Materials Physics and Mechanics 42 (2019) 528-534 Received: December 4, 2018

MODELING OF ELASTOPLASTIC DEFORMATION OF STRUCTURAL STEEL BY A TRAJECTORY CONTAINING THREE CIRCLES TOUCHING INTERNALLY

V.G. Zubchaninov[1], A.A. Alekseev[1*], V.I. Gultiaev[1], E.G. Alekseeva[2]

[1]Tver State Technical University, nab. Afanasiya Nikitina, 22, Tver, 170026, Russia

[2]Bauman Moscow State Technical University, ul. Baumanskaya 2-ya, 5/1, Moscow, 105005, Russia

*e-mail: alexeew@bk.ru

Abstract. To verify the mathematical model of the theory of processes, numerical calculations of the complex elastoplastic deformation of St3 steel along a flat curvilinear strain trajectory under combined tension-compression with torsion were carried out. The numerical calculations are compared with the experimental data obtained by the authors on the experimental complex SN-EVM on thin-walled tubular specimen. It is shown that the used mathematical model qualitatively and quantitatively satisfactorily describes the main effects of complex plastic deformation for the class of strain trajectories under consideration.
Keywords: plasticity, complex loading, modeling of processes, strain trajectory, mechanical tests, tubular specimen

1. Introduction

Conducting experimental studies with complex loading beyond the elastic limit is an important part of the formulation of new mathematical models of the processes of elastoplastic deformation of the material and their verification. The theory of constitutive relations as one of the main directions in the mechanics of deformable solids was formed due to the experimental and theoretical development of the theory of plasticity. To check the physical reliability of the constitutive relations of the theory of plasticity, it is necessary to compare the numerical calculations with the experimental data available in the literature for various classes of deformation trajectories. Therefore, the study of the laws of complex elastic-plastic deformation of structural materials is important for the development of the fundamental principles of the theory of plasticity. The ideas and results of the construction of the constitutive relations of the theory of plasticity and the conduct of basic experimental studies are presented in the works [1-15] and others.

In the theory of elastoplastic processes [1-3], specific versions of mathematical models [13-2000] are constructed in relation to certain classes of trajectories, which allows us to estimate the limits of applicability (adequacy) of models when compared with experimental data. Previously, the variants of the used model of the theory of processes were used to describe the processes of deformation along two- and multi-link strain trajectories with various lengths of links and angles of breaks [17-19], as well as strain trajectories with curved sections of constant curvature [20-21] and circles centered at the origin [22-23].

Of particular interest for verification of the theory are the strain trajectories, in the sections of which the curvature changes. This paper presents experimental data on a complex deformation trajectory (tension with torsion – *P*-*M* experiments), representing a flat curve of

http://dx.doi.org/10.18720/MPM.4252019_6

five sections containing three circles of different curvatures with an internal touch at one point. The realized strain trajectory is important for demonstrating the very nontrivial connection between stresses and strains during elastic-plastic deformation of the material. To verify the proposed mathematical model, the results of numerical calculations are compared with experimental data. The calculated and experimental results are presented in the vector representation of stresses and strains A.A. Ilyushin [1-3].

2. Constitutive equation

In the theory of elastoplastic processes for numerical calculations of elastoplastic deformation of materials in deviatoric space E_5 at complex flat trajectories with the analytical curvilinear sections used the constitutive relations [3], considering scalar and vector properties of materials

$$\frac{d\overline{\sigma}}{ds} = M_1 \frac{d\overline{Э}}{ds} + \left(\frac{d\sigma}{ds} - M_1 \cos\vartheta_1\right)\frac{\overline{\sigma}}{\sigma}, \qquad \frac{d\vartheta_1}{ds} + \kappa_1 = -\frac{M_1}{\sigma}\sin\vartheta_1, \tag{1}$$

where $\overline{\sigma}, \overline{Э}$ – stress and strain vectors respectively; s – is the arc length of the trajectory deformation; $\vartheta_1 = \vartheta_1(s, \kappa_1, \vartheta_1^0)$ – the angle of delay (functional of the process of vector properties of the material), characterizing the direction of the vector $\overline{\sigma}$ at each point of the strain trajectory. κ_1 – curvature of the strain trajectory; ϑ_1^0 – the break angle at the starting point of the analytical section of the strain trajectory; $\sigma = \sigma(s, \kappa_1, \vartheta_1^0)$ – functional of the process of scalar properties of the material; M_1 and $\frac{d\sigma}{ds}$ – are functionals of the parameters of the intrinsic geometry of the strain trajectory $s, \kappa_1, \vartheta_1^0$. The constitutive equations of the mathematical model theory in plane problems are equations (1) and universal approximation of functionals depending on all the above parameters is a complex loading for a flat curvilinear trajectories

