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STEADY STATE RESPONSE DUE TO MOVING LOAD IN THERMOELASTIC MATERIAL WITH DOUBLE POROSITY

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Abstract. The present study is to focus on the steady state response due to moving load in a homogeneous, isotropic thermoelastic material with double porosity in the context of Lord-Shulman theory [1] of thermoelasticity with one relaxation time. The load is moving at a constant velocity along the one of the coordinate axis. Fourier transform has been applied to obtain normal stress, tangential stress, equilibrated stresses and temperature distribution. The resulting expressions are obtained in the physical domain by using numerical inversion technique. Numerically computed results for these quantities are depicted graphically to study the effect of porosity for normal force and thermal source. Some particular cases are also deduced from the present investigation.
Keywords: thermoelasticity, double porosity, moving load, Fourier transform

1. Introduction

Porous media theories play an important role in many branches of engineering including material science, the petroleum industry, chemical engineering, biomechanics and other such fields of engineering. Representation of a fluid saturated porous medium as a single phase material has been virtually discarded. The material with the pore spaces such as concrete can be treated easily because all concrete ingredients have the same motion if the concrete body is deformed. However the situation is more complicated if the pores are filled with liquid and in that case the solid and liquid phases have different motions. Due to these different motions, the different material properties and the complicated geometry of pore structures, the mechanical behavior of a fluid saturated porous thermoelastic medium becomes very difficult. So researchers from time to time, have tried to overcome this difficulty and we see many porous media in the literature. A brief historical background of these theories is given by de Boer [2,3].

Biot [4] proposed a general theory of three-dimensional deformation of fluid saturated porous salts. Biot theory is based on the assumption of compressible constituents and till recently, some of his results have been taken as standard references and basis for subsequent analysis in acoustic, geophysics and other such fields. Another interesting theory is given by Bowen [5], de Boer and Ehlers [6] in which all the constituents of a porous medium are assumed to be incompressible. The fluid saturated porous material is modeled as a two phase system composed of an incompressible solid phase and incompressible fluid phase, thus meeting the many problems in engineering practice, e.g. in soil mechanics. One important generalization of Biot's theory of poroelasticity that has been studied extensively started with the works by Barenblatt et al. [7], where the double porosity model was first proposed to express the fluid flow in hydrocarbon reservoirs and aquifiers.

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The double porosity model represents a new possibility for the study of important problems concerning the civil engineering. It is well-known that, under super- saturation conditions due to water of other fluid effects, the so called neutral pressures generate unbearable stress states on the solid matrix and on the fracture faces, with severe (sometimes disastrous) instability effects like landslides, rock fall or soil fluidization (typical phenomenon connected with propagation of seismic waves). In such a context it seems possible, acting suitably on the boundary pressure state, to regulate the internal pressures in order to deactivate the noxious effects related to neutral pressures; finally, a further but connected positive effect could be lightening of the solid matrix/fluid system.

Wilson and Aifanits [8] presented the theory of consolidation with the double porosity. Khaled, Beskos and Aifantis [9] employed a finite element method to consider the numerical solutions of the differential equation of the theory of consolidation with double porosity developed by Aifantis[8]. Wilson and Aifantis [10] discussed the propagation of acoustics waves in a fluid saturated porous medium. The propagation of acoustic waves in a fluid-saturated porous medium containing a continuously distributed system of fractures is discussed. The porous medium is assumed to consist of two degrees of porosity and the resulting model thus yields three types of longitudinal waves, one associated with the elastic properties of the matrix material and one each for the fluids in the pore space and the fracture space.

Beskos and Aifantis [11] presented the theory of consolidation with double porosity-II and obtained the analytical solutions to two boundary value problems. Khalili and Valliappan [12] studied the unified theory of flow and deformation in double porous media. Aifantis [13-16] introduced a multi-porous system and studied the mechanics of diffusion in solids. Moutsopoulos et al. [17] obtained the numerical simulation of transport phenomena by using the double porosity/ diffusivity continuum model. Khalili and Selvadurai [18] presented a fully coupled constitutive model for thermo-hydro –mechanical analysis in elastic media with double porosity structure. Pride and Berryman [19] studied the linear dynamics of double-porosity dual-permeability materials. Straughan [20] studied the stability and uniqueness in double porous elastic media. Various authors have [21-26] investigated problems on elastic solids, viscoelastic solids and thermoelastic solids with double porosity based on Darcy's law.

Nunziato and Cowin [27] developed a nonlinear theory of elastic material with voids. Later, Cowin and Nunziato [28] developed a theory of linear elastic materials with voids for the mathematical study of the mechanical behavior of porous solids. Iesan and Quintanilla [29] derived a theory of thermoelastic solids with double porosity structure by using the theory developed by Nunziato and Cowin. Darcy's law is not used in developing this theory. So far not much work has been done on the theory of thermoelasticity with double porosity based on the model proposed by Iesan and Quintanilla [29]. Recently investigations have been started in the theory of thermoelasticity with double porosity [29] which has a significant application in continuum mechanics.

The problem of determining the response of an elastic system subjected to a moving load has received considerable attention. Work in this area has been mostly motivated by the need to analyze the vibrations of such structures as bridges and rail/road tracks caused by moving vehicles. The design of highways, airport runways as well as the foundation problems in soil mechanics, particularly when the earth mass is supporting a heavy structure having a moving load over its free plane surface, lead to the investigation of the dynamic stress distribution associated with the problem. An important problem concerning such diverse fields as wave propagation, contact mechanics and tribology is the rapid motion of a line mechanical and/or thermal load over the surface of a half-space. Indeed, this is the case when (a) ground motion and stresses are produced by the surface blast waves due to explosives or by supersonic aircraft (b) high velocity rockets sleds moving on guide rails. Such dynamical mechanical/thermal loading may produce severe deformation and temperature rise in a thin zone near the half-space surface and thereby causes excessive wear and even cracking near the contact zone. In many cases, the above described problem can be modeled as a plane-strain steady state situation, involving an elastic half-plane under mechanical/thermal loading which moves over the half-plane surface of constant speed.

