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MICROPOLAR MEDIA WITH STRUCTURAL TRANSFORMATIONS – THEORY ILLUSTATED BY AN EXAMPLE PROBLEM

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Abstract. This paper is concerned with a model for an extended theory of micropolar media. The extension concerns the balance for the tensor field of the moment of inertia, which in contrast to previous theories contains a production term. This term becomes important if the micropolar material undergoes structural changes. In the present case we consider an assemblage of hollow spheres, which due to a transient temperature field contract or expand. This leads to a true field for the tensor of the moment of inertia varying in space and time. For this situation the production term can be calculated numerically. In addition, the temporal and spatial change of the macroscopic inertia field influences rotational motion. Based on a numerical solution for the balance of spin we study the evolvement of angular velocity in space and time. The importance of the presence of a volume couple density is stressed and its physical realization will be discussed.
Keywords: micropolar media, production of microinertia, balances of angular momentum and spin, volume couple.

1. Introduction and outline to the paper

Generalized Continuum Theories (GCTs) have gained the attention of the materials science community for a long time. Their idea is to capture the behavior of high performance materials with an inner structure and internal degrees of freedom. Applications range from the small to the large scale and involve solids as well as liquids. Concrete examples are manifold and can be found in light-weight aerospace and automotive structures, liquid crystal panels, and well as micromechanics and microelectronic gadgets. One of the GCTs is the so-called micropolar theory, which emphasizes the aspect of inner rotational degrees of freedom of a material [1]. This theory seems particularly promising for applications to soils, polycrystalline and composite matter, granular and powder-like materials, and even to porous media and foams.

Continuum mechanics of solids is typically formulated in the Lagrangian way, a.k.a. material description, where the concept of an indestructible "material particle" prevails, identifiable by its reference position. Hence a bijective mapping for describing the particle's path through three-dimensional space in time uniquely can be used. Note that this requires the neighboring material particles to remain "close" to each other during the motion. Furthermore note that a material particle in the continuum sense is composed of myriads of atoms or molecules, so that statistical fluctuations play no role in a macroscopic continuum. Furthermore there is no exchange of the atoms and molecules between material particles: The mass of a material particle is conserved.

Traditionally this concept is also used in micropolar theory [1,2]. One may say that the corresponding material particle consists of a statistically significant number of subunits on a mesoscopic scale, for confusion often also called "particles." Now, if the Lagrangian idea of a material particle is followed, the material particles must stay together during the motion and there should be no exchange of subunits between them. Also note that within the material description of a micropolar continuum, each material point is phenomenologically equivalent to a rigid body, such that its microinertia does not change [2].

However, there is a catch. As an example consider a granular medium which is milled. This effects the material particle, because its subunits will be crushed. They will change their mass and their moment of inertia and, what is more, during the milling process there might even be an exchange of crushed subunits between neighboring material particles, which are then no longer material in the original sense. Consequently, on a macroscopic scale the moments of inertia will change as well. It is for that reason that the authors of [3] have departed from the idea of following the Lagrangian way and turn to the Eulerian perspective (a.k.a. spatial description) instead. Originally the Eulerian description stems from fluid mechanics. It does not impose strict constraints on the motion of mass-conserved material points. Rather it embraces the idea of an open system, allowing a priori for exchange of mass, momentum, energy, moment of inertia, etc., between the cells of the Eulerian grid.

Moreover, the authors in [3] proposed a kinetic equation for microinertia (the field of the local inertia tensor), which in contrast to former theories contains a production term. For a better understanding of this new concept they also present an underlying mesoscopic theory. Their idea is to connect information on a mesoscale by taking the intrinsic microstructure within a spatial grid cell into account with the macroscopic world, i.e., with the balances of micropolar continua in combination with suitable constitutive equations.

These new ideas have been illustrated by several examples in previous papers [3-5], in particular: (a) A homogeneous mix of pressurized hollow spherical particles undergoing a uniform change of external pressure so that their diameter and moment of inertia changes; (b) Particles of type (a) but initially inhomogeneously distributed in an isothermal atmosphere subjected to a barometric pressure distribution falling down and thereby transporting a flux of into new observational points; (c) Changes of anisotropy due to reorientation of initially randomly oriented ellipsoidal particles; (d) Fragmentation of spherical particles in a crusher, analytically as well as numerically. What has been missing so far were examples that show the impact of a changing moment of inertia onto rotational motion.

