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MODELING OF ELASTOPLASTIC STEEL DEFORMATION IN TWO-LINK BROKEN TRAJECTORIES AND DELAYING OF VECTOR AND SCALAR MATERIAL PROPERTIES

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Abstract. On the basis of a mathematical model of A. A. Il'yushin's theory of elastoplastic processes, numeric calculations are given for programs of experiments in the form of two-link broken strain trajectory. The results of the numeric simulation are compared with the experimental data obtained by authors on the automated test-machine SN-EVM. Delaying of vector and scalar properties is investigated. The instability of magnitude of the trace of delay as a characteristic of the material is shown.

Keywords: plasticity; complex loading; two-link trajectories; loading process; vector and scalar properties; trace of delay

1. Introduction

The theory of constitutive relations is one of the important directions in the mechanics of deformable solids. It was formed due to the experimental and theoretical development of the theory of plasticity. The solution of elastoplastic boundary value problems, which are based on physically nonlinear constitutive relations, designed to reliably describe the properties of materials in a wide variety of operating conditions, is not easy for theoretical analysis. For verification of the physical integrity of the constitutive relations of the theory of plasticity, it is necessary to carry out calculations and compare the obtained numerical results with experimental data on a wide class the trajectories of deformation. The main mathematical models of the theory of plasticity and the results of numerous experiments with complex loading of materials are partially presented in [1-11] and others.

In this paper, to verify the mathematical model of elastoplastic processes theory, calculations of the deformation processes of steel 45 are carried out on two-link broken trajectories (tension with torsion; P–M tests). The results of numeric simulation are compared with the experimental data obtained by the authors on the automated test-machine SN-EVM. Previously, variants of the mathematical model showed good agreement with experimental data on two-link strain trajectories containing curved sections, both in the presence and absence of break points [12-14].

Experimental test procedure is based on the theory of elastoplastic processes by A.A. Il'yushin [1,2], in which stress and strain variation history with the course of time is represented by the corresponding trajectories in the five-dimensional vector spaces. In this case, the bonding between stress and strains is described as scalar properties characterizing the bonding between invariants of stress and strain deviators, and vector properties characterizing the disalignment of stress and strains deviators, and their increments.

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Il'yushin further proposed the principle of de1ay [2], which states that the stress response to a strain trajectory is not defined by the whole strain trajectory, but by the recent past known as the trace of delay of the vector properties of the material λ. Studies of the value λ under complex loading are devoted to the work [3,15-18] in which the instability λ is noted. This paper also shows the instability of the trace of delay of the scalar properties of the material λ_{sc}.

2. Main equations

In a linear combined space E_6 of stresses and strains with an orthonormal basis $\left\{\hat{i}_k\right\}$ for stress σ_{ij} and strain ε_{ij} tensors

$$\sigma_{ij} = \sigma_0\delta_{ij} + S_{ij}, \quad \sigma_0 = \sigma_{ij}\delta_{ij}/3, \quad \varepsilon_{ij} = \varepsilon_0\delta_{ij} + Э_{ij}, \quad \varepsilon_0 = \varepsilon_{ij}\delta_{ij}/3 \tag{1}$$

vectors are assigned

$$\overline{S} = S_0\hat{i}_0 + \overline{\sigma}, \quad \overline{\sigma} = S_k\hat{i}_k, \quad \overline{\varepsilon} = Э_0\hat{i}_0 + \overline{Э}, \quad \overline{Э} = Э_k\hat{i}_k, \quad (k = 1, 2, ...5), \tag{2}$$

where vectors coordinates S_k, $Э_k$ related to tensor σ_{ij}, ε_{ij} and deviator components S_{ij}, $Э_{ij}$ noted by unequivocal transformation [1,2]. The volume strain in E_6 is assumed to be elastic according to the law $\sigma_0 = 3K\varepsilon_0$, where K is modulus of volume elasticity (Bulk modulus). According A.A. Il'yushin's isotropy postulate [1], stress $\overline{\sigma}$ and strain $\overline{Э}$ vectors are bound by constitutive relations, which for a planar rectilinear trajectories are:

$$\frac{d\overline{\sigma}}{ds} = M_1\frac{d\overline{Э}}{ds} + \left(\frac{d\sigma}{ds} - M_1\cos\vartheta_1\right)\frac{\overline{\sigma}}{\sigma}, \qquad \frac{d\vartheta_1}{ds} = -\frac{M_1}{\sigma}\sin\vartheta_1, \tag{3}$$

where M_1, $\dfrac{d\sigma}{ds}$ – functionals, depending on the following parameters of complex loading: s – the arc length of the trajectories of strain and its break angle ϑ_1^0. The angle of delay ϑ_1 characterizes the direction of the vector $\overline{\sigma}$ towards the shearing to strain trajectory at each point. This angle reflects the influence of the vector properties of the material on the deformation process.