The steady state assumption employed here has its own justification in the dynamic analysis of moving sources [30-33] and may yield reliable results when the mechanical/thermal load in question has been applied and moving for a long time. Kumar and Deshwal [34] investigated the steady state response due to moving loads in a micropolar generalized thermoelastic half space without energy dissipation. Kumar and Ailawalia [35] discussed the moving load response at thermal conducting fluid and micropolar solid interface. Sharma et al. [36] discussed the steady state response due to moving load in thermoelastic solid Media. Malekzadeh and Heydarpour [37] studied the response of functionally graded cylindrical shells under moving thermo-mechanical loads. Chatterjee et al. [38] studied the response of moving load due to irregularity in slightly compressible, finitely deformed elastic media. The role of shear deformation in dynamic behavior of a fully saturated poroelastic beam traversed by a moving load was studied by Kiani et al. [39].

The present investigation is to determine the components of stress and temperature distribution in a homogenous, isotropic, thermoelastic half-space with double porosity due to moving mechanical/thermal sources. Fourier transform technique has been applied to obtain the components of stress and temperature distribution. Numerical inversion technique has been applied to recover the resulting quantities in the physical domain.The resulting quantities are depicted graphically to study the effect of porosity for normal force and thermal source. Some particular cases are also deduced.

2. Basic equations

Following Iesan and Quintanilla [29] and Lord and Shulman [1], the constitutive relations and field equations for homogeneous thermoelastic material with double porosity structure without body forces, extrinsic equilibrated body forces and heat source can be written as:

Constitutive relations:

$$t_{ij} = \lambda e_{rr}\delta_{ij} + 2\mu e_{ij} + b\delta_{ij}\varphi + d\delta_{ij}\psi - \beta\delta_{ij}T\,, \tag{1}$$

$$\sigma_i = \alpha\varphi_{,i} + b_1\psi_{,i}\,, \tag{2}$$

$$\chi_i = b_1\varphi_{,i} + \gamma\psi_{,i}\,, \tag{3}$$

Equation of motion:

$$\mu\nabla^2\vec{u} + (\lambda+\mu)\nabla\nabla\cdot\vec{u} + b\nabla\varphi + d\nabla\psi - \beta\nabla T = \rho\frac{\partial^2 u}{\partial t^2}, \tag{4}$$

Equilibrated stress equations of motion:

$$\alpha\nabla^2\varphi + b_1\nabla^2\psi - b\nabla\cdot\vec{u} - \alpha_1\varphi - \alpha_3\psi + \gamma_1 T = \kappa_1\frac{\partial^2\varphi}{\partial t^2}, \tag{5}$$

$$b_1\nabla^2\varphi + \gamma\nabla^2\psi - d\nabla\cdot\vec{u} - \alpha_3\varphi - \alpha_2\psi + \gamma_2 T = \kappa_2\frac{\partial^2\psi}{\partial t^2}, \tag{6}$$

Equation of heat conduction:

$$K^*\nabla^2 T = \left(1+\tau_0\frac{\partial}{\partial t}\right)\left(\beta T_0\nabla.\dot{\vec{u}} + \gamma_1 T_0\dot{\varphi} + \gamma_2 T_0\psi + \rho C^*\dot{T}\right), \tag{7}$$

where λ and μ are Lame's constants, ρ is the mass density; $\beta=(3\lambda+2\mu)\alpha_t$; α_t is the linear thermal expansion; C^* is the specific heat at constant strain, u_i is the displacement components; t_{ij} is the stress tensor; κ_1 and κ_2 are coefficients of equilibrated inertia; φ and ψ are the volume fraction fields corresponding to pores and fissures respectively; σ_i is the equilibrated stress corresponding to pores; χ_i is the equilibrated stress corresponding to fissures; K^* is the coefficient of thermal conductivity and $b, d, b_1, \gamma, \gamma_1, \gamma_2$ are constitutive coefficients; δ_{ij} is the Kronecker's delta; T is the temperature change measured form the absolute temperature $T_0\,(T_0 \neq 0)$; a superposed dot represents differentiation with respect to time variable t.

$$\nabla=\hat{i}\frac{\partial}{\partial x_1}+\hat{j}\frac{\partial}{\partial x_2}+\hat{k}\frac{\partial}{\partial x_3}, \quad \nabla^2=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+\frac{\partial^2}{\partial x_3^2}$$

are the gradient and Laplacian operators, respectively.

3. Formulation of the problem

We consider a homogeneous, isotropic, thermoelastic material with double porosity structure in the undeformed state at uniform temperature T_0.The rectangular Cartesian coordinate system (x_1, x_2, x_3) having origin on the surface $x_3=0$ with x_3-axis pointing vertically into the medium is introduced. A moving normal force or thermal source is assumed to be acting at the origin of the rectangular Cartesian coordinates. It follows from the description of the problem that all the considered functions will depend upon (x_1, x_3, t). We thus obtain the displacement vector $\vec{u}$ of the form $\vec{u}=(u_1, 0, u_3)$.

Following Fung [40], Galilean transformation is introduced as

$$x_1^*=x_1+Ut, \quad x_3^*=x_3, \quad t^*=t. \tag{8}$$

4. Solution of the problem

To transform equations (4)-(7) to non-dimensional form, we define the following non-dimensional quantities as:

$$x_1'=\frac{\omega_1}{c_1}x_1^*, \; x_3'=\frac{\omega_1}{c_1}x_3^*, u_1'=\frac{\omega_1}{c_1}u_1, u_3'=\frac{\omega_1}{c_1}u_3, t_{ij}'=\frac{t_{ij}}{\beta t_0}, \tau_0'=\omega_1\tau_0,$$

$$\varphi'=\frac{k_1\omega_1^2}{\alpha_1}\varphi, \psi'=\frac{k_1\omega_1^2}{\alpha_1}, T'=\frac{T}{T_0}, t'=\omega_1 t, \sigma_i'=\left(\frac{c_1}{\alpha\omega_1}\right)\sigma_i, \chi_i'=\left(\frac{c_1}{\alpha\omega_1}\right)\chi_i, \tag{9}$$

where $c_1^2=\dfrac{\lambda+2\mu}{\rho}, \omega_1=\dfrac{\rho C^* c_1^2}{K^*}$.