Therefore, in this paper we will, first, present the foundations of the extended continuum approach to micropolar media and make a few remarks regarding the underlying mesoscopic interpretation. In particular, we will motivate and explain the necessity for a kinetic equation describing the temporal development of the field for the moment of inertia. Second, we will study the change of the state of rotation of a homogeneous mix of pressurized hollow spherical particles undergoing a nonuniform change of external temperature affecting their moment of inertia. Note that within the classical framework of micropolar theory a change of temperature would not influence rotation. However, within the to-be-presented theory changes in temperature will influence the inertia tensor and hence couple to rotational speed.

The paper will conclude with an outlook of how the developed models can be used for complex engineering applications, which will require a fully numerical investigation. In this context the problems studied so far may provide a first orientation.

2. Theoretical background

If we refrain from taking an interaction between linear and angular kinetic energies into account, the objective of micropolar theory is to determine the following thirteen primary fields: (a) the scalar field of mass density, $\rho(\boldsymbol{x},t)$; (b) the vector field of linear velocity, $\boldsymbol{\upsilon}(\boldsymbol{x},t)$; (c) the symmetric, second rank, positive definite specific moment of inertia tensor field, $\boldsymbol{J}(\boldsymbol{x},t)$, in units of m^2; and (d) the spin (a.k.a. angular velocity) field, $\boldsymbol{\omega}(\boldsymbol{x},t)$, in all points, $\boldsymbol{x}$, and at all times, t, within a region of space, $\mathfrak{B}$, which can be either a material volume, *i.e*., it consists of the same matter at all times, or be a region through which matter is flowing.

The determination of these fields relies on field equations for the primary fields. The field equations are based on balance laws and need to be complemented by suitable constitutive relations. In regular points these macroscopic balances read as follows:

- balance of mass:

$$\frac{\delta\rho}{\delta t}+\rho\nabla\cdot\boldsymbol{\upsilon}=0\,, \tag{1}$$

- balance of momentum:

$$\rho\frac{\delta\boldsymbol{\upsilon}}{\delta t}=\nabla\cdot\boldsymbol{\sigma}+\rho\boldsymbol{f}\,, \tag{2}$$

- balance of moment of inertia and coupling moment of inertia tensors:

$$\frac{\delta\boldsymbol{J}}{\delta t}+\boldsymbol{J}\times\boldsymbol{\omega}-\boldsymbol{\omega}\times\boldsymbol{J}=\boldsymbol{\chi}_J\,, \tag{3}$$

- balance of spin:

$$\rho\boldsymbol{J}\cdot\frac{\delta\boldsymbol{\omega}}{\delta t}+\boldsymbol{\omega}\times\boldsymbol{J}\cdot\boldsymbol{\omega}=\nabla\cdot\boldsymbol{\mu}+\boldsymbol{\sigma}_\times+\rho\boldsymbol{m}\,, \tag{4}$$

with

$$\frac{\delta(\cdot)}{\delta t}=\frac{\mathrm{d}(\cdot)}{\mathrm{d}t}+(\boldsymbol{\upsilon}-\boldsymbol{w})\cdot\nabla(\cdot)\,, \tag{5}$$

denoting the substantial (a.k.a. material) derivative of a field quantity, and $\mathrm{d}(\cdot)/\mathrm{d}t$ being the total time derivative including the mapping velocity $\boldsymbol{w}$ of the observational point. Moreover, $\boldsymbol{\sigma}$ is the (non-symmetric) Cauchy stress tensor, $\boldsymbol{f}$ is the specific body force, $\boldsymbol{\chi}_J$ (a second rank symmetric tensor) is the production related to the moment of inertia tensor, $\boldsymbol{J}$; $\boldsymbol{\mu}$ is the couple stress tensor, $\boldsymbol{\sigma}_\times:=\boldsymbol{\varepsilon}\bullet\boldsymbol{\sigma}$ is the Gibbsian cross applied to the (non-symmetric) Cauchy stress tensor (where "$\bullet$" is supposed to denote the outer double scalar product), $\boldsymbol{\varepsilon}$ being the Levi-Civita tensor, and $\boldsymbol{m}$ refers to the specific volume couple density.