In the mathematical model of materials plastic deformation process theory in addition to the equations (3) uses universal approximations of functionals proposed by V.G. Zubchaninov [3]

$$\sigma(s) = \Phi(s) - Af^{p}\left(\vartheta_1^0\right)\Omega(\Delta s), \quad M_1 = 2G_p + \left(2G - 2G_p^0\right)f^{q}, \tag{4}$$

where $\Phi(s)$ – universal function of Odkvist-Il'yushin for simple (proportional) loading; G, G_p – elastic and secant shear modulus; $\Delta s = s - s_K^{\text{т}}$; $s_K^{\text{т}}$ – arc length at the break point K of trajectory;

$$f = \frac{1-\cos\vartheta_1}{2}, \qquad \Omega(\Delta s) = \gamma\Delta s\, e^{-\gamma\Delta s} + b\left(1 - e^{-\gamma\Delta s}\right) \tag{5}$$

– complex loading function; A, b, γ, p, q – observed approximation parameters [3]. The index «zero» refers to the quantities at the break point of trajectory. The numerical solution of the defining relations (3) for the acquainted functionals (4) is implemented by the Runge-Kutta method of the fourth order of accuracy using the MathWorks Matlab software package. The strain vector trajectory is specified and the stress vector trajectory is obtained by integrating equations.

3. Comparison of numerical results with experimental ones

Using the foregoing mathematical model, experimental studies and numerical calculations of the processes of deformation of thin-walled tubular specimens of steel 45 on the SN-EVM testing complex along two-link broken strain trajectory in the plane $Э_1 - Э_3$, producing a plane state of stress in the wall of the tube. Specimens on the first link were twisted to a value $Э_3^0 = 2\,\%$ and on the second link had been tested under combined tension and torsion.

Figures 1-4 show the results of calculations and experimental data for two-link strain trajectory with the break angle 45º. The experimental data are marked with circles. Figure 1 shows the implemented strain trajectory in the plane $Э_1 - Э_3$, and on Fig. 2 the response to it in the stress space on the plane $S_1 - S_3$ is discribed. Figure 3 the diagram of tracing the straining process $\sigma - s$, characterizing the scalar properties of the material is presented, and on Fig. 4 chart $\vartheta_1 - \Delta s$ characterizing vector properties of the material.