$$\begin{gathered}\sigma(s) = \Phi\left(s, \vartheta_1^0, \kappa_1\right) = \Phi(s) + A f_0^p\,\Omega - B\Delta s\kappa_1 - \Delta\sigma, \\ \frac{d\sigma}{ds} = \frac{d\Phi}{ds} + A f_0^p \frac{d\Omega}{ds} - B\frac{d}{ds}\left(\Delta s\kappa_1\right),\end{gathered} \tag{2}$$

$$M_1 = 2G_p + \left(2G - 2G_p^0\right) f^q, \tag{3}$$

where $\Delta s = s - s_K^{\text{т}}$ – the increment of the arc of the trajectory deformation; $s_K^{\text{т}}$ – the length of the arc at the break point of the trajectory, or change its curvature; $\Phi(s)$ – a universal hardening function of Odquist-Ilyushin for processes close to simple (proportional) loading, without regard to their history; $\Delta\sigma = \Phi(s_{\text{к}}^{\text{т}}) - \sigma_{\text{к}}^{\text{т}}$ – the difference between the values of the universal functions of Odquist-Ilyushin and the real value of the stress vector module at the point of change sections of the strain trajectory;

$$\Omega = -\left[\gamma\Delta s\, e^{-\gamma\Delta s} + b\left(1 - e^{-\gamma\Delta s}\right)\right] \tag{4}$$

is a function of a complex loading, describing scalar «dive» stresses in complex unloading and subsequent plastic deformation;

$$f = f(\vartheta_1) = \frac{1-\cos\vartheta_1}{2}; \quad f_0 = f(\vartheta_1^0) = \frac{1-\cos\vartheta_1^0}{2} \tag{5}$$

is a function of a complex loading taking into account the orientation of the stress vector in the process of deformation and its value at the break point of the strain trajectory;

A, B, b, γ, p, q – parameters approximations. To approximate a universal function of Odquist-Ilyushin in simple (proportional) loading is used [3]

$$\sigma = \Phi(s) = \begin{cases} \dfrac{2G}{\alpha}\left(1-e^{-\alpha s}\right), \text{ if } 0 \le s \le s^{\text{т}}, \\ \sigma^{\text{т}} + 2G_*(s-s^{\text{т}}) + \sigma_*\left(1-e^{-\beta(s-s^{\text{т}})}\right), \text{ if } s > s^{\text{т}}; \end{cases} \quad (6)$$

$\sigma^{\text{т}} = \sqrt{2/3}\sigma_{\text{т}}$; $\sigma_{ò}$ – yield strength tensile; $s^{\text{т}}$ – is the border of sites of the chart $\sigma = \Phi(s)$ dividing elastic region and yielding ($0 \le s \le s^{\text{т}}$) from the site of strain hardening ($s > s^{\text{т}}$); G – shear modulus; σ_*, G_*, α, β are material constants determined from experiments under simple (proportional) loading.

Under the given initial conditions for the components $Э_k$ $(k = 1, 3)$ and angle ϑ_1^0, the basic equations of the model are reduced to the Cauchy problem, for the numerical solution of which the Runge-Kutta method of the fourth order of accuracy in the package of linear algebra MathWorks MATLAB was used.

3. Comparison of numerical results with the experimental ones

Experimental research was performed on SN-EVM testing complex that implements the three-parameter impact on the specimens (axial tension-compression, torsion and internal pressure). Thin-walled tubular specimens made of initially isotropic mild steel St3 had the following dimensions: gage-length $l = 110$ mm, wall thickness $h = 1$ mm and diameter of the middle surface $d = 31$ mm. The initial isotropy of the steel St3 with a sufficient degree of accuracy was confirmed in test under proportional loading (tension test and torsion test), where after processing these experimental stress–strain diagrams were taken the following values of the material parameters for steel St3 in (6): $\sigma^{\text{т}} = 220$ MPa, $s^{\text{т}} = 1 \cdot 10^{-2}$, $2G = 1.57 \cdot 10^5$ MPa, $\beta = 70$, $\alpha = 1500$, $\sigma_* = 54.9$ MPa, $2G_* = 1181.5$ MPa.

The program of the test under kinematic loading is a plane curve in the Ilyushin's deviatoric strain space $Э_1 - Э_3$ consisting of five sections, including circles of different curvature with an internal touch at one point offset from the origin. (Fig. 1).