Additional information on this extended set of equations can be found in [3-5]. Nevertheless, since Eq. (3) is non-standard several comments are in order. In its present form it was introduced for the first time in [3]. There is a precedent relation to it called "conservation of microinertia" in [2], pg.15. Note that this equation does not contain a production term, $\boldsymbol{\chi}_J$. On the macroscopic continuum level this new term must be interpreted suitably. In [3] it was referred to as a constitutive quantity characteristic of the to-be-processed material. However, a deeper analysis shows that this is not as clear cut as we would wish it to be. Indeed, in [4] evidence was provided that it also takes process characteristics into account. On a more general note it must be asked as to whether Eq. (3) is truly a balance equation, because its counterpart in [2] is a purely kinematic relation. Therefore, one could be tempted to characterize it as a kinetic equation for $\boldsymbol{J}$, which would turn all of Eq. (3) into a constitutive relation. On the other hand, in view of Eq. (1), which balances translational inertia, i.e., mass, it is equally tempting to interpret (3) as a balance of rotational inertia.

One of the purposes of this and other of our papers [4,5] is to present explicit relations for χ_J, always in context with illustrative problems. Typically such relations are based on a mesoscopic model which is then applied on the macroscopic level. We shall illustrate this approach in the next sections. It should also be said that for certain situations it is possible to give additional explanations for the necessity of occurrence of a production of inertia. For example in [5] in was embedded in a mixture theory and related to reaction rates and excess velocities. In [7] statistical mechanics based on the transfer equation procedure introduced by Irving and Kirkwood was used in order to relate it to the effect of the non-material transport of rotational inertia on a microscopic scale through an open system. May it suffice to say that situations where the particle number and the associated moment of inertia (but not the mass) change require us to look at the problem from the Eulerian point-of-view (a.k.a. spatial description) and not from the Lagrangian one (a.k.a. material description). An example (the crusher) is presented in [4]. However, in the case study of this paper, this distinction is not necessary. Here the change of rotational inertia is based on internal shape changes as we shall now proceed to explain.

3. A model problem: Turning heat conduction into space-varying rotational motion

The general problem is as follows. We consider a medium consisting of empty hollow elastic spheres homogenously distributed within a one-dimensional region $x \in [0, L]$. Their initial inner and outer radii are R_i and R_o, respectively. However, the temperature of this medium changes within time from an initially constant value T_ini because reservoirs kept at temperatures T_0 and T_l are attached at positions $x = 0$ and $x = l$ of the region, respectively. The development of temperature, T, is therefore governed by the following initial boundary value problem:

$$\frac{\partial T}{\partial t} = D \frac{\partial^2 T}{\partial x^2}\,,\ D = \frac{\kappa}{\rho_0 c_\upsilon}\,,\ T(x, t = 0) = T_\mathrm{ini}\,,\ T(x = 0, t) = T_0\,,\ T(x = l, t) = T_l\,, \tag{6}$$

where D is thermal diffusivity, κ thermal conductivity, ρ_0 the (constant) mass density of the medium, and c_υ specific heat at constant volume. According to [8], Sect. 3.4 the solution to this problem in dimensionless form is given by:

$$\bar{T}(\bar{x}, \bar{t}) = \bar{T}_0 + (\bar{T}_l - \bar{T}_0)\bar{x} + \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{\bar{T}_l \cos(n\pi) - \bar{T}_0}{n} \sin(n\pi\bar{x}) \exp\left(-n^2\pi^2\bar{t}\right) + \tag{7}$$

$$\frac{2}{\pi}\sum_{n=1}^{\infty} \frac{1 - \cos(n\pi)}{n} \sin(n\pi\bar{x}) \exp\left(-n^2\pi^2\bar{t}\right),$$

with $\bar{x} = x/l$, $\bar{t} = D/l^2\, t$, $\bar{T} = T/T_\mathrm{ini}$. It is assumed that the temperature is instantaneously adopted by the linear elastic spheres. In order to find the current inner and outer radii, r_i and r_o, respectively we have to solve the problem of thermal expansion of a hollow linear elastic sphere when subjected to a uniform temperature change. The solution for a free, fully radially symmetric expansion is:

$$r_\mathrm{i} = \left[1 + \alpha T_\mathrm{ini}\left(\bar{T} - 1\right)\right] R_\mathrm{i}\,,\ r_\mathrm{o} = \left[1 + \alpha T_\mathrm{ini}\left(\bar{T} - 1\right)\right] R_\mathrm{o}\,. \tag{8}$$