Making use of dimensionless quantities given by (9) in equations (4)-(7), we obtain (suppressing the primes for convenience)

$$\frac{\partial e}{\partial x_1}+a_1\nabla^2 u_1+a_2\frac{\partial\varphi}{\partial x_1}+a_3\frac{\partial\psi}{\partial x_1}-a_4\frac{\partial T}{\partial x_1}=a_5\frac{\partial^2 u_1}{\partial x_1^2}, \tag{10}$$

$$\frac{\partial e}{\partial x_3}+a_1\nabla^2 u_3+a_2\frac{\partial\varphi}{\partial x_3}+a_3\frac{\partial\psi}{\partial x_3}-a_4\frac{\partial T}{\partial x_3}=a_5\frac{\partial^2 u_1}{\partial x_3^2}, \tag{11}$$

$$a_6\nabla^2\varphi + a_7\nabla^2\psi - a_8 e - a_9\varphi - a_{10}\psi + a_{11}T = \frac{\partial^2\varphi}{\partial x_1^2}, \tag{12}$$

$$a_{12}\nabla^2\varphi + a_{13}\nabla^2\psi - a_{14} e - a_{15}\varphi - a_{16}\psi + a_{17}T = \frac{\partial^2\psi}{\partial x_3^2}, \tag{13}$$

$$\nabla^2 T = a_{18}\frac{\partial e}{\partial x_1} + a_{19}\frac{\partial \varphi}{\partial x_1} + a_{20}\frac{\partial \psi}{\partial x_1} + a_{21}\frac{\partial T}{\partial x_1} + a_{22}\frac{\partial^2 e}{\partial x_1^2} + a_{23}\frac{\partial^2 \varphi}{\partial x_1^2} + a_{24}\frac{\partial^2 \psi}{\partial x_1^2} + a_{25}\frac{\partial^2 T}{\partial x_1^2}, \tag{14}$$

where

$$a_1 = \frac{\mu}{\lambda+\mu}, a_2 = \frac{b\alpha_1}{k_1\omega_1^2(\lambda+\mu)}, a_3 = \frac{d\alpha_1}{k_1\omega_1^2(\lambda+\mu)}, a_4 = \frac{\beta T_0}{(\lambda+\mu)}, a_5 = \frac{\rho U^2}{(\lambda+\mu)},$$

$$a_6 = \frac{\alpha}{\kappa_1 U^2}, a_7 = \frac{b_1}{\kappa_1 U^2}, a_8 = \frac{bc_1^2}{\alpha_1 U^2}, a_9 = \frac{\alpha_1 c_1^2}{\kappa_1\omega_1^2 U^2}, a_{10} = \frac{\alpha_3 c_1^2}{\kappa_1\omega_1^2 U^2}, a_{11} = \frac{\gamma_1 T_0}{\alpha_1},$$

$$a_{12} = \frac{b_1}{\kappa_2 U^2}, a_{13} = \frac{\gamma c_1^2}{\kappa_2 U^2}, a_{14} = \frac{d\kappa_1 c_1^2}{\alpha_1\kappa_2 U^2}, a_{15} = \frac{\alpha_3 c_1^2}{\kappa_2\omega_1^2 U^2}, a_{16} = \frac{\alpha_2 c_1^2}{\kappa_2\omega_1^2 U^2},$$

$$a_{17} = \frac{\gamma_2 T_0\kappa_1 c_1^2}{\alpha_1\kappa_2 U^2}, a_{18} = \frac{\beta U c_1}{K^*\omega_1}, a_{19} = \frac{\gamma_1\alpha_1 U c_1}{K^*\kappa_1\omega_1^3}, a_{20} = \frac{\gamma_2\alpha_1 U c_1}{K^*\kappa_1\omega_1^3}, a_{21} = \frac{\rho C^* U c_1}{K^*\omega_1},$$

$$a_{22} = \frac{\tau_0 U^2\beta c_1}{K^*\omega_1}, a_{23} = \frac{\tau_0 U^2\gamma_1\alpha_1}{K^*\kappa_1\omega_1^3}, a_{24} = \frac{\tau_0 U^2\gamma_2\alpha_1}{K^*\kappa_1\omega_1^3}, a_{25} = \frac{\tau_0 U^2\rho C^*}{K^*\omega_1}$$

The displacement components u_1 and u_3 are related by potential functions φ_1 and ψ_1 as

$$u_1 = \frac{\partial\varphi_1}{\partial x_1} - \frac{\partial\psi_1}{\partial x_3}, \qquad u_3 = \frac{\partial\varphi_1}{\partial x_3} + \frac{\partial\psi_1}{\partial x_1}. \tag{15}$$

We define Fourier transform by

$$\tilde f(\xi) = \int_{-\infty}^{\infty} f(x_1) e^{i\xi x_1} dx_1. \tag{16}$$

Using Eqs. (15) and (16) in the Eqs. (10)-(14) and assuming that $\tilde\varphi_1, \tilde\varphi, \tilde\psi, \tilde T, \tilde\psi_1 \to 0$ as $x_3 \to \infty$, after simplification, we obtain

$$\left(\tilde\varphi_1, \tilde\varphi, \tilde\psi, \tilde T\right) = \sum_{i=1}^{4}(1, g_{1i}, g_{2i}, g_{3i}) B_i e^{-m_i x_3}, \tag{17}$$

$$\tilde\psi_1 = B_5 e^{-m_5 x_3}, \tag{18}$$

where $m_i\ (i = 1, 2, 3, 4)$ are the roots of the equation

$$D^8 + E_1 D^6 + E_2 D^4 + E_3 D^2 + E_4 = 0 \tag{19}$$

$$D = \frac{d}{dx_3}$$

and $m_5 = \sqrt{\left(1 - \frac{a_5}{a_1}\right)\xi}$, (20)

where E_1, E_2, E_3, E_4 and E_5 are given in the Appendix I.

