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INFLUENCE OF PATCH PARAMETERS ON DAMAGE AND RESIDUAL STRENGTH OF ADHESIVELY BONDED COMPOSITE REPAIR UNDER FATIGUE LOADING

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Abstract. A progressive fatigue damage analysis method was developed to predict damage propagation and residual strength of the adhesively bonded repairs in composites subjected to constant amplitude cyclic loading. The fatigue damage models of adhesively bonded repairs are respectively established with different shapes of the patches. Based on the theory of the residual strength and residual stiffness in composites, the material property degradation technique for modifying element stiffness and strength and materials ultimate failure criteria were proposed. This program is able to simulate progressive fatigue damage and predict residual strength of the adhesively bonded repairs in composites subjected to constant amplitude cyclic loading with different shapes of the patches. The calculation results of damage model show that the patch shape and size are important fatigue design parameters of adhesively bonded repairs. The effect of repair with tilted square patch is the best than others and the residual strength improves by 27.6 % than the unrepaired composite laminates.

1. Introduction

Compared with the traditional mechanical repair technology, the adhesively bonded repairs in composites have the obvious advantages of less increment of structure weight, strong design ability, easy forming and low cost. Therefore, the technology of adhesively bonded repairs in composites has been successfully applied to some damaged structures of aircraft. Since aircraft structures are often subjected to fatigue loading condition, analysis of the fatigue behavior for composite patches repair is thus essential to evaluate the repair performances. Several works have been dedicated to study the fatigue life of repairs involving composites. The majority of the studies are dedicated to repairs of metallic structures with composite patches [1-2]. The majority of the studies involving composite are focused on fatigue analysis of bonded joints [3-5]. Only few works have been dedicated to fatigue analysis of the adhesively bonded repairs. Shokrieh et al. [6] built a progressive fatigue damage model which is used to simulate the process of damaged composite laminates under fatigue loading at first. The composite fatigue life prediction system is made up of the stress analysis, failure analysis and material degradation. Butkus et al. [7] studied the effect of environmental conditions (temperature and humidity) the

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fatigue life of the adhesively bonded repairs. Tserpes et al. [8] predicted the residual strength and fatigue life of the composite laminates with initial damage assessments by the progressive fatigue damage model. Botelho et al. [9] evaluated the effects of the fatigue behavior on repaired aramid fiber/epoxy composites and verified a decrease of 10 % in the low cycle fatigue resistance values and 18 % for high- cycle fatigue relatively to the non-repaired structure. They concluded that the repair procedure can be used in aerospace applications. Moreira et al. [10] researched the high-cycle fatigue behavior under three-point bending of single-strap repairs of carbon–epoxy composite laminates by experimental and numerical analyses.

In the present work, the residual strength model and Hashin failure criterion were used to fatigue damage analysis in composite laminates. The third strength theory was used to predict the failure of the adhesive layer. The fatigue damage models of adhesively bonded repairs are respectively established with different shapes of the patches. This program is able to simulate progressive fatigue damage and predict residual strength of the adhesively bonded repairs in composites subjected to constant amplitude cyclic loading with different shapes of the patches.

2. Model analysis

The models of adhesively bonded repairs are respectively established with different shapes of the patches by using the software of ANSYS. The laminated plates of dimensions (230*40*3.0 mm) with the lamina orientations: [(90/±45/0)3]s were prepared with 24 plies as shown in Fig. 1. The radius of the damage hole (r) was 3.175 mm. The radius of the patch with the lamina orientations: (0/0) was represented by R, which provided a thickness of 0.25 mm.

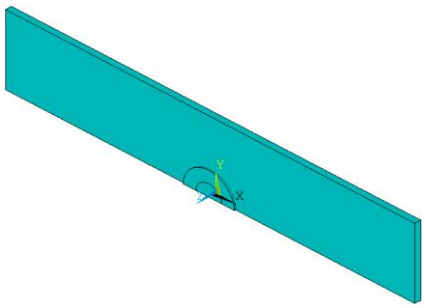


Fig. 1. The model of adhesively bonded repair.

Table 1. Mechanical properties of T300/1034-C.

properties	means	properties	means
E_{11}/GPa	146.90	X_t/MPa	1730.60
E_{22}/GPa	11.38	X_c/MPa	1379.0
E_{33}/GPa	12.38	Y_t/MPa	66.54
G_{12}/GPa	6.185	Y_c/MPa	268.2
G_{13}/GPa	6.185	Z_t/MPa	52.0
G_{23}/GPa	5.780	Z_c/MPa	280.0
v_{12}	0.30	S_{12}/MPa	133.8
v_{13}	0.30	S_{13}/MPa	133.8
v_{23}	0.42	S_{23}/Mpa	100

The laminates and patches were manufactured by T300/1034-C. The mechanical properties of the materials are illustrated in Table 1 [11]. The mechanical properties of the adhesive (*Eadh*=1 GPa, v=0.3 and S=85 MPa) were defined in Table 1.