Now recall that the current specific moment of inertia tensor of a hollow sphere with corresponding inner and outer radii is isotropic and given by:

$$\boldsymbol{J} = J\,\mathbf{I}\,,\ J = \frac{2}{5}\frac{r_\mathrm{o}^5 - r_\mathrm{i}^5}{r_\mathrm{o}^3 - r_\mathrm{i}^3}\,. \tag{9}$$

Hence:

$$J = J_0\left[1+\alpha T_{\text{ini}}\left(\bar{T}-1\right)\right]^2 \;,\; J_0 := \frac{2}{5}R_{\text{o}}^2\frac{1-\beta^5}{1-\beta^3} \;,\; \beta := \frac{R_{\text{i}}}{R_{\text{o}}} \;. \tag{10}$$

We now argue that after homogenization the macroscopic field for the tensor of inertia stays isotropic, $\boldsymbol{J}\left(\boldsymbol{x},t\right) = J\left(\boldsymbol{x},t\right)\mathbf{I}$ (hence the second term on the left of Eq. (4) vanishes), and is given by the last relation where the normalized temperature $\bar{T}$ is replaced by the expression shown in Eq. (7). Turning to Eq. (3) we are now in a position to specify the production for the moment of inertia, if we assume that the translational velocity vanishes, $\boldsymbol{\upsilon} = \mathbf{0}$ (see Eq. (5)):

$$\chi_J = \chi_J\mathbf{I} \,,\; \chi_J \equiv \frac{\partial J}{\partial t} = \chi_0\left[1+\alpha T_{\text{ini}}\left(\bar{T}\left(\bar{x},\bar{t}\right)-1\right)\right]\frac{\partial\bar{T}\left(\bar{x},\bar{t}\right)}{\partial\bar{t}} \,,\; \chi_0 := 2\alpha T_{\text{ini}}J_0\frac{D}{l^2} \,. \tag{11}$$

How does this affect the development of translational and angular velocities? For an answer we turn to the macroscopic balances of linear and angular momentum shown in Eqs. (2/4). There are no body forces (we ignore gravity), $\boldsymbol{f} = \mathbf{0}$, the stress tensor is zero (we consider the medium to be “dust”), $\boldsymbol{\sigma} = \mathbf{0}$, the translational velocity is initially zero. Then Eq. (2) is telling us that it stays zero, $\boldsymbol{\upsilon} = \mathbf{0}$. Regarding the balance of angular momentum (4) we assume that the couple stress tensor vanishes, $\boldsymbol{\mu} = \mathbf{0}$, and initially start with a constant volume couple density different from zero, $\boldsymbol{m} = m_0\boldsymbol{e}_z$, $m_0 = \text{const.}$, $\boldsymbol{e}_z$ being the unit normal in z-direction. Hence $\boldsymbol{\omega} = \omega(x,t)\boldsymbol{e}_z$ and

$$\frac{\partial\omega}{\partial t} = \frac{m_0}{J(x,t)} \,. \tag{12}$$

This differential equation can be solved numerically so that we obtain an angular velocity field $\omega = \omega(x,t)$ different from zero decreasing or increasing in space and time. We conclude that if the temperature changes in space and time due to the presence of external heat reservoirs, the moment of inertia will change accordingly, and we may harvest “good” macroscopic rotational energy by using this “heat engine,” provided there is an agent of transfer in terms of a non-vanishing (constant) specific volume couple density m_0. Note that even if the temperature remained totally constant we would obtain a homogeneously distributed angular velocity increasing linearly in time as follows,

$$\omega\left(t\right) = \frac{m_0}{J_0}t \,, \tag{13}$$

if we assume it initially to be zero.

