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EXPERIMENTAL VERIFICATION OF POSTULATE OF ISOTROPY AND MATHEMATICAL MODELING OF ELASTOPLASTIC DEFORMATION PROCESSES FOLLOWING THE COMPLEX ANGLED NONANALYTIC TRAJECTORIES

V.G. Zubchaninov[1], A.A. Alekseev[1*], E.G. Alekseeva[2], V.I. Gultiaev[1]

[1]Tver State Technical University, nab. Afanasiya Nikitina, 22, Tver, 170026, Russia

[2]Bauman Moscow State Technical University, ul. Baumanskaya 2-ya, 5/1, Moscow, 105005, Russia

*e-mail: alexeew@bk.ru

Abstract. The results of numerical simulation for 45 steel deformation with the use of general and linearized models of the theory of processes along flat smooth trajectories with curvilinear sectors are presented. The results of calculation are compared with the data of thin-walled cylindrical specimen deformation experiment, carried out on SN-EVM testing complex. It is shown that for the realized types of experimental trajectories symmetric with respect to the bisector of a right angle Ilyushin's postulate of isotropy is fulfilled in the proper way.
Keywords: elastoplastic deformation; postulate of isotropy; numerical simulation.

1. Introduction

Systematic tests on metals and alloys mechanical behavior behind elastic limit at the deflected mode were conducted to verify key provisions and a reasonable creation of elastoplastic deformation mathematical models of materials [1-10]. A. A. Ilyushin's postulate of isotropy [11-12] is one of the most important laws of the theory of elastoplastic processes, it was checked by many researchers for different materials on different trajectories [8-11]. In particular, in [10] trajectories, considered to be smooth on A. A. Ilyushin terminology [11], are symmetric relatively bisectors of the right angle. On the first sites of flat trajectories [10] deformation on the quarter of circle was realized, and then the trajectory without break, but with curvature change, passed into straight section. In the work verification of reliability of A. A. Ilyushin's postulate of isotropy and results of mathematical modeling is considered at complex deformation on the flat nonanalytic trajectories having the curvilinear site and point of break.

2. Main equations

In the linear-combined space E_6 of tension and deformations with orthonormalized basis $\left\{ \hat{i}_k \right\}$ stress tensors σ_{ij} and deformations ε_{ij}

$$\sigma_{ij} = \sigma_0 \delta_{ij} + S_{ij}, \quad \sigma_0 = \sigma_{ij}\delta_{ij}/3, \quad \varepsilon_{ij} = \varepsilon_0 \delta_{ij} + Э_{ij}, \quad \varepsilon_0 = \varepsilon_{ij}\delta_{ij}/3 \tag{1}$$

vectors are put in compliance

$$\bar{S} = S_0 \hat{i}_0 + \bar{\sigma}, \quad \bar{\sigma} = S_k \hat{i}_k, \quad \bar{\varepsilon} = Э_0 \hat{i}_0 + \bar{Э}, \quad \bar{Э} = Э_k \hat{i}_k, \quad (k = 1, 2, \ldots 5), \tag{2}$$

where coordinates of vectors S_k, $Э_k$ are connected with components of tensors σ_{ij}, ε_{ij} and deviators S_{ij}, $Э_{ij}$ tension, and deformations, by the known biunique transformations [1, 2]. Volumetric strain in E_6 is supposed elastic according to the law of elastic change of the volume $\sigma_0 = 3K\varepsilon_0$, where K is the modulus of volume elasticity.

According to A. A. Ilyushin's postulate of isotropy, vectors of tension $\bar{\sigma}$ and deformations $\bar{Э}$ of forming are connected by the defining ratios [1, 2]

$$\frac{d\bar{\sigma}}{ds} = M_1 \frac{d\bar{Э}}{ds} + \left(\frac{d\sigma}{ds} - M_1 \cos\vartheta_1\right)\frac{\bar{\sigma}}{\sigma}, \quad \frac{d\vartheta_1}{ds} + \kappa_1 = -\frac{M_1}{\sigma}\sin\vartheta_1, \tag{3}$$

where, M_1, $\frac{d\sigma}{ds}$ is the functionalities of the process of deformation depending on parameters of complex loading: s is lengths of the arc of the trajectory of deformation, its curvature κ_1, and corners of break ϑ_1^0. The approach angle ϑ_1 characterizes the direction of the vector $\bar{\sigma}$ in relation to the tangent to deformation trajectory in each its point and reflects the influence on the process of deformation of vector material properties.