2.1. Finite element model of laminates. The laminates are made up of many single plates, whose thickness is much smaller than other dimensions. In this paper, the finite element model is established in APDL language. The element of SOLID46 is used to simulate the laminates and patches. This paper develops fatigue failure criterion for the above five kinds of failure modes. Comparing the fatigue failure criterion and the Hashin failure criterion, the main difference is the value of strength. In the Hashin failure criterion, the material properties are constants. But in the fatigue failure criterion, the material properties change with fatigue load parameters, such as the number of fatigue cycles (n), the stress ratio (k), and the fatigue stress (σ). The following criteria are used.

Matrix crushing fatigue failure mode ($\sigma_{yy} \geq 0$)

$$\left(\frac{\sigma_{yy}}{Y_T(n,\sigma,k)}\right)^2 + \left(\frac{\sigma_{xy}}{S_{xy}(n,\sigma,k)}\right)^2 + \left(\frac{\sigma_{yz}}{S_{yz}(n,\sigma,k)}\right)^2 \geq 1. \quad (1)$$

Matrix compression fatigue failure mode ($\sigma_{yy} < 0$)

$$\left(\frac{\sigma_{yy}}{Y_C(n,\sigma,k)}\right)^2 + \left(\frac{\sigma_{xy}}{S_{xy}(n,\sigma,k)}\right)^2 + \left(\frac{\sigma_{yz}}{S_{yz}(n,\sigma,k)}\right)^2 \geq 1. \quad (2)$$

Fiber tension fatigue failure mode ($\sigma_{xx} \geq 0$)

$$\left(\frac{\sigma_{xx}}{X_T(n,\sigma,k)}\right)^2 + \left(\frac{\sigma_{xy}}{S_{xy}(n,\sigma,k)}\right)^2 + \left(\frac{\sigma_{xz}}{S_{xz}(n,\sigma,k)}\right)^2 \geq 1. \quad (3)$$

Fiber compression fatigue failure mode ($\sigma_{xx} < 0$)

$$\left(\frac{\sigma_{xx}}{X_C(n,\sigma,k)}\right)^2 \geq 1. \quad (4)$$

Matrix-fiber shearing fatigue failure mode ($\sigma_{xx} < 0$)

$$\left(\frac{\sigma_{xx}}{X_C(n,\sigma,k)}\right)^2 + \left(\frac{\sigma_{xy}}{S_{xy}(n,\sigma,k)}\right)^2 + \left(\frac{\sigma_{xz}}{S_{xz}(n,\sigma,k)}\right)^2 \leq 1. \quad (5)$$

Normal tension fatigue failure mode ($\sigma_{zz} \geq 0$)

$$\left(\frac{\sigma_{zz}}{Z_T(n,\sigma,k)}\right)^2 + \left(\frac{\sigma_{xz}}{S_{xz}(n,\sigma,k)}\right)^2 + \left(\frac{\sigma_{yz}}{S_{yz}(n,\sigma,k)}\right)^2 \geq 1. \quad (6)$$

Normal compression fatigue failure mode ($\sigma_{zz} < 0$)

$$\left(\frac{\sigma_{zz}}{Z_C(n,\sigma,k)}\right)^2 + \left(\frac{\sigma_{xz}}{S_{xz}(n,\sigma,k)}\right)^2 + \left(\frac{\sigma_{yz}}{S_{yz}(n,\sigma,k)}\right)^2 \geq 1, \quad (7)$$

where $X_i(n,\sigma,k)$, $Y_i(n,\sigma,k)$ and $Z_i(n,\sigma,k)$ are the longitudinal, the transverse and the normal residual fatigue strength of unidirectional plies under uniaxial fatigue loading conditions, respectively. The subscripts "T" and "C" represent tension and compression. $S_{xy}(n,\sigma,k)$ is the in-plane shear residual fatigue strength of a unidirectional ply under uniaxial shearing fatigue loading conditions. $S_{xz}(n,\sigma,k)$ and $S_{yz}(n,\sigma,k)$ are the out-plane shear residual fatigue strengths of unidirectional plies under uniaxial shearing fatigue loading conditions.

Based on the mentioned criteria above, the stiffness degradation criteria is determined. The sudden material property degradation rules for the fatigue failure modes of a unidirectional ply under a multiaxial state of fatigue stress are developed as shown in Table 2.