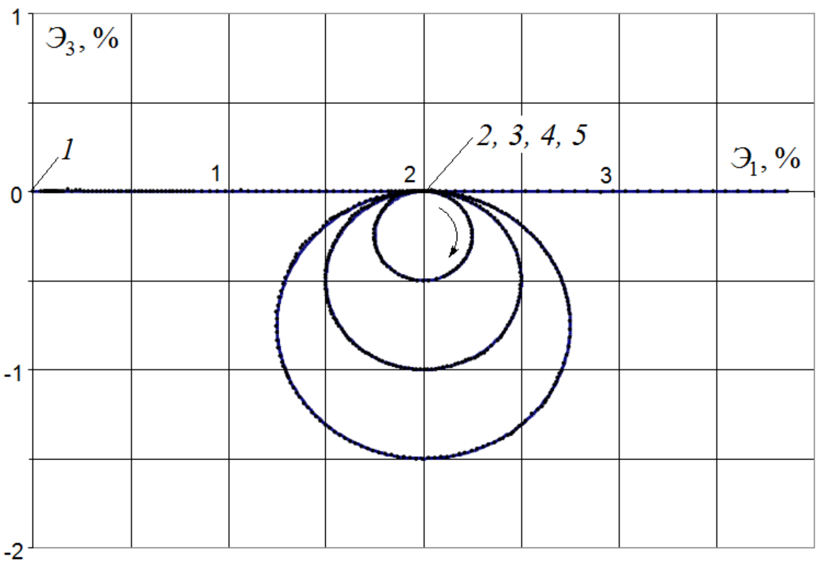

Fig. 1. Strain trajectory

The specimen has been tested under combined tension and torsion, producing a plane state of stress in the wall of the tube. The specimen was subjected to the following strain history: the first section implemented proportional stretching to $Э_1^* = 2\,\%$, the second section (without break) implemented a trajectory of constant curvature as a circle of radius $R = 0.25\,\%$ and curvature $\kappa_1 = 400$; for the circle one complete rotation. The third section has a circle of radius $R = 0.5\,\%$ and curvature $\kappa_1 = 200$, and the fourth – a circle of radius $R = 0.75\,\%$ and curvature $\kappa_1 = 133.33$. On the last fifth section specimen was stretching to $Э_1^* \approx 3.9\,\%$ under constant $Э_3$. In Fig. 2 shows the response to the strain trajectory in the plane of the $S_1 – S_3$ deviatoric stress space, Figs. 3 and 4 show chart $\sigma - s$ and $\vartheta_1 - \Delta s$, characterizing scalar and vector properties of the materials, respectively. On Figures. 5, 6 show the local stress–strain chart. Numbers 1, 2, 3, 4, 5 on Figures 1-6 are designated the start point of the corresponding sections of the realized trajectory. Experimental data in the figures are indicated by dots, and the numerical calculations – the solid line (blue).

In the terminology of A.A. Ilyushin [1], the realized strain trajectory (Fig. 1) is smooth, since there are no break points on it. That is, for the function $\bar{Э} = \bar{Э}(s)$ that describes the strain trajectory in linear space, the first derivatives are continuous, and the second derivatives are discontinuous. Experiments show [3,21] that on smooth trajectories the change of curvature at the junction of the sections is equivalent to a break the trajectory with the formation of stress «dives». However, as can be seen from the experimental data (Fig. 3), the effect of complex loading does not appear immediately upon transition to the curved section, but after about a quarter of a circle revolution. This circumstance was taken into account in the numerical calculation by dividing the sections of the circles into two parts (with lengths equal to a quarter and three quarters of a circle) and the use of fictitious break angles to describe scalar stress «dives» according to formulas (2), (4). As shown by experimental data on two-link broken strain trajectories [3,17-18], the influence of complex loading begins to significantly occur at the break angle $\vartheta_1^0 \geq 90^\circ$. So for the first parts of sections of circles, where the influence of complex loading slightly, was adopted $\vartheta_1^0 = 45^\circ$. For the second part of the circles, where the influence of complex loading significantly on it $\vartheta_1^0 = 90^\circ$.