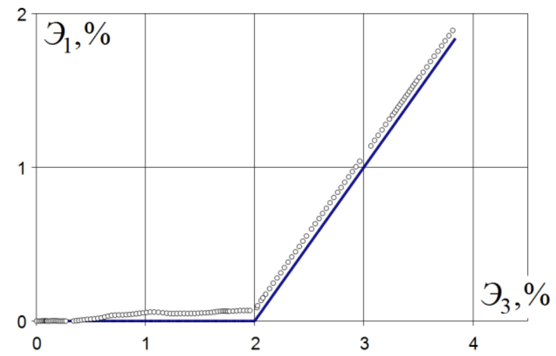

Fig. 1. Strain trajectory with the break angle 45º

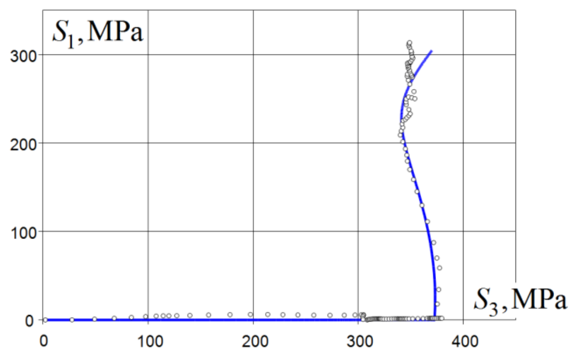

Fig. 2. The response $S_1 - S_3$

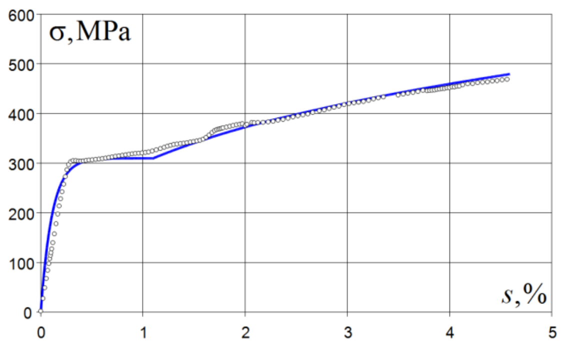

Fig. 3. Chart of deformation $\sigma - s$

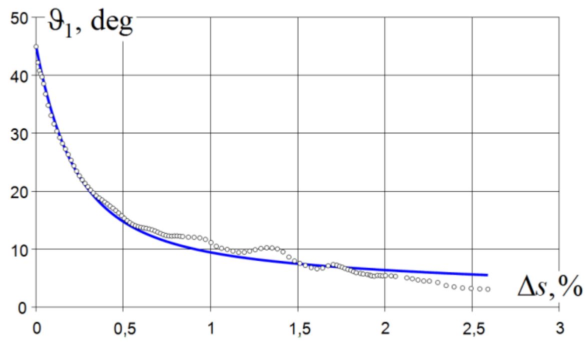

Fig. 4. Chart $\vartheta_1 - \Delta s$

Figures 5-8 present the results of calculation and experimental data for two-link strain trajectory with the break angle 90º and Fig. 9-12 – with the break angle 135º.