Table 2. The material property degradation rules.

Failure mode	degradation rules
Matrix cracking	$E_{xx}^{D}=0.2E_{xx}$ $G_{xy}^{D}=0.2G_{xy}$ $G_{yz}^{D}=0.2G_{yz}$
Matrix compression	$E_{yy}^{D}=0.4E_{yy}$ $G_{xy}^{D}=0.4G_{xy}$ $G_{yz}^{D}=0.4G_{yz}$
Matrix-fiber shearing	$G_{xy}^{D}=0$ $v_{xy}^{D}=0$
Delamination	$E_{zz}^{D}=0$ $G_{yz}^{D}=0$ $G_{xz}^{D}=0$ $v_{yz}^{D}=0$ $v_{xz}^{D}=0$
Fiber fracture	$E_{xx}^{D}=0.07E_{xx}$ $E_{yy}^{D}=0.07E_{yy}$ $G_{xy}^{D}=0.07G_{xy}$ $G_{yz}^{D}=0.07G_{yz}$ $G_{xz}^{D}=0.07G_{xz}$ $v_{xy}^{D}=0.07v_{xy}$ $v_{yz}^{D}=0.07v_{yz}$ $v_{xz}^{D}=0.07v_{xz}$

Note: E_{ij} and G_{ij} are the Elastic Modulus and the Shear modulus of unidirectional plies under uniaxial fatigue loading conditions; E_{ij}^{D} and G_{ij}^{D} are the Elastic Modulus and the Shear modulus of unidirectional plies under uniaxial fatigue loading conditions after the failure of material.

Base on the principle of equivalent damage in Ref. [12], this paper proposes that the strength and stiffness reduction induced by impact damage can be equal to the strength and stiffness reduction induced by fatigue loading. So the rules are also suitable for impact-damage zone. The rules are as follows:

$$R(n,\sigma,k)=\left[1-\left(\frac{\log n-\log 0.25}{\log N_f-\log 0.25}\right)^{\beta}\right]^{\frac{1}{\alpha}}(R_0-\sigma)+\sigma, \tag{8}$$

$$E(n,\sigma,k)=\left[1-\left(\frac{\log n-\log 0.25}{\log N_f-\log 0.25}\right)^{\lambda}\right]^{\frac{1}{\gamma}}\left(E_0-\frac{\sigma}{\varepsilon_f}\right)+\frac{\sigma}{\varepsilon_f}, \tag{9}$$

where R_0 and E_0 are the initial strength and initial stiffness of the laminate; α, β, λ and γ are four curve fitting coefficients. ε_f is the strain of failure which can be obtained by the test; N_f is the fatigue life that can be got by an empirical fatigue life model [13].

$$\frac{\ln(a/f)}{\ln[(1-m)(c+m)]}=A+B\log N_f, \tag{10}$$

where $a=\frac{\sigma_a}{\sigma_t}$, $m=\frac{\sigma_m}{\sigma_t}$, $c=\frac{\sigma_c}{\sigma_t}$; σ_t and σ_c are tensile and compressive strength; σ_a is the fatigue load amplitude; σ_m is the average stress of fatigue load; f, A and B are the curve fitting coefficients with nothing to do with the stress state, which are obtained by the uniaxial test.

2.2. Adhesive model. The element of SOLID45 is used to simulate the adhesive layer which is isotropic. For the element of the adhesive, the most important failure mode is the deboned damage. The maximum shear stress strength criterion is used to predict the failure of the adhesive element as follows:

$$\tau_{max} = \frac{(\sigma_1 - \sigma_3)}{2} \geq S, \tag{11}$$

where σ_1 and σ_3 are maximum principal stress and minimum principal stress of the adhesive element.

If the adhesive layer has the failure element, the stiffness degradation criteria has to be used in Eq. (12)

$$E_{adh}^{D} = \lambda E_{adh}, \tag{12}$$

where E_{adh}^{D} is the elastic modulus of the failure of adhesive. E_{adh} is the initial elastic modulus of the adhesive. λ is the adhesive degradation coefficient. When it is the first time to begin the shear failure in the adhesive, λ is defined as 0.1. When the same element of the adhesive begins the second shear failure, λ is defined as 0.001. When the number of the shear failure element in the adhesive layer accounts for 3/4 of the total number of the element in the adhesive layer, the ultimate failure in the adhesive layer occurs.

2.3. Validation model. By using the method and the program, the static strength and the residual strength under fatigue loading conditions for T300/BMP-316 laminates are studied. The stacking sequence is [45/-45/0/0/-45/90/0/90/45/0] s. The prediction results are compared with the experimental results in Ref. [3]. The results are as shown in Table 3.

