L = l_{11n}\left(u_{n0}^L\right) + l_{12n}\left(v_{n0}^L\right),\ s^2 v_{n0}^L = l_{21n}\left(u_{n0}^L\right) + l_{22n}\left(v_{n0}^L\right); \tag{5}$$

$$\begin{gathered} s^2 u_{nm}^L = l_{11n}\left(u_{nm}^L\right) + l_{12n}\left(v_{nm}^L\right) + g_u\left(E_{rn,m-1}^L, \rho_{n,m-1}^L\right), \\ s^2 v_{nm}^L = l_{21n}\left(u_{nm}^L\right) + l_{22n}\left(v_{nm}^L\right) + g_v\left(E_{\theta n,m-1}^L, H_{n,m-1}^L\right),\ (n\ge 1,\ m\ge 1); \end{gathered}$$

$$\begin{gathered} \eta_e^2(s+\gamma)E_{\theta nm}^L = -\frac{1}{r}\frac{\partial\left(rH_{nm}^L\right)}{\partial r} - \eta_e^2 s\rho_{e0}v_{nm}^L, \\ \eta_e^2(s+\gamma)E_{rnm}^L = \frac{n(n+1)}{r}H_{nm}^L - \eta_e^2\rho_{e0}u_{nm}^L; \end{gathered} \tag{6}$$

$$(s+\gamma)\rho_{nm}^L = -sl_{n\rho}\left(u_{nm}^L, v_{nm}^L\right),\ (n\ge 1,\ m\ge 1) \tag{7}$$

with the following boundary value conditions:

$$u_{n0}^{L}\Big|_{r=r_1}=U_{1n}^{L}(s)\ (n\geq 0), v_{n0}^{L}\Big|_{r=r_1}=V_{1n}^{L}(s)\ \ (n\geq 1); \tag{8}$$

$$u_{nm}^{L}\Big|_{r=r_1}=v_{nm}^{L}\Big|_{r=r_1}=0\ (n\geq 0,\ m\geq 1),\ v_{nm}^{L}\Big|_{r=r_1}=0\ (n\geq 1,\ m\geq 1);$$

$$\frac{1}{r}\frac{\partial\left(rH_{n0}^{L}\right)}{\partial r}\Bigg|_{r=r_1}=-\eta_e^2 h_0^L\left[V_{1n}^{L}(s),e_{01n}^{L}(s)\right]\Big|_{r=r_1}\ \ (n\geq 1); \tag{9}$$

$$\frac{1}{r}\frac{\partial\left(rH_{nm}^{L}\right)}{\partial r}\Bigg|_{r=r_1}=0\ (n\geq 1,\ m\geq 1),\ h_0^L(v,e)=s\rho_{e0}v+(s+\gamma)e, \tag{10}$$

where $U_{1n}^{L}(s)\ (n\geq 0)$ и $V_{1n}^{L}(s),\ e_{01n}^{L}(s)\ (n\geq 1)$ - images on Laplace of coefficients of decomposition on series on Legendre and Gegenbauer polynomials according to the right parts in (1).

4. Integral expression of solutions

Pursuant to [3], let the solutions of the boundary value problems (4), (9), (10) with known right-hand parts, as well as functions $E_{\theta nm}^{L}$ and E_{rnm}^{L} in (6) be written as follows:

$$H_{nm}(r,\tau)=-\eta_e^2\int_0^{r_1}\rho_{e0}(\xi)\left[G_{Hun}^{c}(r,\xi)\dot{u}_{nm}(\xi,\tau)+G_{Hvn}^{c}(r,\xi)\dot{v}_{nm}(\xi,\tau)\right]d\xi, \tag{11}$$