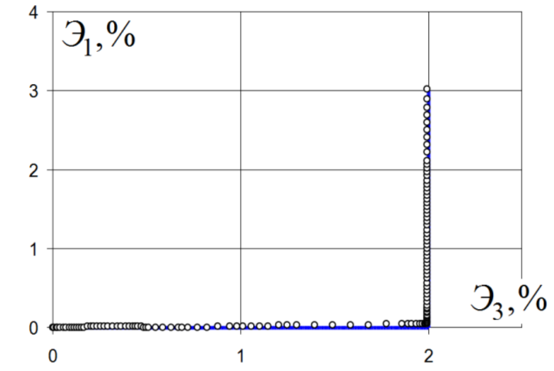

Fig. 5. Strain trajectory with the break angle 90°

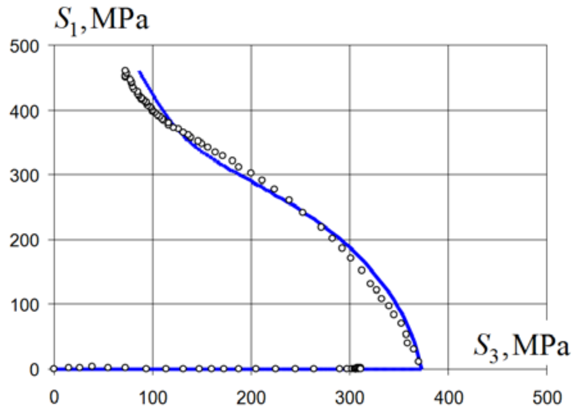

Fig. 6. The response $S_1 - S_3$

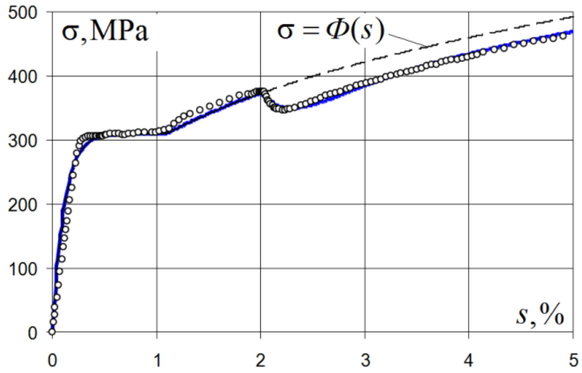

Fig. 7. Chart of deformation $\sigma - s$

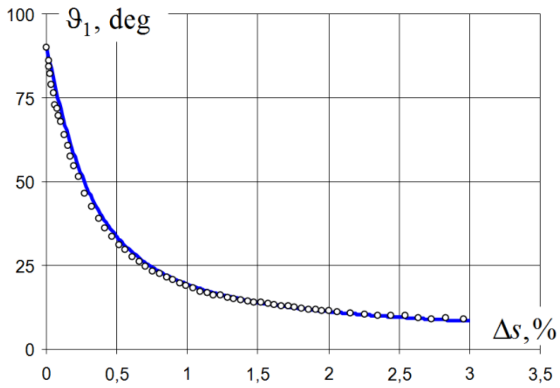

Fig. 8. Chart $\vartheta_1 - \Delta s$

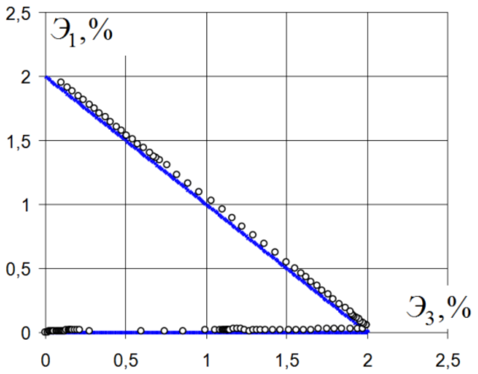

Fig. 9. Strain trajectory with the break angle 135°

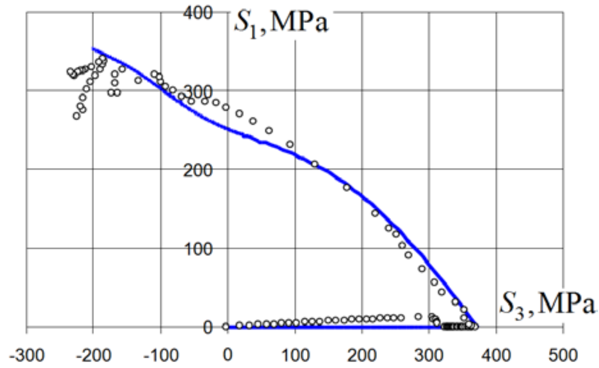

Fig. 10. The response $S_1 - S_3$

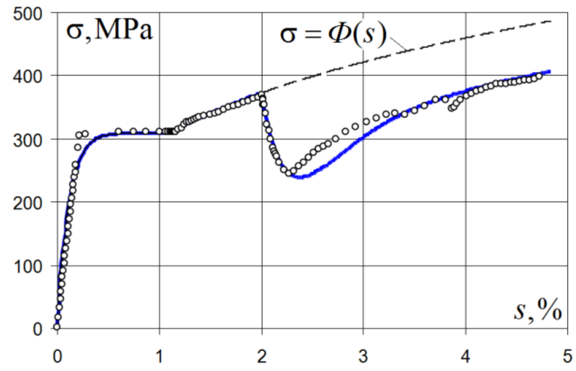

Fig. 11. Chart of deformation $\sigma - s$

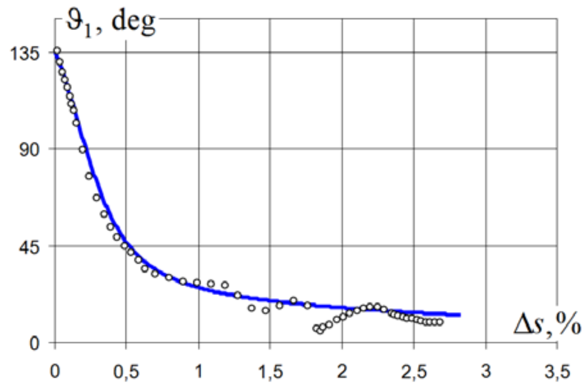

Fig. 12. Chart $\vartheta_1 - \Delta s$

As we can see, the numerical calculations according to the mathematical model are in good agreement with the experimental data on scalar and vector properties, both qualitatively and quantitatively. This shows the integrity of the calculated data, sufficient for practical problems and the acceptable accuracy of the constructed approximations of the functional processes in the mathematical model of the processes theory applied to this class of trajectories. The modified version of the model has been successful at multi-link broken trajectories [19].

4. The property of delay vector and scalar characteristics of the materials

In [4] A.A. Ilyushin noted that the delay of vector properties is a common property of all plastic materials. The experiments show that the direction of the stress vector at some point K relative to the strain trajectory does not depend on the entire previous path, but only on its limited part, called the trace of delay λ. Usually, the trace of delay λ is determined on two-link broken trajectories at different break angles of the trajectory ϑ_1^0 [2,3,15,20-22].