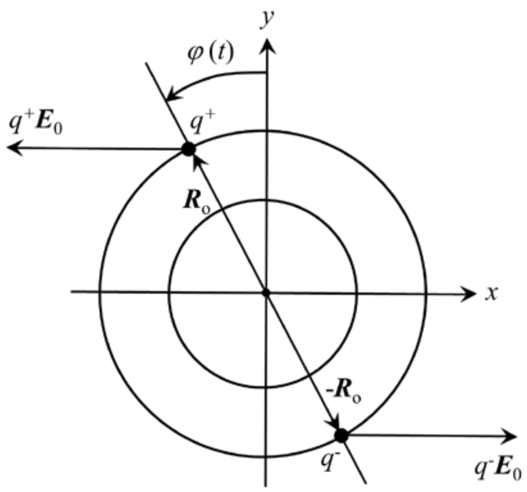

Fig. 1. Model of an electrically polarized sphere.

But how do we realize specific volume couples, at least theoretically? Unfortunately the pertinent literature on micropolar solids is rather taciturn regarding this issue. In [9] we read "Ordinarily, we expect gravitational or electromagnetic fields to produce the forces [***f*** and ***m***] ... The question of how best to describe the effects of electromagnetic fields is not so easily settled. It is of some importance since they are observed to influence the orientation of liquid crystals." And in [10] we hear: "But the modern notion of volume couple, a continuum density of torque, is more elusive having, it is true, no direct physically visible realization ... But what about a density of magnetic couple, a thing that also played an important role in the construction of wide classes of GCM [Generalized Continuum Mechanics] with nonsymmetric Cauchy stress tensor? ..." Indeed, the theory of nematic crystals, which are perceived as electric dipoles, provides a clue. Imagine, we manage to polarize the spheres electrically as indicated in Fig. 1. The net charge, $q^{+} + q^{-}$, would be zero. Now we apply a constant external electric field, $\boldsymbol{E}_0 = E_0 \boldsymbol{e}_x$, in negative *x*-direction. The total Coulomb force, and therefore (after homogenization) the body force (in *x*-direction), would vanish. However, the moment couple acting on the sphere would not. Rather it points in *z*-direction and is given by:

$$\boldsymbol{M} = q^{+} \boldsymbol{R}_{\mathrm{o}} \times \boldsymbol{E}_0 - q^{-} \boldsymbol{R}_{\mathrm{o}} \times \boldsymbol{E}_0 = 2qR_{\mathrm{o}} E_0 \cos\varphi(t)\, \boldsymbol{e}_z \,, \tag{14}$$

q being the magnitude of the dipole charge. Similar reflections can be found in [11]. Hence in this model the volume moment couple density is time-dependent as follows:

$$\boldsymbol{m} = m_0 \cos \int_{\bar{t}=0}^{\bar{t}=t} \omega(\bar{t})\, \mathrm{d}\bar{t}\; \boldsymbol{e}_z \,,\; m_0 = 2\frac{q}{m_{\mathrm{p}}} R_{\mathrm{o}} E_0 \tag{15}$$

provided $\varphi(0) = 0$. m_{p} is the mass of one particle. In this case the balance of angular momentum (12) would change to:

$$J(x,t)\frac{\partial \omega(x,t)}{\partial t} = m_0 \cos \int_{t'=0}^{t'=t} \omega(x,t')\, \mathrm{d}t' \,. \tag{16}$$

This can be rewritten as

$$\frac{\partial}{\partial t}\left[\frac{J}{m_0}\frac{\partial \omega}{\partial t}\right] = -\omega \sqrt{1 - \left(\frac{J}{m_0}\frac{\partial \omega}{\partial t}\right)^2} \,, \tag{17}$$

a differential equation leading to oscillating motion as to-be-expected.

In summary we may say that the harvesting of rotational energy becomes possible because of the presence of a specific volume couple density, i.e., because of an electric field, and the amount of harvesting in a point of space depends on the development of the temperature field therein.