General mathematical model of the theory of processes. For the case of flat trajectories ($\vartheta_2 = 0$, $\kappa_2 = 0$) the defining ratios can be given to the system of the equations of the task of Cauchy in the scalar form [1]:

$$\begin{cases} \dfrac{dS_k}{ds} = M_1 \dfrac{dЭ_k}{ds} + M\dfrac{S_k}{\sigma}, \quad (k = 1, 3), \\ \dfrac{d\vartheta_1}{ds} + \kappa_1 = -\dfrac{M_1}{\sigma}\sin\vartheta_1, \end{cases} \tag{4}$$

for which solution in work the numerical method of Runge-Kutta of the fourth order of accuracy in the realized application for the system of computer mathematics MATLAB was used.

At the numerical solution of system (4), V. G. Zubchaninov's universal approximations of functionalities were used [1, 2]

$$\begin{gathered} M_1 = 2G_p + \left(2G - 2G_p^0\right) f^q e^{-\gamma_1 \Delta s}, \quad M = \frac{d\sigma}{ds} - M_1 \cos\vartheta_1, \\ \sigma = \Phi(s) + A f_0^p \Omega(\Delta s), \quad \frac{d\sigma}{ds} = \frac{d\Phi}{ds} + A f_0^p \frac{d\Omega}{ds}, \end{gathered} \tag{5}$$

where G, G_p is elastic and plastic modules of shearing; G_p^0 is value of G_p in trajectory break-point; $\Delta s = s - s_к^т$ is increment of length of the arc of trajectory after its break in some point K;

$$\Omega(\Delta s) = -\left[\gamma \Delta s\, e^{-\gamma \Delta s} + b\left(1 - e^{-\gamma \Delta s}\right)\right] \tag{6}$$

– function which after break of trajectory describes scalar dive of tension at difficult unloading and the subsequent secondary plastic deformation;

$$f = \frac{1 - \cos\vartheta_1}{2}; \quad f_0 = f(\vartheta_1^0) = \frac{1 - \cos\vartheta_1^0}{2} \tag{7}$$

– the function considering the orientation of vector of tension in the course of deformation and its value in break-point at the value of approach angle for nonanalytic trajectory.

For approximation of Odqvist-Ilyushin's general function of hardening $\Phi(s)$ at simple loading expressions were used

$$\sigma = \Phi(s) = \begin{cases} \dfrac{2G}{\alpha}\left(1 - e^{-\alpha s}\right), \text{ при } 0 \le s \le s_*^{\text{т}}, \\ \sigma^{\text{т}} + 2G_*(s - s_*^{\text{т}}) + \sigma_*\left(1 - e^{-\beta(s - s_*^{\text{т}})}\right), \text{ при } s > s_*^{\text{т}}; \end{cases} \quad (8)$$

where $\sigma^{\text{т}} = \sqrt{2/3}\sigma_{\text{т}}$; $\sigma_{\text{т}}$ is limit of stretching strain; $s_*^{\text{т}}$ is the border of sites of the chart of deformation dividing elastic part of the chart and site of flowability ($0 \le s \le s_*^{\text{т}}$) from the site of self-hardening of material ($s > s_*^{\text{т}}$); σ_*, G_*, α, β, A, b, γ, γ_1, p, q are experimentally determined parameters of the structural material.

Linearized model. In the simplified linearized option [1, 13-14] of mathematical model for flat trajectories in the assumption of trifle of size ϑ_1 ($\sin\vartheta_1 \approx \vartheta_1$, $\cos\vartheta_1 \approx 1$) follows the differential equation

$$\frac{d\vartheta_1}{ds} + \kappa_1 = -n\vartheta_1, \quad (9)$$

in which, for active processes at $0 < \vartheta_1 < 90^\circ$ it is possible to accept

$$n(s) = \frac{M_1}{\sigma} \approx \frac{\alpha_1 2G}{\sigma_{\text{к}}^{\text{т}}} = k, \quad (10)$$

where $k = \text{const}$; α_1 is constant coefficient ($0 < \alpha < 1$); $\sigma_{\text{к}}^{\text{т}}$ is the value of the module $\overline{\sigma}$ in deformation trajectory break point. The solution of the equation (9) leads to expression