The coupling constants are given by

$$g_{1i} = -\frac{D_{1i}}{D_{0i}}, \ g_{2i} = \frac{D_{2i}}{D_{0i}}, \ g_{3i} = -\frac{D_{3i}}{D_{0i}}; i = 1, 2, 3, 4$$

and $D_{0i}, D_{1i}, D_{2i}, D_{3i}$ are given in the Appendix II.

5. Boundary conditions

We consider a moving normal force/thermal source acting at $x_3 = 0$ along with the vanishing of tangential and equilibrated stresses. Mathematically, these boundary conditions on the surface $x_3 = 0$ can be written as

(i) $t_{33} = -F_1\delta\left(x_1^*\right)$, (21)

(ii) $t_{31} = 0$, (22)

(iii) $\sigma_3 = 0$, (23)

(iv) $\chi_3 = 0$, (24)

(v) $T = F_2\delta\left(x_1^*\right)$, (25)

where F_1 and F_2 are the magnitude of force and constant temperature applied on the boundary respectively and $\delta()$ is the Dirac delta function.

Substituting the values of $\tilde{\varphi}_1, \tilde{\varphi}, \tilde{\psi}, \tilde{T}$ and $\tilde{\psi}_1$ from (17) and (18) in the boundary conditions (21)-(25) and with the aid of Eqs. (1)-(3), (9), (15) and (16), we obtain the corresponding expressions for components of stress and temperature distribution as

$$\tilde{t}_{33} = \frac{1}{\Delta}\left[Q_1\Delta_1 e^{-m_1x_3} + Q_2\Delta_2 e^{-m_2x_3} + Q_3\Delta_3 e^{-m_3x_3} + Q_4\Delta_4 e^{-m_4x_3} + Q_5\Delta_5 e^{-m_5x_3}\right], \tag{26}$$

$$\tilde{t}_{31} = \frac{1}{\Delta}\left[R_1\Delta_1 e^{-m_1x_3} + R_2\Delta_2 e^{-m_2x_3} + R_3\Delta_3 e^{-m_3x_3} + R_4\Delta_4 e^{-m_4x_3} + R_5\Delta_5 e^{-m_5x_3}\right], \tag{27}$$

$$\tilde{\sigma}_3 = \frac{1}{\Delta}\left[U_1\Delta_1 e^{-m_1x_3} + U_2\Delta_2 e^{-m_2x_3} + U_3\Delta_3 e^{-m_3x_3} + U_4\Delta_4 e^{-m_4x_3}\right], \tag{28}$$

$$\tilde{\chi}_3 = \frac{1}{\Delta}\left[V_1\Delta_1 e^{-m_1x_3} + V_2\Delta_2 e^{-m_2x_3} + V_3\Delta_3 e^{-m_3x_3} + V_4\Delta_4 e^{-m_4x_3}\right], \tag{29}$$

$$\tilde{T} = \frac{1}{\Delta}\left[g_{31}\Delta_1 e^{-m_1x_3} + g_{11}\Delta_2 e^{-m_2x_3} + g_{11}\Delta_3 e^{-m_3x_3} + g_{11}\Delta_4 e^{-m_4x_3}\right], \tag{30}$$

where

$$\Delta = \begin{vmatrix} Q_1 & Q_2 & Q_3 & Q_4 & Q_5 \\ R_1 & R_2 & R_3 & R_4 & R_5 \\ U_1 & U_2 & U_3 & U_4 & 0 \\ V_1 & V_2 & V_3 & V_4 & 0 \\ g_{31} & g_{32} & g_{33} & g_{34} & 0 \end{vmatrix}. \tag{31}$$

$\Delta_i\left(i = 1, 2, 3, 4, 5\right)$ are obtained by replacing i^{th} column of (31) with $\left[-F_1\ 0\ 0\ 0\ F_2\right]^{tr}$, where $Q_i, R_i (i = 1, 2, 3, 4, 5)$ and $U_j, V_j\ (j = 1, 2, 3, 4)$ are given in the Appendix III.

The transformed components of stress and temperature distribution are functions of the parameter of Fourier transform ξ is of the form $\tilde{f}\left(\xi, x_3\right)$.To obtain the solution of the problem in the physical domain; we invert the Fourier transform by using the method described by Press et al. [41].

Case 5.1 Normal force acting on the surface

If $F_2 \to 0$ in Eqs. (26)-(30), it corresponds to the resulting expressions for normal force.

Case 5.2 Thermal source acting on the surface

If $F_1 \to 0$ in Eqs. (26)-(30), yields the resulting expressions for thermal source.

Particular cases

(i) If $b_1 = \alpha_3 = \gamma = \alpha_2 = \gamma_2 = d \to 0$ in the Eqs. (26)-(30), we obtain the corresponding expressions for thermoelastic material with single porosity for thermomechanical sources.

(ii) If $\tau_0 = 0$, in the Eqs. (26)-(30) yield the corresponding expressions for thermoelastic material with double porosity in context of coupled theory of thermoelasticity.

6. Numerical results and discussion

The material chosen for the purpose of numerical computation is copper, whose physical data is given by Sherief and Saleh [42] as:

$\lambda = 7.76\times10^{10}\,Nm^{-2},\; C^* = 3.831\times10^3 m^2 s^{-2} K^{-1},\; \mu = 3.86\times10^{10}\,Nm^{-2},$

$K^* = 3.86\times10^3\,Ns^{-1}K^{-1},\; T_0 = 0.293\times10^3 K,\; \rho = 8.954\times10^3\,Kgm^{-3},$

$\alpha_t = 1.78\times10^{-5} K^{-1}$.

The double porous parameters are taken as:

$\alpha_2 = 2.4\times10^{10}\,Nm^{-2},\; \alpha_3 = 2.5\times10^{10}\,Nm^{-2},\; \gamma = 1.1\times10^{-5}\,N,\; \alpha = 1.3\times10^{-5}\,N$

$\gamma_1 = 0.16\times10^5\,Nm^{-2},\; b_1 = 0.12\times10^{-5}\,N,\; d = 0.1\times10^{10}\,Nm^{-2}$

$\gamma_2 = 0.219\times10^5\,Nm^{-2},\; \kappa_1 = 0.1456\times10^{-12}\,Nm^{-2}s^2,\; b = 0.9\times10^{10}\,Nm^{-2}$

$\alpha_1 = 2.3\times10^{10}\,Nm^{-2},\; \kappa_2 = 0.1546\times10^{-12}\,Nm^{-2}s^2$.