4. Results and discussion

We choose the following normalized temperature values for our numerical simulations of the developing temperature profile: $\bar{T}_0 = 0.5$, $\bar{T}_l = 2.0$. Fig. 2 shows the development of temperature at three dimensionless times, $\bar{t} = 0.001$ (blue), $\bar{t} = 0.01$ (red), and $\bar{t} = 1.0$ (green), according to Eq. (7) where infinity was replaced by $n_{\max} = 100$. The transition to the stationary linear temperature profile becomes quite obvious. These profiles are then used in Eq. (10) in order to calculate the temporal development of the moment of inertia, $\bar{J}$, for $\alpha T_{\mathrm{ini}} = 0.5$. For this large value we get a certain departure from linearity, even at $\bar{t} = 1.0$: Fig. 3. The two plots are very similar. As to-be-expected the moment of inertia decreases if temperature goes down and vice versa.

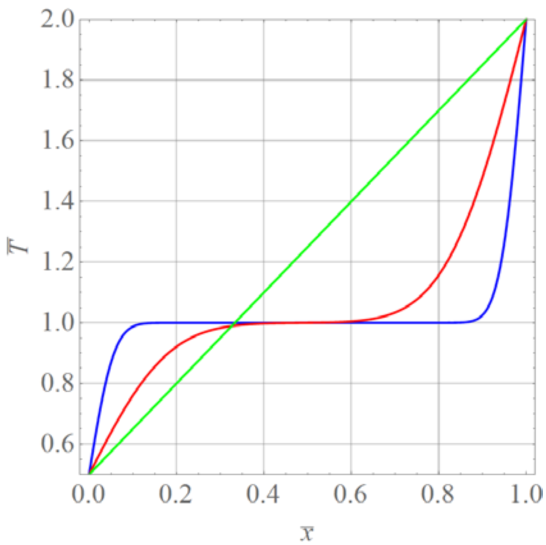

Fig. 2. Temperature profiles.

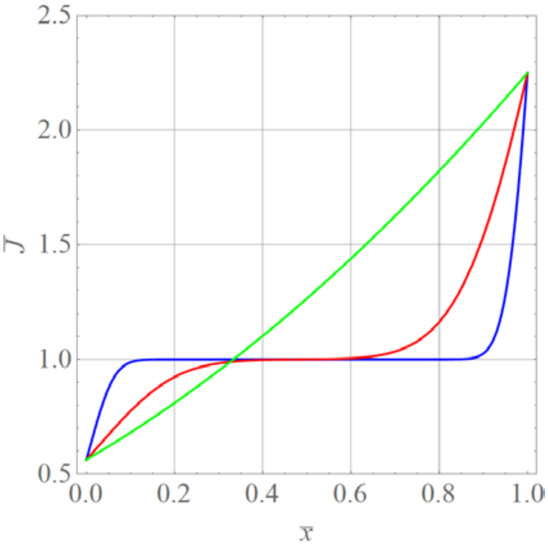

Fig. 3. Development of local moment of inertia.

The reason for the changing moment of inertia is presented in Fig. 4. It shows the temporal development of the normalized productions of moment of inertia, $\bar{\chi} = \chi / \chi_0$, Four positions are examined at $\bar{x} = 0.1$ (blue), $\bar{x} = 0.5$ (red), $\bar{x} = 0.9$ (green), and $\bar{x} = 1/3$ (magenta). The latter is the position for which $\bar{T} = 1$ after an infinite time (see Eq. (7)). As demonstrated in Fig. 3 the spheres shrink for small values of $\bar{x}$ (and expand for large ones), so that the moment of inertia decreases (and increases) accordingly, first fast and, as time goes on, slower and slower. The production behaves accordingly. The effect is less pronounced around the position $\bar{x} = 1/3$ where the normalized temperature is close to one and only little change of moment of inertia occurs. For all cases the effect vanishes completely for $\bar{t} \to \infty$ when the size of the spheres hardly changes any more.