$$\begin{gathered}E_{rnm}(r,\tau)=-\frac{n(n+1)}{r}\int_0^{r_1}\rho_{e0}(\xi)\left[G_{Hun}^{c}(r,\xi)u_{nms}(\xi,\tau)+G_{Hvn}^{c}(r,\xi)v_{nms}(\xi,\tau)\right]d\xi,\\ E_{\theta nm}(r,\tau)=\rho_{e0}(r)v_{nms}(r,\tau)+\int_0^{r_1}\rho_{e0}(\xi)\left[\Gamma_{Hun}^{c}(r,\xi)u_{nms}(\xi,\tau)+\Gamma_{Hvnr}^{c}(r,\xi)v_{nms}(\xi,\tau)\right]d\xi\ ,\end{gathered} \tag{12}$$

here

$$G_{Hun}^{c}(r,\xi)=\xi\left[\tilde{G}_{Hn}^{c}(r,\xi)H(\xi-r)+\tilde{G}_{Hn}^{c}(\xi,r)H(r-\xi)\right],$$
$$G_{Hvn}^{c}(r,\xi)=\xi\left[G_{Hvn1}^{c}(r,\xi)H(\xi-r)+G_{Hvn2}^{c}(r,\xi)H(r-\xi)\right],$$
$$\Gamma_{Hunr}^{c}(r,\xi)=\Gamma_{Hun1}^{c}(r,\xi)H(\xi-r)+\Gamma_{Hun2}^{c}(r,\xi)H(r-\xi),$$
$$\Gamma_{Hvnr}^{c}(r,\xi)=\Gamma_{Hvn1}^{c}(r,\xi)H(\xi-r)+\Gamma_{Hvn2}^{c}(r,\xi)H(r-\xi),$$

where $\Gamma_{Hun1}^{c}(r,\xi),\Gamma_{Hun2}^{c}(r,\xi),G_{Hvn1}^{c}(r,\xi),G_{Hvn2}^{c}(r,\xi),\Gamma_{Hvn1}^{c}(r,\xi),\ \Gamma_{Hvn2}^{c}(r,\xi),\tilde{G}_{Hn}^{c}(r,\xi)$ - rational functions of the arguments r,ξ . In the formulas (11), (12) and later on $H(\xi)$ is Heaviside function with the additional lower index s indicating the result of application of operator to such function (asterisk means time convolution)

$f_s(\tau)=f(\tau)-\gamma e^{-\gamma\tau}*f(\tau)$.

Notably, the kernels of these integral representations were obtained in quasi-static approximation ($\eta_e=0$).

As the problems (9), (11), (14) were thoroughly studied in the publication [3], [17], further on in boundary value conditions (1) let us assume that

$U_1(\theta,\tau)\equiv 0, V_1(\theta,\tau)\equiv 0$.

Then these problems become homogeneous. Therefore, their solutions are trivial:

$u_{n0}(r,\tau)\equiv 0,\ v_{n0}(r,\tau)\equiv 0\ (n\geq 0)$.

The solution of "mechanical" part of the problems (9) – (16) with known right-hand pars in accordance with the conclusions of the publication [4] is also represented in integral form:

$$\begin{aligned}
u_{nm}(r,\tau) &= \int_0^{r_1} G_{uun}(r,\xi,\tau) * f_{un,m-1}(\xi,\tau)\,d\xi + \int_0^{r_1} G_{uvn}(r,\xi,\tau) * f_{vn,m-1}(\xi,\tau)\,d\xi,\\
v_{nm}(r,\tau) &= \int_0^{r_1} G_{vun}(r,\xi,\tau) * f_{un,m-1}(\xi,\tau)\,d\xi + \int_0^{r_1} G_{vvn}(r,\xi,\tau) * f_{vn,m-1}(\xi,\tau)\,d\xi;
\end{aligned} \tag{13}$$

$$\begin{aligned}
\dot{u}_{nm}(r,\tau) &= \int_0^{r_1} \Pi_{uun}(r,\xi,\tau) * f_{un,m-1}(\xi,\tau)\,d\xi + \int_0^{r_1} \Pi_{uvn}(r,\xi,\tau) * f_{vn,m-1}(\xi,\tau)\,d\xi,\\
\dot{v}_{nm}(r,\tau) &= \int_0^{r_1} \Pi_{vun}(r,\xi,\tau) * f_{un,m-1}(\xi,\tau)\,d\xi + \int_0^{r_1} \Pi_{vvn}(r,\xi,\tau) * f_{vn,m-1}(\xi,\tau)\,d\xi,
\end{aligned} \tag{14}$$