The software MATLAB has been used to find the values of normal stress t_{33}, tangential stress t_{31}, equilibrated stresses σ_3, χ_3 and temperature distribution T. The variations of these values with respect to distance x_1 have been shown in figures (1)-(8) respectively. In all these figures, solid line corresponds to thermal double porous material (TDP) and small dashed line corresponds to thermal single porous material (TSP).

Normal force. Figures 1-4 depicts the variation of normal stress t_{33}, tangential stress t_{31}, equilibrated stress σ_3, temperature distribution T with respect to distance x_1 due to normal force.

In Figure 1, the behavior and variation of normal stress t_{33} for TDP and TSP is opposite to each other near the application of the source while the behavior is same as moving away from the source. It is also noticed that for TDP, the magnitude value of t_{33} is higher for theregion $0 < x \le 1$, become smaller for the region $1 < x \le 3$ than that of TSP and becomes almost same for both, in the remaining region.From Figure 2, it is noticed that the values of tangential stress t_{31} are oscillatory in nature for both TDP and TSP. Also, it is found that due to porosity effect the magnitude values are higher for TSP in comparison to TDP near the application of the source while as moving away from the source application point, the values becomes higher for TDP than that of TSP. From Figure 3, it is clear that the behavior of equilibrated stress σ_3 is oscillatory for both the materials while the amplitude of oscillations is more for TDP as compared to that of TSP. The magnitude values of σ_3 are larger for TDP as compared to TSP for the regions $1 < x < 1.8$, $4 < x < 7.8$ whereas an opposite trend is noticed in the subsequent regions.

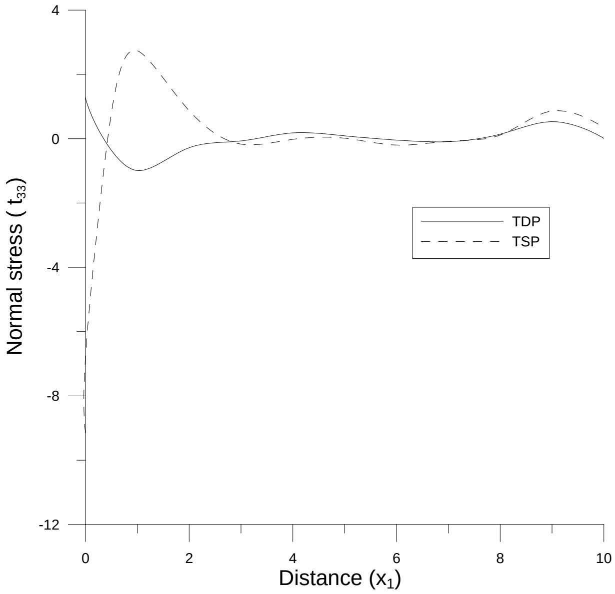

Fig. 1. Variation of normal stress t_{33} w.r.t. x_1

(Normal force)

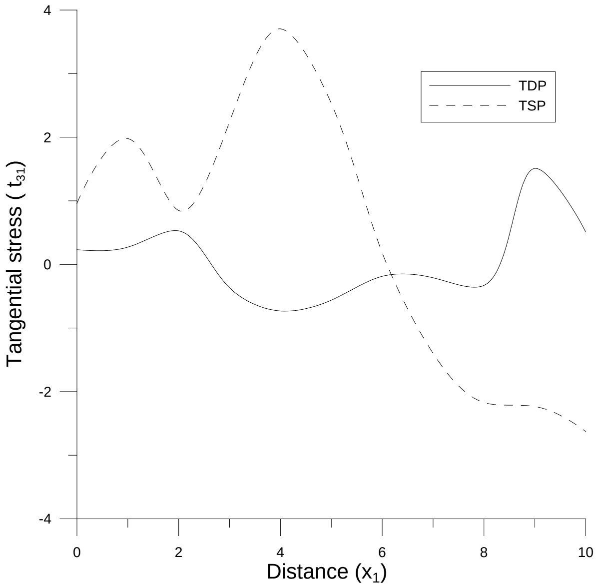

Fig. 2. Variation of tangential stress t_{31} w.r.t. x_1
(Normal force)

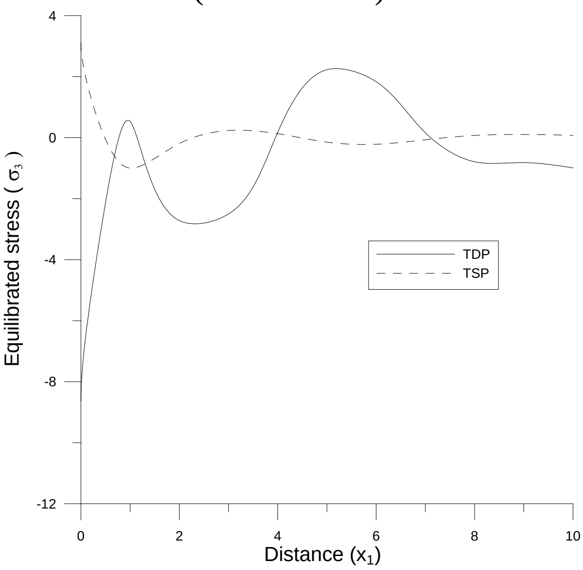

Fig. 3. Variation of equilibrated stress σ_3 w.r.t. x_1
(Normal force)