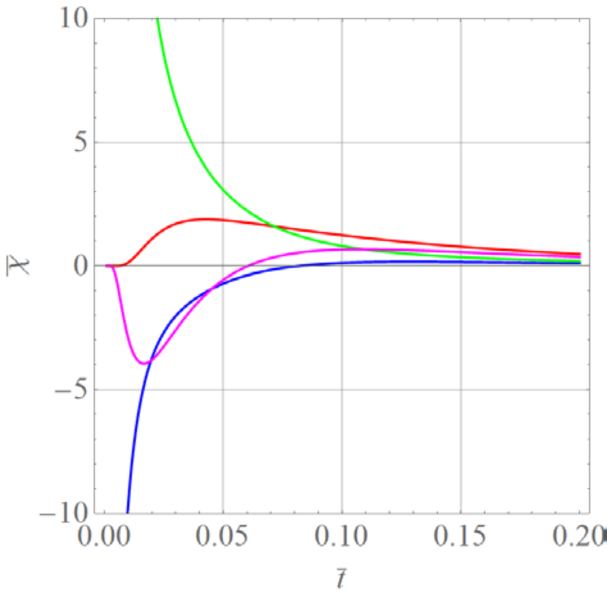

Fig. 4. Productions of moment of inertia at various positions (see text) over time.

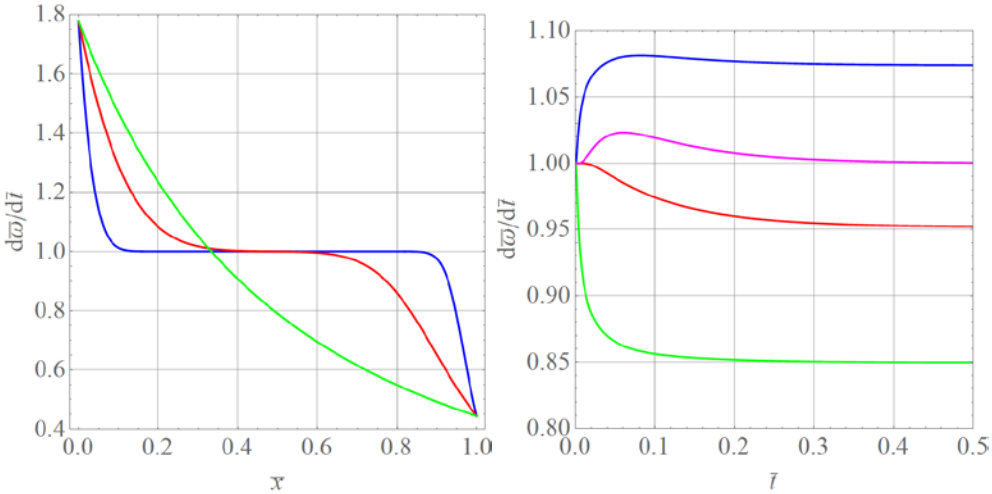

Fig. 5. Angular accelerations at various times over position and vice versa.

Now we turn to the balance of angular velocity for the case of a constant volume couple density, m_0, as shown in Eq. (12). In normalized form it reads:

$$\frac{\partial \bar{\varpi}}{\partial \bar{t}} = \frac{1}{\bar{J}(x,t)} , \; \bar{\varpi} = \frac{\omega}{\omega_0} , \; \omega_0 = \frac{m_0}{J_0} \frac{l^2}{D} , \; \bar{J} = \frac{J}{J_0} = \left[1 + \alpha T_{\text{ini}} \left(\bar{T} - 1 \right) \right]^2 . \tag{18}$$

The angular accelerations are shown in Fig. 5. On the left they are presented in normalized form as a function of position at times $\bar{t} = 0.001$ (blue), $\bar{t} = 0.01$ (red), and $\bar{t} = 1.0$ (green). On the right they are shown as a function of time for various positions at $\bar{x} = 0.1$ (blue), $\bar{x} = 0.5$ (red), $\bar{x} = 0.9$ (green), and $\bar{x} = 1/3$ (magenta). Several features are remarkable: (i) After a certain while the accelerations assume constant values no matter which position is studied. (ii) Positions left of $\bar{x} = 1/3$ show normalized accelerations greater than one, because the temperatures and, correspondingly, the moments of inertia go down and vice versa. (iii) For positions left of 0.5 the accelerations reach a maximum. A numerical integration of Eq. (18) yields the angular velocities as functions of time shown in Fig. 6 for a temporally constant volume couple density. Again four positions are examined at $\bar{x} = 0.1$ (blue), $\bar{x} = 0.5$ (red), $\bar{x} = 0.9$ (green), and $\bar{x} = 1/3$ (magenta). It rotates slightly faster than the angular velocity obtained for the case of a constant moment of inertia, which according to Eq. (13) is $\bar{\varpi}(\bar{t}) = \bar{t}$. This case is shown as a black dashed line. As it should be material points on the left of $\bar{x} = 1/3$ rotate faster and those which are on the right rotate slower, because they gain a smaller or larger moment of inertia, respectively.