where

$$\begin{aligned}
f_{un,m-1}(\xi,\tau) &= \rho_{e0}(\xi) E_{rn,m-1}(\xi,\tau) + E_0(\xi)\rho_{n,m-1}(\xi,\tau),\\
f_{vn,m-1}(\xi,\tau) &= \rho_{e0}(\xi) E_{\theta n,m-1}(\xi,\tau) - \gamma E_0(\xi) H_{n,m-1}(\xi,\tau),\\
\Pi_{uun}(r,\xi,\tau) &= \dot{G}_{uun}(r,\xi,\tau),\ \Pi_{uvn}(r,\xi,\tau) = \dot{G}_{uvn}(r,\xi,\tau),\\
\Pi_{vun}(r,\xi,\tau) &= \dot{G}_{vun}(r,\xi,\tau),\ \Pi_{vvn}(r,\xi,\tau) = \dot{G}_{vvn}(r,\xi,\tau).
\end{aligned} \tag{15}$$

An explicit form of kernels in (13) is specified in [5]. It is omitted here because of its awkwardness.

The function ρ_{nm} being part of (15), according to (7), is defined as follows:

$$\rho_{nm}(r,\tau) = -\rho'_{e0}(r) u_{nms}(r,\tau) - \rho_{e0}\chi_{nms}(r,\tau),$$

where:

$$\chi_{nm}(r,\tau) = \int_0^{r_1} \mathrm{X}_{un}(r,\xi,\tau) * f_{un,m-1}(\xi,\tau)\,d\xi + \int_0^{r_1} \mathrm{X}_{vn}(r,\xi,\tau) * f_{vn,m-1}(\xi,\tau)\,d\xi,$$

$$\mathrm{X}_{un}(r,\xi,\tau) = \chi_n(G_{uun}, G_{vun}),\ \mathrm{X}_{vn}(r,\xi,\tau) = \chi_n(G_{uvn}, G_{vvn}),$$

$$\chi_n(u_n, v_n) = \frac{1}{r^2}\frac{\partial\left(r^2 u_n\right)}{\partial r} + \frac{n(n+1)}{r}.$$

The meaning of the newly introduced function $\chi_n(u,v)$ is a coefficient of series expansion in terms of Legendre polynomials by coefficient of volumetric expansion for displacement field with components u and v.

The relations (11) – (14) are m-recurrent sequence of relations with respect to coefficients of series Legendre and Gegenbauer polynomials and (2) for displacements, strengths of electric and magnetic fields, as well as charge densities. As if follows from the publication [4], the following equations will be the initial conditions for such sequence:

$$\begin{aligned}
&u_{n0}(r,\tau)\equiv 0,\ v_{n0}(r,\tau)\equiv 0\ (n\geq 0),\ H_{n0}(r,\tau) = -\eta_e^2 G_{Hn1}^c(r)\left[\gamma e_{01n}(\tau) + \dot{e}_{01n}(\tau)\right],\\
&E_{rn0}(r,\tau) = -\frac{n(n+1)}{r} G_{Hn1}^c(r) e_{01n}(\tau), E_{\theta n0}(r,\tau) = \Gamma_{Hn1}^c(r) e_{01n}(\tau)\ (n\geq 1),\ \rho_{n0}(r,\tau)\equiv 0,
\end{aligned} \tag{16}$$

where

$$G_{Hn1}^{c}\left(r\right)=\frac{r^{n}}{\left(n+1\right)r_{1}^{n-1}},\ \Gamma_{Hn1}^{c}\left(r\right)=\frac{r^{n-1}}{r_{1}^{n-1}}.$$

Based on the resultant components of displacement field and electromagnetic field one can obtain coordinates of the vector of current density by using the general system of the equations from [14].