$$\vartheta_1 = \vartheta_1^* + \left(\vartheta_1^0 - \vartheta_1^*\right)e^{-k\Delta s}, \quad (11)$$

where $\vartheta_1^* = -\kappa_1 / k$. For piecewise and broken trajectories if to accept more difficult approximation

$$n(s) = \frac{1}{s} + k, \quad (12)$$

that from (9) can be received [15]

$$\vartheta_1 = \frac{s_0}{s}e^{-k\Delta s}\left\{\vartheta_1^0 - \vartheta_1^*\left(1 - \frac{1}{ks_0}\right)\right\} + \vartheta_1^*\left(1 - \frac{1}{ks}\right), \quad (13)$$

where s_0 is length of the arc in trajectory break point. In particular, for two-unit broken lines at $\vartheta_1^* = 0$ from (13) follows

$$\vartheta_1 = \frac{s_0}{s}\vartheta_1^0 e^{-k\Delta s}. \quad (14)$$

3. Results of tests and mathematical modeling

As samples for tests on the SN-EVM rated and experimental complex of A. A. Ilyushin thin-walled barrel shells from steel 45 having in working part have been used: $l = 110$ mm length, the $h = 1$ mm thickness, and diameter of the median surface of $d = 31$ mm. The initial isotropy of material of samples with sufficient precision ratio has been confirmed in experiences on simple loading (stretching, compression, and torsion) and when processing these charts, the following values of material parameters for steel 45 have been accepted in approximations (8): $\sigma^{\text{т}} = 310$ MPa, $s_*^{\text{т}} = 1{,}1 \cdot 10^{-2}$, $2G = 1{,}57 \cdot 10^5$ MPa, $\beta = 70$, $\alpha = 900$, $\sigma_* = 82$ MPa, $2G_* = 2700$ MPa.

Two broken nonanalytic trajectories with preliminary elastoplastic deformation on the site of constant curvature which are mirroring of each other rather direct, slanting to coordinate axes $Э_1$ and $Э_3$ (fig. 1) are realized to verify A. A. Ilyushin's postulate of isotropy and assessment of reliability of results on the mathematical models described above in space of deformations $Э_1 - Э_3$.

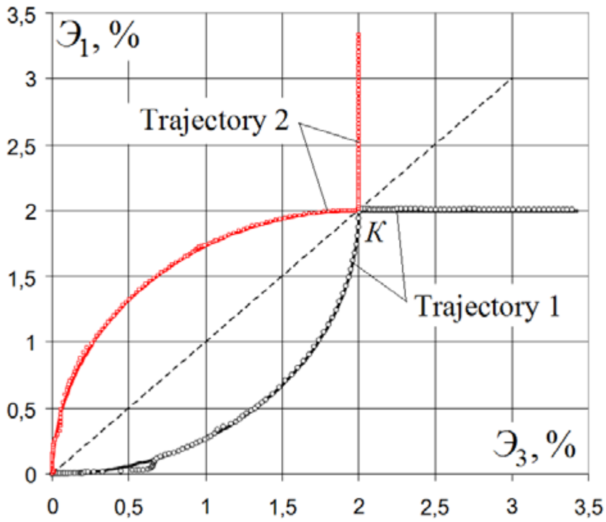

Fig. 1. Deformation trajectories.

The first of two trajectories of deformation (trajectory 1, in fig. 1 is designated in black color) on the first site represents a quarter of a circle radius $\rho = 2\%$ with a center of curvature $Э_1^0 = 2\%$, $Э_3^0 = 0$, on which joint stretching with torsion of a sample to a point *K*. Then, with a break on the corner 90° the trajectory passes to the second straight section where torsion on coordinate $Э_3$ was implemented. In specularly reflected trajectory (trajectory 2 in fig. 1) on the first site to the point *K* joint stretching and torsion on a quarter of a circle were also carried out at $\rho = 2\%$, $Э_1^0 = 0$, $Э_3^0 = 2\%$, and on the second site - only stretching on $Э_1$. In the point *K* joint of the first and second sites of trajectories in addition to the existence of salient points their curvature changes, means the first and second derivatives of the functions $\bar{Э} = \bar{Э}(s)$, describing trajectories in a vector space, undergo a gap.