By hypothesis of R.A. Vasin, it is considered [15] that the trace of delay is exhausted when the angle of delay 6º÷7º is achieved, which is equivalent to the Hencky-Il'yushin theory of small elastic-plastic deformations. The magnitude estimation of the trace of delay of the vector characteristics of the material λ from the break angle of the trajectory ϑ_1^0 could not be carried out, according to the experimental data in the realized experiments, because the specimens were destroyed before the angle of delay achieved $\vartheta_1^* \approx 7$º, so the evaluation is made according to the calculated data of the model, which is well within the experiments.

Figure 13 shows the combined dependency graphs $\vartheta_1 - \Delta s$ for the implemented two-link strain trajectories, and on Fig. 14 the calculated dependence of the trace of delay of the vector characteristics of the material λ on the break angle of the trajectory is given. As you can see, the trace of delay λ significantly depends on the break angle of the trajectory ϑ_1^0, which is indicative of the complexity of the process. This instability means that the trace of delay λ cannot be taken as a constant characteristic of the material.

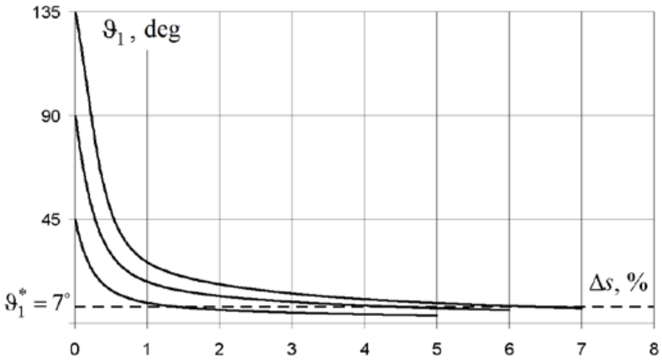

Fig. 13. Chart $\vartheta_1 - \Delta s$ for different break angles of trajectories

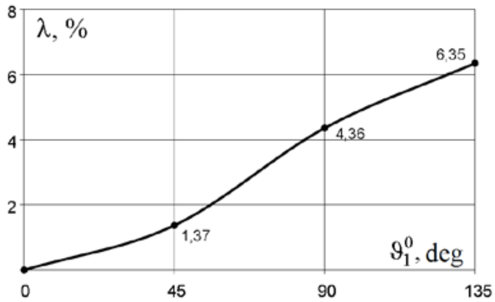

Fig. 14. Chart $\lambda - \vartheta_1^0$

If we divide the current value of the angle of delay ϑ_1 by the value of the angle of delay at the break point of trajectory ϑ_1^0, we get a relative angle of delay $\tilde{\vartheta}_1 = \vartheta_1 / \vartheta_1^0$. However, it is impossible to postpone the value $\vartheta_1^* = 6^\circ - 7^\circ$ on the axis with a dimensionless quantity. In [20] proposed to consider the trace of delay λ exhausted when the ϑ_1^0 decreases by 15 times. In dimensionless quantities to determine the trace of delay, we can take the value of the angle of delay equal to $1/15 \approx 0,067$. When $\vartheta_1^0 = 90^{\circ}$ this corresponds to the value $\vartheta_1^* = 6^{\circ}$, i.e. accuracy Hencky-Il'yushin theory.

Figure 15 shows the dependence of the relative angle of delay $\tilde{\vartheta}_1$ on the increment of the arc length of the strain trajectory after a break. It can be seen that for all the break angles, the calculated charts practically coincide with each other. Thus, it is fair to assumed that the dependence of the relative angle of delay $\tilde{\vartheta}_1$ on the increment of the arc length of the trajectory of deformation after the break Δs is universal and does not depend on the break angle of the trajectory (for the same length of the first link of strain trajectory)

V.S. Lensky in his work [23] introduced the concept of trace of delay of the scalar properties of materials. Fig. 16 shows the dependence on the break angle of the trajectory for the scalar trace of delay of material λ_{sc}. For the value λ_{sc} on the diagram $\sigma - s$ was equal to the distance along the arc length from the break point of trajectory with the value $\sigma_{\text{к}}^{\text{т}}$ to the point at which the stress after the «dive» again achieved significance $\sigma_{\text{к}}^{\text{т}}$.

This dependence (Fig. 16) shows that the characteristic λ_{sc} is also unstable, and when compared with Fig. 14, it is seen that $\lambda_{sc} \neq \lambda$, while the trace of delay of vector characteristics λ was several times larger than the trace of delay of the scalar characteristics λ_{sc}. In the future, the graphs (Figs. 14, 16) can be used to determine the values of the parameters of approximations of the process functionals responsible for the vector and scalar properties of the materials.