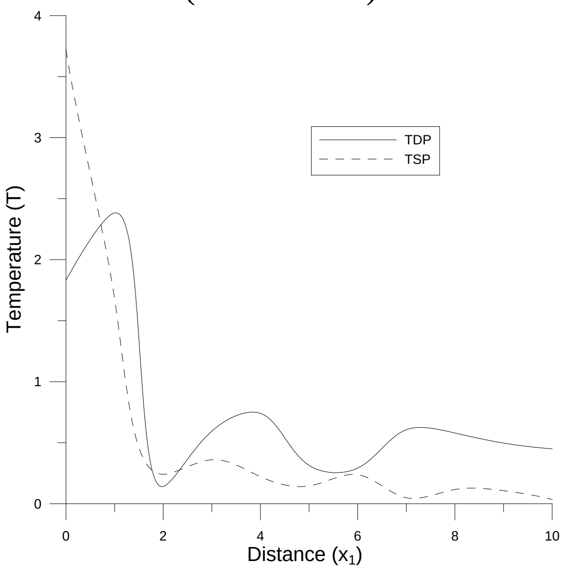

Fig. 4. Variation of temperature distribution T w.r.t. x_1
(Normal force)

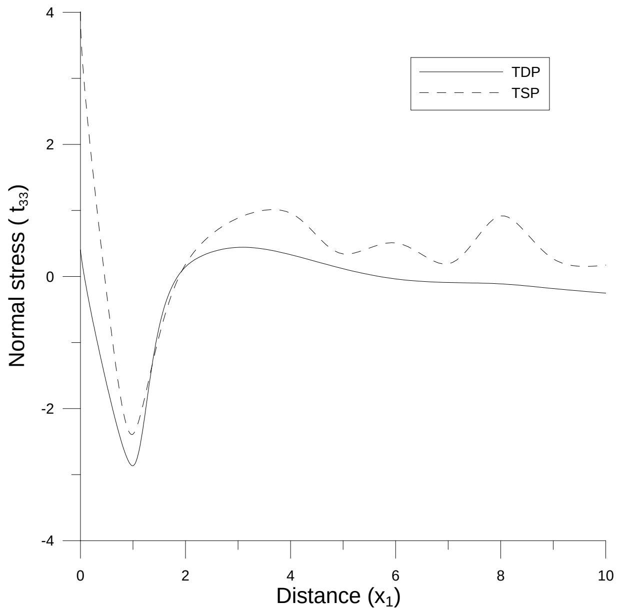

Fig. 5. Variation of normal stress t_{33} w.r.t. x_1

(Thermal source)

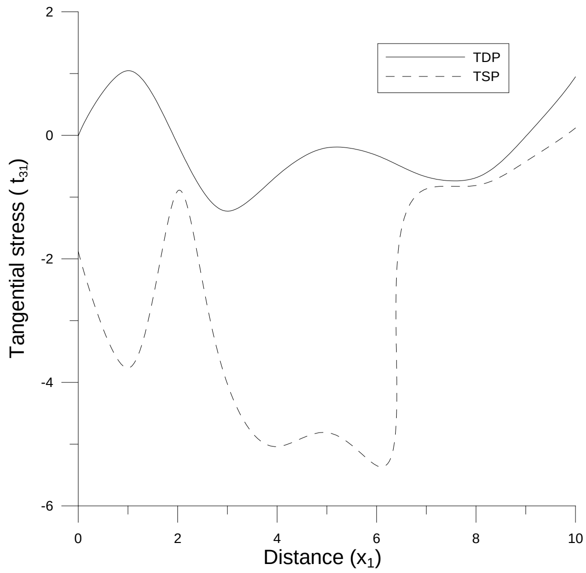

Fig. 6. Variation of tangential stress t_{31} w.r.t. x_1

(Thermal source)

From Figure 4, the trend of variation of temperature distribution T is oscillatory in nature for both TDP and TSP. It is noticed that the magnitude values of T are more for TSP than that of TDP for the region $0 < x < 2$ whereas the values are higher for TDP in comparison to TSP as $x \geq 2$. It is also found that T attains maxima near the point of application of the source for both the materials.

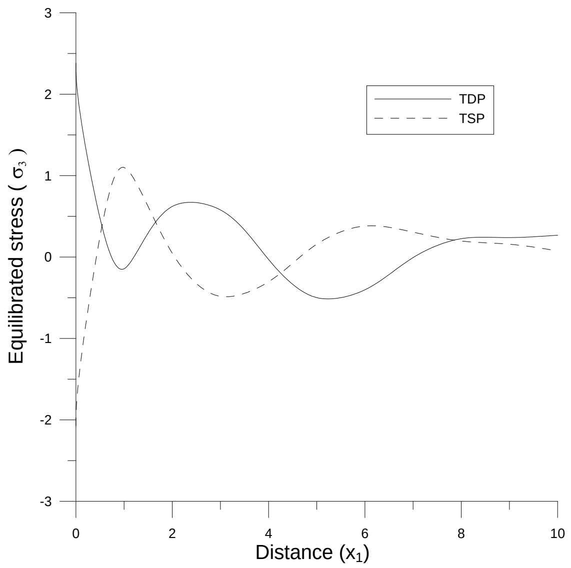

Fig. 7. Variation of equilibrated stress σ_3 w.r.t. x_1

(Thermal source)

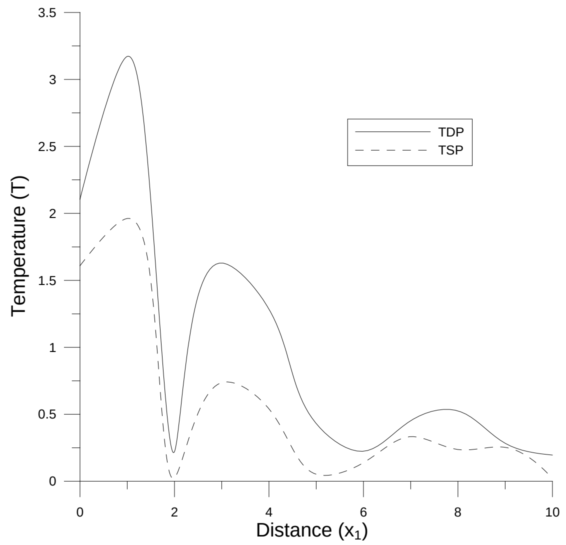

Fig. 8. Variation of temperature distribution T w.r.t. x_1

(Thermal source)

Thermal source. Figures 5-8 depict the variation of normal stress t_{33}, tangential stress t_{31}, equilibrated stresses σ_3 and temperature distribution T with respect to distance x_1 due to thermal source.