Results of numerical calculations and experimental data for a trajectory 1 (see fig. 1) are given in fig. 2-7, and results and the experimental data for reflected trajectory 2 are given in fig. 8-13. Experimental data in drawings are noted by points: for a trajectory 1 – circles of black color; for a trajectory 2 – small squares of red color. Curves 1 (blue color) are results of calculations for the general mathematical model of the theory of processes, curves 2 (black color) – of the linearized model. Results of calculations and experiments rated are given in representation of deformations and tension according to A. A. Ilyushin's postulate through coordinates of the corresponding vectors of forming [1-2, 11-12].

In calculations, identical values of material parameters in approximations of functionalities of plasticity for both trajectories were taken. On the first site was taken: $q = 0.1$, $\gamma_1 = 25$; on the second: $q = 0.3$, $\gamma_1 = \gamma = 50$, $p = 4$, $b = 0.35$. For the initial value of ϑ_1^0 on the second site after a salient point of a trajectory was taken $\vartheta_1^0 = 90^\circ - \vartheta_1^к \approx 72^\circ$, where $\vartheta_1^к$ – the value of an approach angle on the first site in a trajectory breaking point K in the calculation of the general model ($\vartheta_1^{\kappa} \approx 18^{\circ}$). On the linearized model in calculations was taken $\alpha_1 = 0.7$, $k = 258$; on the first site the corner ϑ_1 was defined on (11) where $\vartheta_1^0 = 0$, ϑ_1^* at $\kappa_1 = 1/\rho = 50$ was taken $\vartheta_1^* \approx 0{,}193\,\text{rad}$. On the second site of a trajectory, the approach angle ϑ_1 was defined on (14) at $\vartheta_1^0 = 90^{\circ} - \vartheta_1^{\kappa} \approx 79^{\circ}$, where $\vartheta_1^{\kappa} \approx 11^{\circ}$ is a calculated value of a corner on the linearized model in a point the end of the first site of a trajectory of deformation.

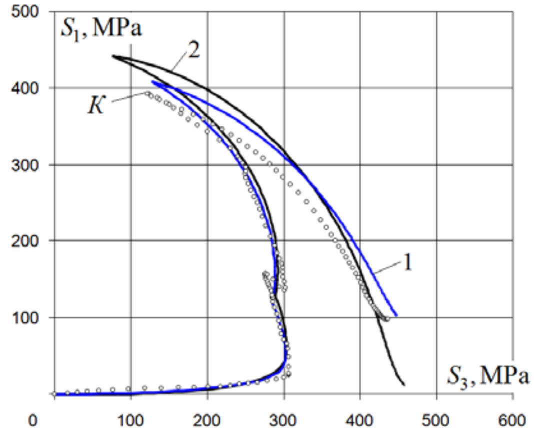

Fig. 2. Trajectory 1. Response $S_1 - S_3$.

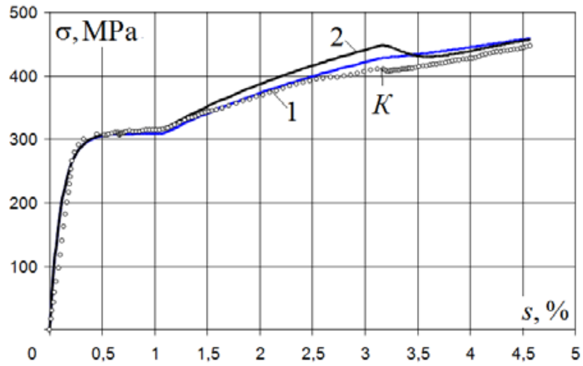

Fig. 3. Trajectory 1. Chart of deformation $\sigma - s$.

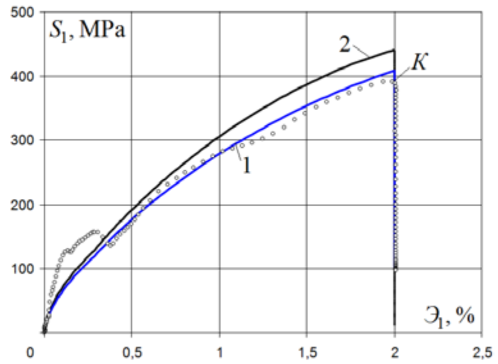

Fig. 4. Trajectory 1. Chart $S_1 - Э_1$.