Table 3. Comparison of the residual strength and error of impacted laminated composites.

Cycles	The diameter of hole, mm	The average residual strength of test [14], MPa	Predicted result in Ref. [15], MPa	Predicted result in this paper, MPa	Error compared with test, %	Error compared with existing literature, %
10^4	9	442.05	458.63	460	4.06	0.3
10^5	9	459.33	482.72	486	5.8	0.6
10^6	9	477.28	511.19	520	8.9	1.7

The comparison of the predicted results and the test results [3] is listed in Table 3. The most relative error between the predicted results and the test results is 8.9 %.

2.4. The flow chart of the fatigue damage analysis. The fatigue damage models of adhesively bonded repairs are respectively established to predict the ultimate failure of adhesively bonded repairs. On the ANSYS plat form, a progressive damage of laminated structure simulating program is developed in APDL language. Fig. 2 demonstrates its flow chart.

3. Influence of patch shape on damage and residual strength of adhesively bonded composite repair under fatigue loading

The reasonable patch shape can make adhesively bonded repairs better bearing capacity under external force. The round, square, hexagon and lozenge four patch shapes are selected to research the influence of patch shape on fatigue damage of adhesively bonded composite repair. Patch shapes are shown in Fig. 3.

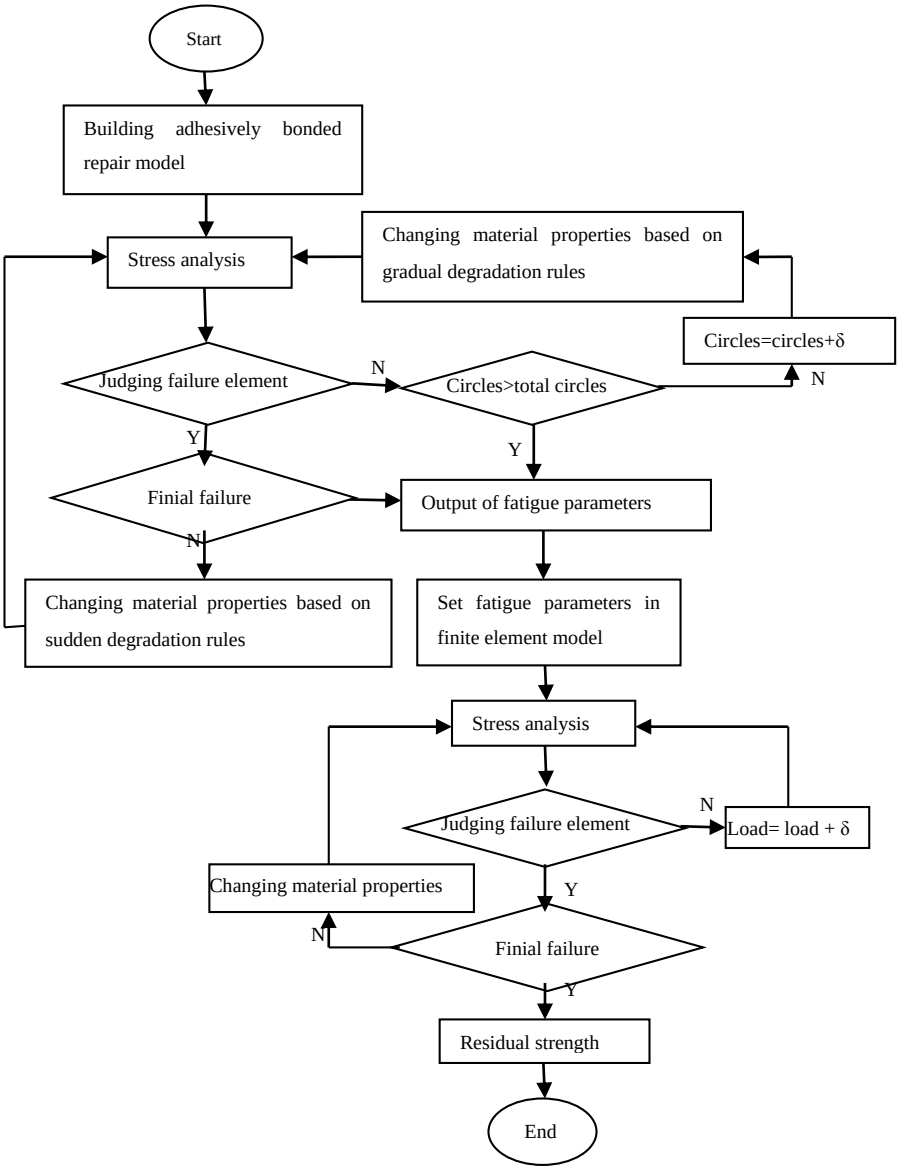


Fig. 2. The flow chart of the fatigue damage analysis.