5. Example

Assume that radius of a ball is $r_1 = 2$, and its material has the following non-dimensional parameters: $\eta = 2,04;\ \eta_e = 0,111\cdot 10^{-4};\ \gamma = 5,06;\ \alpha = 0,0806$, where $E_* = 100\, w/m$. The electrical field has the following initial parameters: $E_0 = 1,\ \rho_{0e} = 2/r$. The strength of electrical field on the cavity boundary has the form: $e_{01} = -\tau_+ \sin\theta$, $\tau_+ = \tau\ H(\tau)$, which corresponds to the following coefficients of expansion in series right parts of equalities (1): $e_{001} = -\tau_+$, $e_{00n} \equiv 0\ (n \geq 2)$. Therefore, only coefficients of the series on Legendre and Gegenbauer polynomials with number $n = 1$ will be distinct from-zero.

The calculations were made by relations (11) - (16) with taking account of the terms of series (2) of order α^3. The integrals in recurrent relations were found numerically. Figure 1 demonstrate dependencies of coefficients of series Legendre polynomials with number $n = 1$ for displacements (y-axis) on radius r: solid lines indicate the moment of time $\tau = 0,2$, dashed lines, $\tau = 0,3$, and dash-and-dot, $\tau = 0,4$.

Similar relation H is a non-zero component of magnetic field vector, but with respect of time τ are depicted in Fig. 2: solid lines indicate point $r = 0,5$, dashed, $r = 1$, and dash-and-dot, $r = 1,5$.

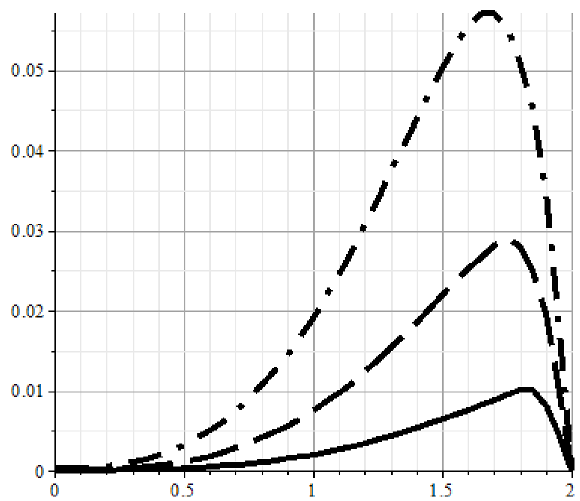

Fig. 1. u_1 vs. radius r

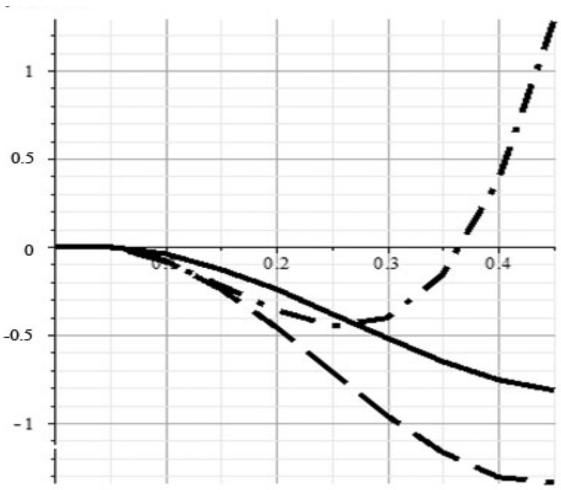

Fig. 2. $H\cdot 10^{11}$ vs. time τ

6. Conclusions

The algorithm for solving the coupled electromagnetoelasticity problem for conducting sphere allows us to find the mechanical and electromagnetic components of the problem at any point of the ball at an arbitrary instant time under the action of surface mechanical or electromagnetic disturbances. Mathematical modeling of the proposed problem allows us clearly and without the expensive physical experiment to see the mutual influence of the electromagnetic and mechanical fields. The constructed exact solution may have applications for predicting the behavior of conductor materials in various tasks of modern technology such as electroacoustics, automation, microelectronics.

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