From Figure 5, it is noticed that the value of normal stress t_{33} decreases sharply for $0 < x \le 1.2$, again increases sharply for $1.2 < x \le 2$ and then decreases slowly and steadily as $x > 2$. Due the effect of porosity, the magnitude values of TSP are more in comparison to TDP for all x although the trend of variation is similar for both the materials. From Figure 6, it is evident that the behavior and variation of tangential stress t_{31} is oscillatory in nature for both TDP and TSP. The magnitude values of t_{31} are more for TDP than that of TDP for all the values of x due to the effect of porosity. In Figure 7, an oscillatory behavior of variation of equilibrated stress σ_3 is noticed for both TDP and TSP. It is found that the amplitude of oscillation is maximum at the application point of source and start to decrease as moving away from the source. Figure8 shows that the magnitude values of temperature distribution T are more for TDP as compared to that of TSP due to effect of porosity for all the values of x_1. Although the trend and behavior of variation is similar for both the materials for all x_1 while the amplitude of oscillation decreases with the increase in value of x_1.

7. Conclusions

In this paper, the deformation due to moving load in thermoelastic medium with double porosity in context of Lord-Shulman theory of thermoelasticity has been investigated. It is concluded that analysis of elastodynamics deformation in thermoelastic materials with double porosity structure due to moving load is a significant problem of mechanics. The behavior of components of stress and temperature distribution in an isotropic homogeneous thermoelastic material with double porosity structure has been investigated for thermoelastic interactions due to moving load by using Fourier transform technique. All the field quantities are observed to be very sensitive towards the porosity parameter. Graphical representation indicated that double porosity and single porosity have both the increasing and decreasing effects on the numerical values of the physical quantities.

This type of study is useful due to its application in geophysics and rock mechanics. The results obtained in this investigation should prove to be beneficial for the researchers working on the theory of thermoelasticity with double porosity structure. The introduction of double porous parameter to the thermoelastic medium represents a more realistic model for these studies.

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APPENDIX I

$$E_1 = \left(\delta_1 r_1 - a_2 r_5 + a_3 r_9 + a_4 r_{13} + a_7 r_2\right)/a_7 r_1,$$

$$E_2 = \left(\delta_1 r_2 - a_2 r_6 + a_3 r_{10} + a_4 r_{14} + a_7 r_3\right)/a_7 r_1,$$

$$E_3 = \left(\delta_1 r_3 - a_2 r_7 + a_3 r_{11} + a_4 r_{15} + a_7 r_4\right)/a_7 r_1,$$

$$E_4 = \left(\delta_1 r_4 - a_2 r_8 + a_3 r_{12} + a_4 r_{16}\right)/a_7 r_1,$$

$$\delta_1 = \left(a_5 - a_{27}\right)\xi^2,$$

$$\delta_2 = a_8 \xi^2,$$

$$\delta_3 = \left(1 - a_6\right)\xi^2 - a_9,$$

$$\delta_4 = -\left(a_7 \xi^2 + a_{10}\right),$$

$$\delta_5 = a_{14}\xi^2,$$

$$\delta_6 = -\left(a_{12}\xi^2 + a_{15}\right),$$

$$\delta_7 = \left(1 - a_{13}\right)\xi^2 - a_{16},$$

$\delta_8 = -a_{28}\xi^2,$

$r_1 = a_6 a_{13} - a_7 a_{12},$

$r_2 = \delta_3 a_{13} + a_6(a_{13}a_{31} + \delta_7) - \delta_4 a_{12} - a_7(a_{12}a_{31} + \delta_6),$

$r_3 = a_6(\delta_7 a_{31} - a_{17}a_{30}) + \delta_3(a_{13}a_{31} + \delta_7) - a_7(\delta_6 a_{31} - a_{17}a_{29})$

$\quad - \delta_4(a_{12}a_{31} + \delta_6) + a_{11}(a_{12}a_{30} - a_{13}a_{29}),$

$r_4 = \delta_3(\delta_7 a_{31} - a_{17}a_{30}) - \delta_4(\delta_6 a_{31} - a_{17}a_{29}) + a_{11}(\delta_6 a_{30} - \delta_7 a_{29}),$

$r_5 = a_7 a_{14} - a_8 a_{13},$

$r_6 = a_7(a_{14}a_{31} + a_{17}a_{28} - \delta_5) - a_8(a_{13}a_{31} + \delta_7) + a_{13}(\delta_2 - a_{11}a_{28}) + \delta_4 a_{14},$

$r_7 = a_8(a_{17}a_{30} - \delta_7 \mathrm{a}_{31}) + \delta_2(a_{13}a_{31} + \delta_7) - a_7(\delta_5 a_{31} - a_{17}\delta_8)$

$-\delta_4\left(\delta_5 - a_{14}a_{31} - a_{17}a_{28}\right) - a_{11}\left(a_{13}\delta_8 + \delta_7 a_{28} + a_{14}a_{30}\right),$

$r_8 = \delta_2(\delta_7 a_{31} - a_{17}a_{30}) - \delta_4(\delta_5 a_{31} - \delta_8 a_{17}) + a_{11}(\delta_5 a_{30} - \delta_7\delta_8),$

$r_9 = a_6 a_{14} - a_8 a_{12},$

$r_{10} = a_{12}\left(\delta_2 - a_{11}a_{28}\right) - a_8(a_{12}a_{31} + \delta_6) + a_6(a_{14}a_{30} + a_{17}a_{28} - \delta_5) + \delta_3 a_{14},$

$r_{11} = a_8(a_{17}a_{29} - \delta_6 \mathrm{a}_{31}) + \delta_2(a_{12}a_{31} + \delta_6) - a_6(\delta_5 a_{31} - a_{17}\delta_8)$