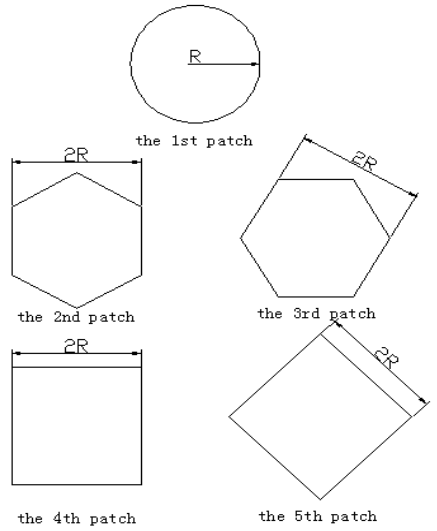


Fig. 3. Five different patch shapes.

3.1. Influence of patch shape on damage of adhesively bonded composite repair under fatigue loading. The fatigue cumulative damage of each layer in the adhesively bonded repairs with the different patch shapes is calculated under the 85 % stress level after the fatigue load ($n=10^4$). It is defined as the ultimate failure that fiber fracture propagates to plate edge in 0° ply. It only lists the change of fatigue damage with the increase of fatigue circles in all 0° plies.

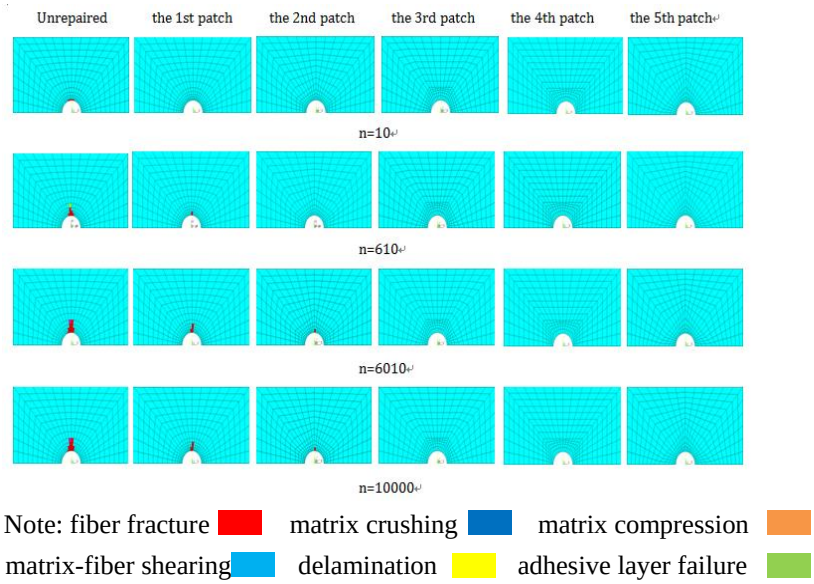


Fig. 4. Fatigue damage process of the repaired laminates with the different patch shapes in 0° ply.

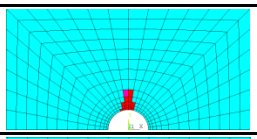
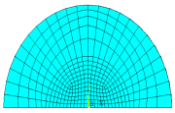
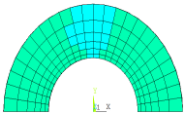
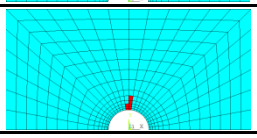
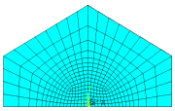
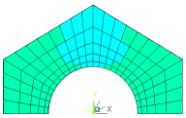
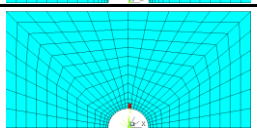
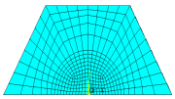
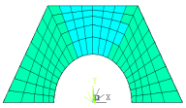
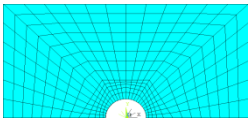
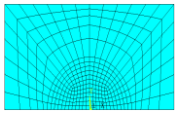
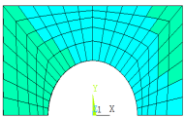
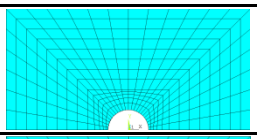
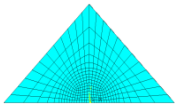
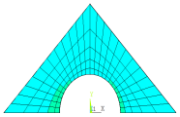
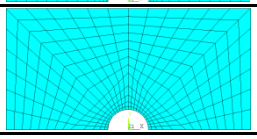
Figure 4 shows that the fatigue damage of the adhesively bonded composite repair in 0° ply changes with the increase in the number of cycles. Fiber tension is the mainly damage in all 0° plies. As applied 10 cycles, Small amount of fiber tension occurs in repaired laminates, while the fatigue damage doesn't occur in other specimens. As applied 610 cycles, fiber tension begins to occur in the 1st specimen. As applied 6010 cycles, fiber tension begins to occur in the 2nd specimen, and the fatigue damage extends along the width direction of the laminates in the damage specimens. As applied 10000 cycles, the damage form of the specimens is similar with n=6010 and the damage doesn't expand significantly. Fatigue damage extends along the width direction of the laminates in the damage specimens with the increase of fatigue circles.