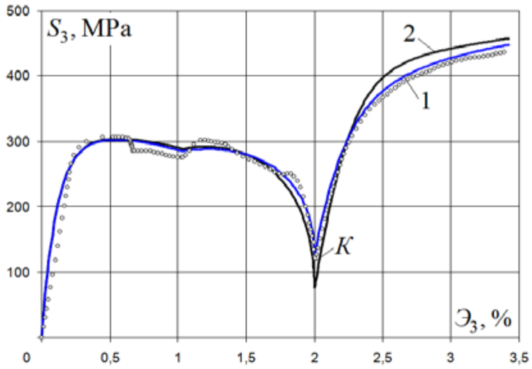

Fig. 5. Trajectory 1. Chart $S_3 - Э_3$.

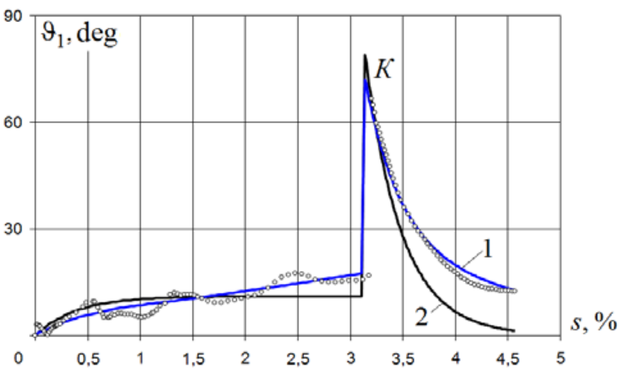

Fig. 6. Trajectory 1. Chart $\vartheta_1 - s$.

The results received on the general mathematical model for charts $\sigma - s$ (curves 1 in fig. 3, 8) and $\vartheta_1 - s$ (curves 1 in fig. 6, 11), the scalar and vector material properties reflecting respectively are well coordinated with data of tests. Good compliance of rated results on the general model with the experimental data is also observed on a response in space of tension $S_1 - S_3$ (fig. 2, 7) and to local charts of deformation at stretching and compression $S_1 - Э_1$ (fig. 4, 9) and torsion $S_3 - Э_3$ (fig. 5, 10). On the basis of it is possible to claim that the V. G. Zubchaninov's general mathematical model of yields adequate results at the description of patterns of elastoplastic behavior of material for the considered types of difficult nonanalytic trajectories with a breaking point.

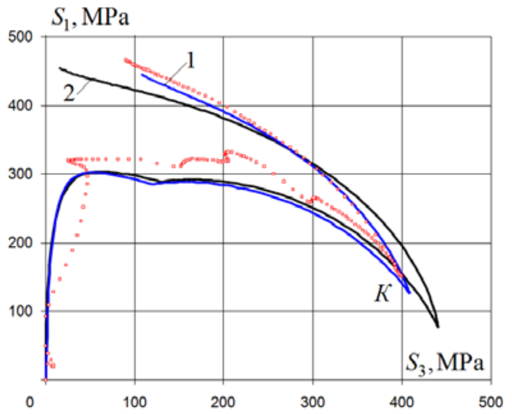

Fig. 7. Trajectory 2. Response $S_1 - S_3$.

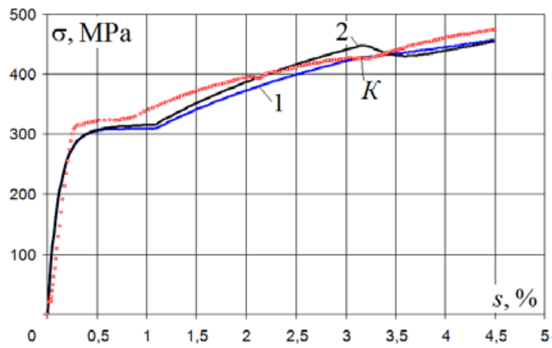

Fig. 8. Trajectory 2. Chart of deformation $\sigma - s$.