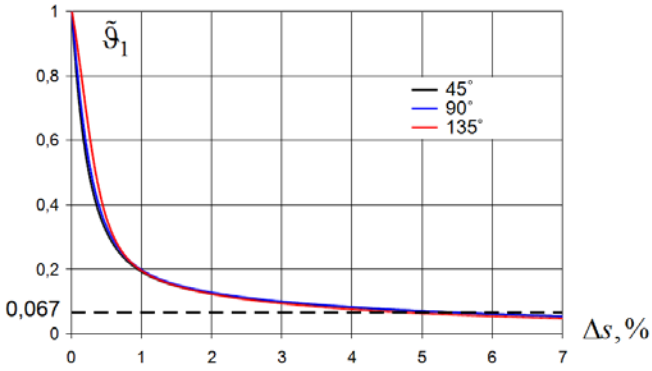

Fig. 15. Chart $\tilde{\vartheta}_1 - \Delta s$ for different break angles of trajectories

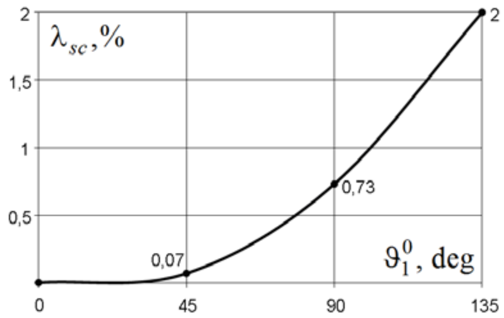

Fig. 16. Chart $\lambda_{sc} - \vartheta_1^0$

To determine the effect of the length of the first link of the strain trajectory s_0 on the scalar and vector properties of the material, numerical and experimental studies were conducted on the strain programs of testing are shown on Fig. 17. The specimens were subjected to the following strain history: specimens had been twisted to $s_0 = Э_3 = 1.5,\ 2,\ 2.5\,\%$, and then stretched along the component $Э_1$ under constant $Э_3$. The dependence $\vartheta_1 - \Delta s$ characterizing vector characteristics of the material for model and experimental data is shown in Fig. 18.

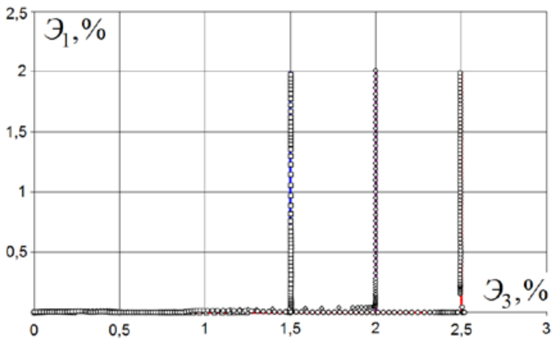

Fig. 17. Strain trajectories with different lengths of the first link

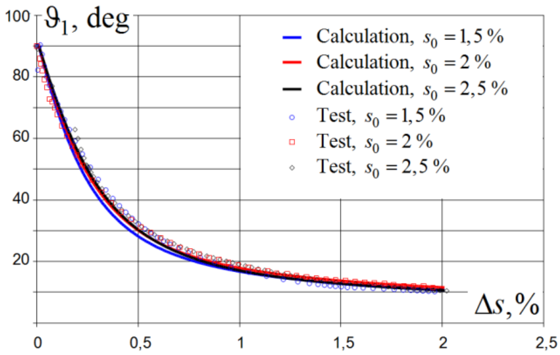

Fig. 18. Charts $\vartheta_1 - \Delta s$

It is seen that the experimental points practically coincide with each other and are consistent with the numerical calculations of the proposed mathematical model. This means that the length of the first link of the strain trajectory, in the case of a simple (proportional) loading on it, practically does not affect the vector properties of the material and the value of the trace of delay λ. In case of replacing the trajectory to the break point by another section, for example, a circle, the trace of delay changes significantly [24].

5. Conclusion

The theoretical positions of the mathematical model of the theory of processes during deformation along two-link strain trajectories are verified in comparison with experimental data. Verification results indicate the correct modeling of elastoplastic deformation of structural steels for a given class of strain trajectories.

It is established that the trace of delay of vector properties and the trace of delay of the scalar characteristics of the material significantly depend on the break angle of the trajectories. This indicates the instability of the value of the trace of delay as a characteristic of the material.

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