As can be seen from Table 4, the adhesively bonded repairs can improve the fatigue bearing capacity of the laminates. The different shape of patch has different effect on the repaired laminates. As applied 10000 cycles, fiber fracture mainly occurs in 0° ply. Compared with the unrepaired laminates, it's better for the repaired laminates in fatigue damage with the increase of the number of cycles. Fiber fracture does occur in 0° plies of the 3rd specimen, the 4th specimen and the 5th specimen. As for the adhesive layer, all specimens occur in different degrees of fatigue damage, where the adhesive layer of the 5th specimen has minimal damage. Fatigue damage doesn't occur in all patch layers.

3.2. Influence of patch shape on residual strength of adhesively bonded composite repair. Based on the fatigue damage, the influence of different patch shape on the process of progressive damage of adhesively bonded composite repair is simulated after the number of fatigue cycles ($n=10^4$). Before the analysis of residual strength, the material parameters of

fatigue damage state, which are obtained by the prediction of fatigue damage, will be regard as the initial state of the finite element model of residual strength.

Table 4. Fatigue damage in different plies of the laminates with different patch shape under fatigue load (n=10000).

Patch shape	Patch layer	Adhesive layer	0° ply
Unrepaired	nothing	nothing	
The 1st patch			
The 2nd patch			
The 3rd patch			
The 4th patch			
The 5th patch			

Note: fiber fracture matrix crushing matrix compression
matrix-fiber shearing delamination adhesive layer failure

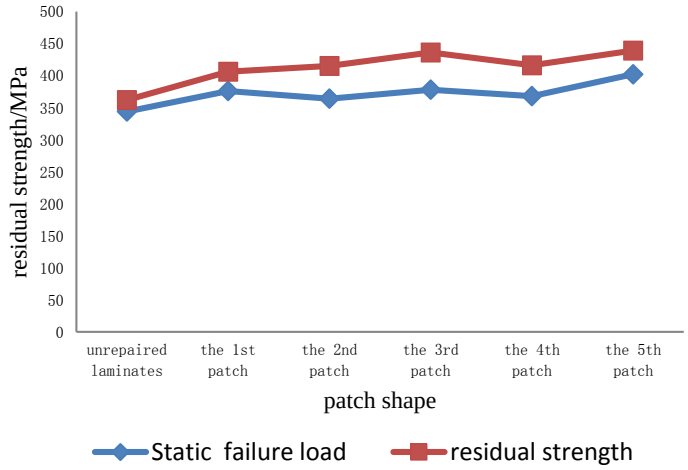


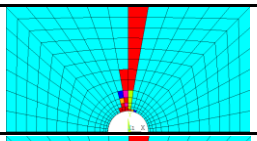
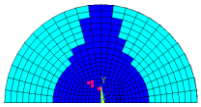
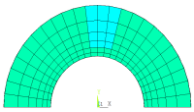
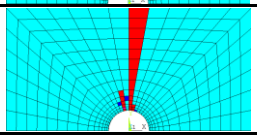
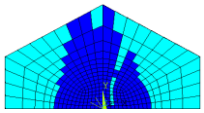
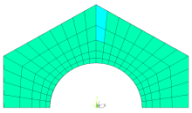
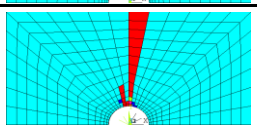
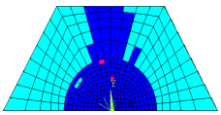
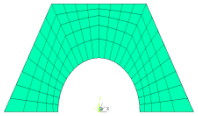
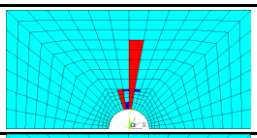
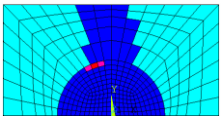
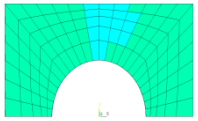
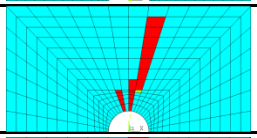
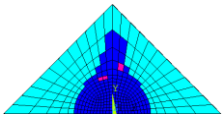
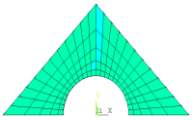
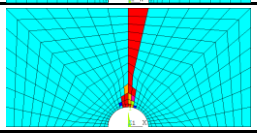
Fig. 5. Residual strength of repaired laminates with different patch after the number of fatigue circles (n=10000).