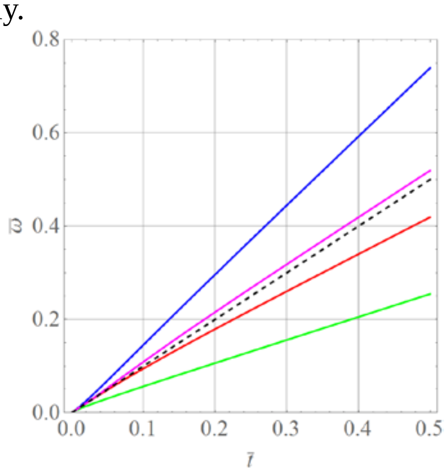

Fig. 6. Angular velocities at various positions.

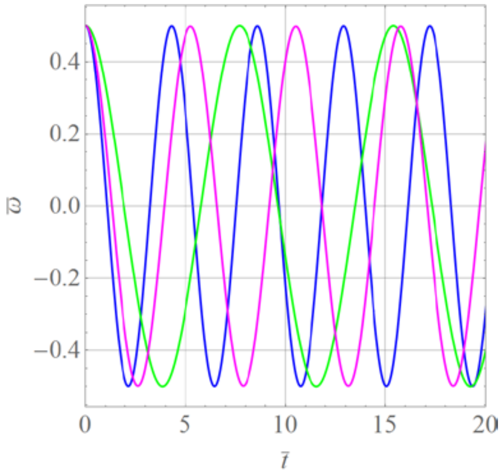

Fig. 7. Periodic movement of dipoles under a constant electric field at different locations.

Fig. 7 shows numerical solutions of the differential equation (17) in dimensionless form:

$$\frac{\partial}{\partial \bar{t}}\left[\bar{J}\,\frac{\partial \bar{\varpi}}{\partial \bar{t}}\right] = -\gamma\,\bar{\varpi}\sqrt{1-\left(\bar{J}\,\frac{\partial \bar{\varpi}}{\partial \bar{t}}\right)^2}\,,\quad \gamma = \frac{m_0}{J_0}\left(\frac{l^2}{D}\right)^2, \tag{19}$$

with a periodically volume couple density at positions $\bar{x} = 0.1$ (blue), $\bar{x} = 0.9$ (green), and $\bar{x} = 1/3$ (magenta). All curves were calculated with the same initial conditions, $\bar{\varpi}(\bar{x},0) = 0.4$, $\partial\bar{\varpi}(\bar{x},0)/\partial\bar{t} = 0.0$ and for $\gamma = 1.5$. The periodic motion is clearly visible. The durations of the period are influenced by location.

5. Conclusions and outlook

In this paper we first repeated the extended balance equations for a micropolar medium allowing for structural transformations. As a recently introduced feature these included a balance for the moment of inertia tensor with a production density. The case of hollow spheres was considered, which under the influence of a one dimensional temperature field would change their moment of inertia in space and time due to thermal expansion and contraction. This model of structural change allowed to calculate the production of the moment of inertia in the extended set of micropolar balance equations. Results for the moment of inertia varying in space and time were used to study the evolvement of an angular velocity field. This necessitated a specific volume couple density to be present. An attempt was made to interpret this quantity based on a mesoscopic model of electrically polarized particles under the influence of an external electric field inducing a moment couple because of the Coulomb force.

In the future the authors will continue to explore the fully coupled set of equations for a micropolar medium. Whilst even the simplified one-dimensional examples presented so far required a numerical approach it is to be expected that the numerical effort for that will increase considerably.

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