$\quad - \delta_3\left(\delta_5 - a_{14}a_{31} - a_{17}a_{28}\right) - a_{11}\left(a_{12}\delta_8 + \delta_6 a_{28} + a_{14}a_{29}\right),$

$r_{12} = \delta_2(\delta_6 a_{31} - a_{17}a_{29}) - \delta_3(\delta_5 a_{31} - \delta_8 a_{17}) + a_{11}(\delta_5 a_{29} - \delta_6\delta_8),$

$r_{13} = a_{28}(\mathrm{a}_6\, a_{13} - a_7 a_{12}),$

$r_{14} = a_8(\mathrm{a}_{13}\, a_{29} - a_{12}a_{30}) + a_6(\delta_7 a_{28} + \delta_8 a_{13} + a_{14}a_{30})$

$+a_{28}(\delta_3 a_{13} - \delta_4\delta_{12}) - a_7(\mathrm{a}_{14}\, a_{29} + \delta_8 a_{12} + \delta_6 a_{28}),$

$r_{15} = a_8(\delta_7 a_{29} - \delta_6 a_{30}) + \delta_2(\mathrm{a}_{12}\, a_{30} - a_{13}a_{29}) - a_6(\delta_5 a_{30} - \delta_7\delta_8)$

$-\delta_3(\mathrm{a}_{14}\, a_{30} + \delta_7 a_{28} + \delta_8 a_{13}) - \delta_4(\mathrm{a}_{14}\, a_{29} + \delta_8 a_{12} + \delta_6 a_{28}) + a_7(\delta_5 a_{29} - \delta_6\delta_8),$

$r_{16} = \delta_2(\delta_6 a_{30} - \delta_7 a_{29}) - \delta_3(\delta_5 a_{30} - \delta_7\delta_8) + \delta_4(\delta_5 a_{29} - \delta_6\delta_8),$

$a_{27} = 1 + a_1,$

$a_{28} = a_{22}\xi^2 + i a_{18}\xi,$

$a_{29} = a_{23}\xi^2 + i a_{19}\xi,$

$a_{30} = a_{24}\xi^2 + i a_{20}\xi,$

$a_{31} = a_{25}\left(\xi^2 - 1\right) + i a_{21}\xi.$

Appendix II

$$D_{0i} = \begin{vmatrix} a_6 m_i^2 + \delta_3 & a_7 m_i^2 + \delta_4 & a_{11} \\ a_{12} m_i^2 + \delta_6 & a_{13} m_i^2 + \delta_7 & a_{17} \\ a_{29} & a_{30} & m_i^2 + a_{31} \end{vmatrix},$$

$$D_{1i} = \begin{vmatrix} -a_8 m_i^2 + \delta_2 & a_7 m_i^2 + \delta_4 & a_{11} \\ -a_{14} m_i^2 + \delta_5 & a_{13} m_i^2 + \delta_7 & a_{17} \\ a_{28} m_i^2 + \delta_8 & a_{30} & m_i^2 + a_{31} \end{vmatrix},$$

$$D_{2i} = \begin{vmatrix} -a_8 m_i^2 + \delta_2 & a_6 m_i^2 + \delta_3 & a_{11} \\ -a_{14} m_i^2 + \delta_5 & a_{12} m_i^2 + \delta_6 & a_{17} \\ a_{28} m_i^2 + \delta_8 & a_{29} & m_i^2 + a_{31} \end{vmatrix},$$

$$D_{3i} = \begin{vmatrix} -a_8 m_i^2 + \delta_2 & a_6 m_i^2 + \delta_3 & a_7 m_i^2 + \delta_4 \\ -a_{14} m_i^2 + \delta_5 & a_{12} m_i^2 + \delta_6 & a_{13} m_i^2 + \delta_7 \\ a_{28} m_i^2 + \delta_8 & a_{29} & a_{30} \end{vmatrix}, i = 1,2,3,4.$$

Appendix III

$$Q_1 = P_1 m_1^2 - \xi^2 P_2 + P_3 g_{11} + P_4 g_{21} - g_{31}, Q_2 = P_1 m_2^2 - \xi^2 P_2 + P_3 g_{12} + P_4 g_{22} - g_{32},$$

$$Q_3 = P_1 m_3^2 - \xi^2 P_2 + P_3 g_{13} + P_4 g_{23} - g_{33}, Q_4 = P_1 m_4^2 - \xi^2 P_2 + P_3 g_{14} + P_4 g_{24} - g_{34},$$

$$Q_5 = i\xi\left(P_2 - P_1\right) m_5, R_1 = 2m_1, R_2 = 2m_2, R_3 = 2m_3, R_4 = 2m_4,$$

$$R_5 = P_5\left(m_5^2 - \xi^2\right), U_1 = -\left(P_6 g_{11} + P_7 g_{21}\right) m_1, U_2 = -\left(P_6 g_{12} + P_7 g_{22}\right) m_2,$$

$$U_3 = -\left(P_6 g_{13} + P_7 g_{23}\right) m_3, U_4 = -\left(P_6 g_{14} + P_7 g_{24}\right) m_3, V_1 = -\left(P_7 g_{11} + P_8 g_{21}\right) m_1,$$

$$V_2 = -\left(P_7 g_{12} + P_8 g_{22}\right) m_2, V_3 = -\left(P_7 g_{13} + P_8 g_{23}\right) m_3, V_4 = -\left(P_7 g_{14} + P_8 g_{24}\right) m_4,$$

$$P_1 = \frac{\lambda + 2\mu}{\beta T_0}, P_2 = \frac{\lambda}{\beta T_0}, P_3 = \frac{b\alpha_1}{\beta T_0 \kappa_1 \omega_1^2}, P_4 = \frac{d\alpha_1}{\beta T_0 \kappa_1 \omega_1^2},$$

$$P_5 = \frac{\mu}{\beta T_0}, P_6 = \frac{\alpha_1}{\alpha \kappa_1 \omega_1^2}, P_7 = \frac{b_1 \alpha_1}{\alpha \kappa_1 \omega_1^2}, P_8 = \frac{\gamma \alpha_1}{\alpha \kappa_1 \omega_1^2}.$$