Figure 5 presents that the adhesively bonded repairs can make the laminates better bearing capacity under static load. The 5th specimen is better than other specimens which is consistent

with the existing literature [16]. The adhesively bonded repairs can improve residual strength of the laminates under fatigue loading. The different shape of patch has different effect on the residual strength. It's the most significant of the 3rd specimen to improve the residual strength for adhesively bonded repairs under fatigue loading. The 5th specimen has the best effect on residual strength for the laminates. Matrix cracking in partial layers of repaired laminates results in the decrease of hole-edge stress concentration factor, which makes the laminates better bearing capacity under static load and the increase of residual strength.

After tensile fatigue load, the stress concentration appears on the edge of the hole in laminates under tensile loading. So the failure elements of intercepting the hole are compared. When the ultimate failure occurs in adhesively bonded composite repair, fiber fracture propagates to plate edge in all 0° plies. The elements of patch are given priority to matrix cracking. So the failure of patch layer and 0° ply are listed in Table 5.

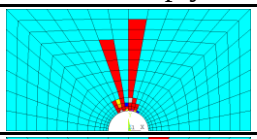
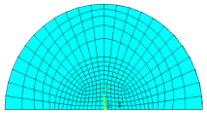
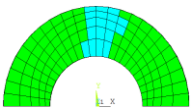
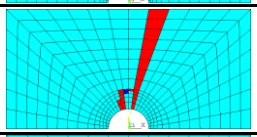
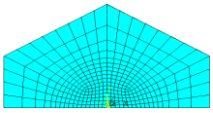
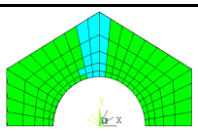
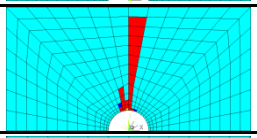
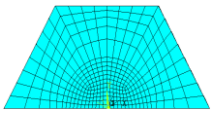
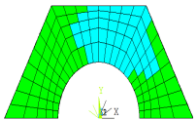
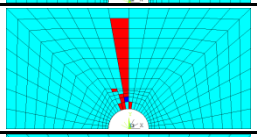
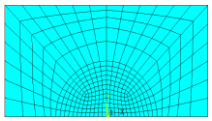
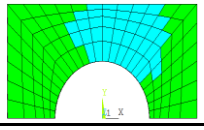
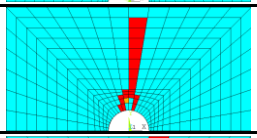
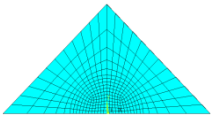
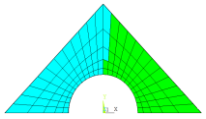
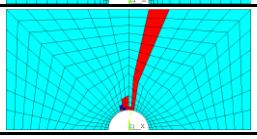
Table 5. The ultimate failure of residual strength in different plies of the laminates with different patch shape.

Patch shape	Patch layer	Adhesive layer	0° ply
Unrepaired	nothing	nothing	
The 1st patch			
The 2nd patch			
The 3rd patch			
The 4th patch			
The 5th patch			

Note: fiber fracture matrix crushing matrix compression
matrix-fiber shearing delamination adhesive layer failure

Table 5 shows that fiber fracture mainly occurs in 0° ply on the basis of fatigue damage, and along with a small amount of matrix cracking and delamination. When the ultimate failure occurs in laminates, the adhesive layer of the 3rd specimen is entirely ineffective, fiber fracture doesn't propagate to plate edge in 0° ply. Fiber fracture propagates to plate edge in 0° ply of other specimens, and the adhesive layers are going to reach ultimate failure. As for the layer of patch, the 2nd specimen only appears matrix cracking, and matrix cracking and fiber fracture occur in other specimens.

Table 6. The ultimate failure of static strength in different plies of the laminates with different patch shape.