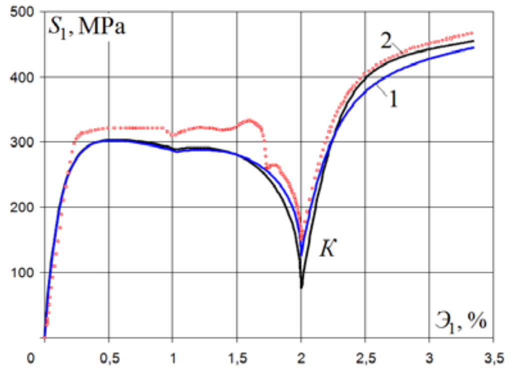

Fig. 9. Trajectory 2. Chart $S_1 - Э_1$.

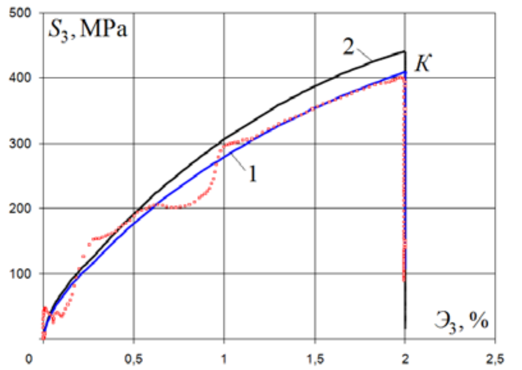

Fig. 10. Trajectory 2. Chart $S_3 - Э_3$.

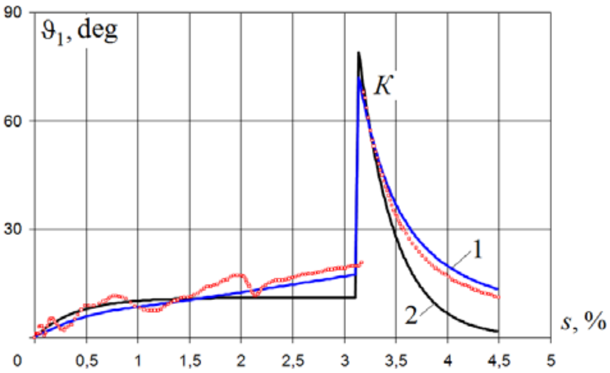

Fig. 11. Trajectory 2. Chart $\vartheta_1 - s$.

Results on the linearized model give essential deviations on vector properties (curves 2 in fig. 6, 11). On scalar properties due to the fact that on the considered trajectories active process of deformation ($0 < \vartheta_1 < 90^\circ$) is carried out, and the parameter of complexity of the process $\vartheta_1^0 < 90^\circ$, the linearized model shows acceptable results.

To verify the postulate of isotropy for trajectories 1 and 2 on fig. 12-13 the reflecting scalar and vector material properties, and also results of mathematical modeling with use of the general model (curves 1 in fig. 12-13) are presented combination of experimental results for charts $\sigma - s$ and $\vartheta_1 - s$.

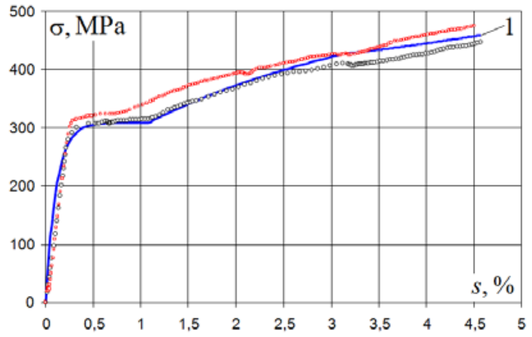

Fig. 12. Trajectories 1 and 2. Charts of deformation $\sigma - s$.

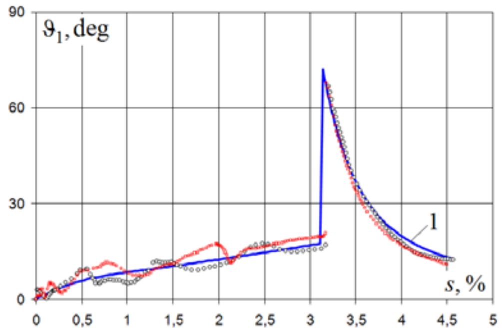

Fig. 13. Trajectories 1 and 2. Charts $\vartheta_1 - s$.

The comparison shows that experimental charts of specularly reflected trajectories of deformation matched among themselves a sufficient precision ratio. Therefore, it is possible to believe that for these types of difficult trajectories with a break and the preliminary elastoplastic deformation preceding it on the site with constant curvature, the postulate of isotropy is carried out on scalar and vector properties.

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