Patch shape	Patch layer	Adhesive layer	0° ply
Unrepaired	nothing	nothing	
The 1st patch			
The 2nd patch			
The 3rd patch			
The 4th patch			
The 5th patch			

Note: fiber fracture matrix crushing matrix compression
matrix-fiber shearing delamination adhesive layer failure

As shown in Tables 5 and 6, the residual strength of adhesively bonded composite repair is significantly higher than the static strength after fatigue load (n=10000) when it reaches ultimate failure. Compared with static load, the failure elements of adhesive layer increase significantly. The adhesive layer of the 3rd specimen is entirely ineffective, and the adhesive layers of other specimens will reach ultimate failure. The elements of patch appear matrix crushing and fiber fracture. The failure of adhesive layer and patch becomes the mainly reason of the ultimate failure of adhesively bonded composite repair.

4. Influence of patch size on residual strength of adhesively bonded composite repair

In the process of bonding repair, patch size directly affect bond area between the patch and the laminates, which has important effect on fatigue repair efficiency. In order to explore the relationship between patch size and residual strength under fatigue load, the radius R of circular patch (1.5 r, 2 r, 2.25 r, 2.5 r, 2.75 r, and 3 r) is selected to analyze and calculate.

Figure 6 shows that residual strength of laminates with different shapes changes with the increase of the radius R of circular patch (1.5 r < R < 3 r). When R is equal to 1.5 r, the adhesive layer of the specimen is entirely ineffective and residual strength of laminates increases by 7.2 %. When R is equal to 2 r, residual strength of laminates increases by 8 %. When R is equal to 2.25 r, residual strength of laminates increases by 9.9 %.When R is equal to 2.5 r, residual strength of laminates increases by 8.7 %. When R is equal to 2.75 r and 3 r, residual strength decreases as compared with static strength. As the radius R (1.5 r < R < 2.25 r) of patch increases, degree in promotion of residual strength gradually increases. While as the radius R (2.25 r < R

< 2.5 r) of patch increases, degree in promotion of residual strength gradually decreases. When R > 2.5 r, residual strength doesn't increase and residual strength decreases as compared with static strength. So we can see that the radius R = 2.25 r of patch is better than others for residual strength.

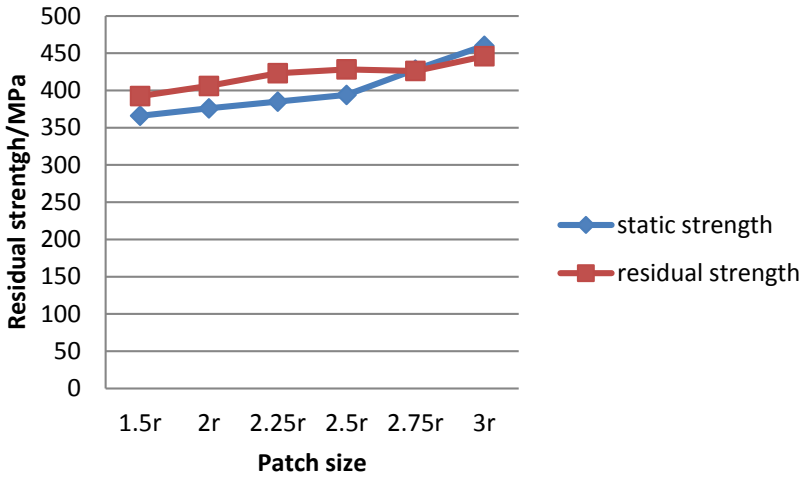


Fig. 6. Influence of patch size on residual strength under fatigue load (n=10000).

5. Conclusion

(1) Three-dimensional progressive fatigue damage model of adhesively bonded composite repair is established, and the accuracy of the prediction in residual stiffness is validated by the experimental data under the fatigue loading.

(2) Fatigue damage of repaired laminates is better than the unrepaired laminates under the fatigue cyclic load. The 3rd specimen and the 5th specimen have better repair efficiently. The matrix cracking damage of some partial plies in repaired laminates results in the decrease of hole-edge stress concentration factor. Therefore, the capacity of static load and residual strength of the laminates increases.

(3) Flat hexagonal patch (the 3rd specimen) has the best effect on the increase of residual stiffness in repaired laminates with the same repaired radius. And it increases by 15.3 % compared with their own static strength. The effect of repair with tilted square patch is the best than others and the residual strength improves by 27.6 % than the unrepaired composite laminates.

(4) In the case of the same patch shape, residual strength changes with the increase of radius under fatigue load. The radius R = 2.25 r of patch is better than others for residual strength for circular patch.

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