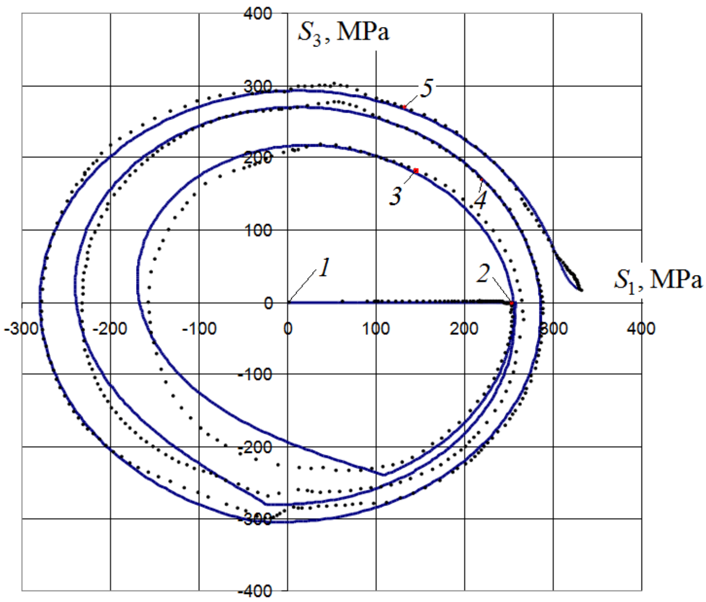

Fig. 2. The response $S_1 - S_3$

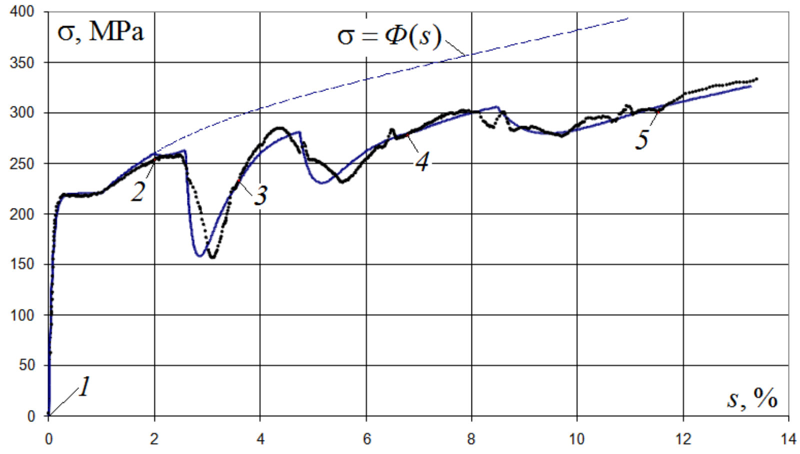

Fig. 3. Chart $\sigma - s$

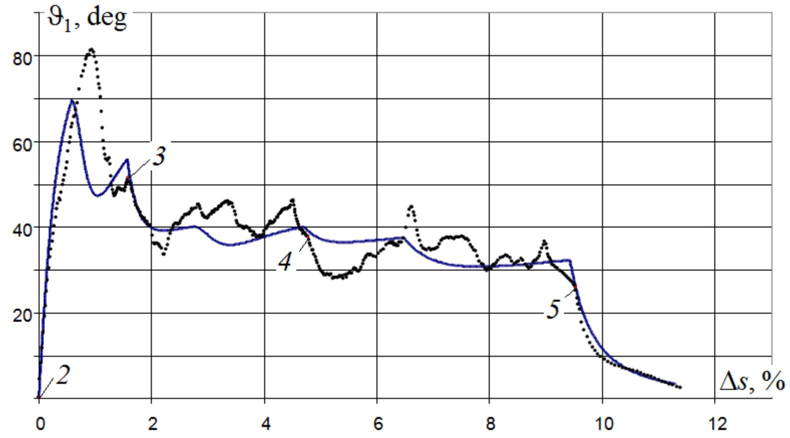

Fig. 4. Chart $\vartheta_1 - \Delta s$

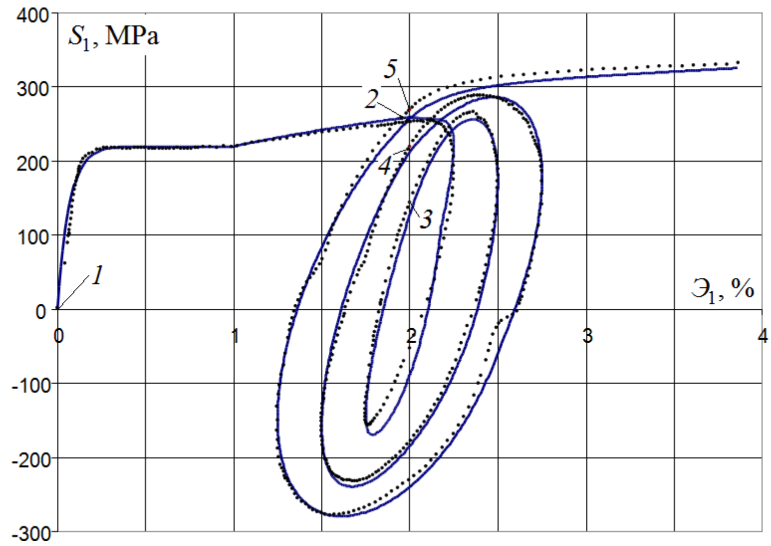

Fig. 5. Chart $S_1 - Э_1$

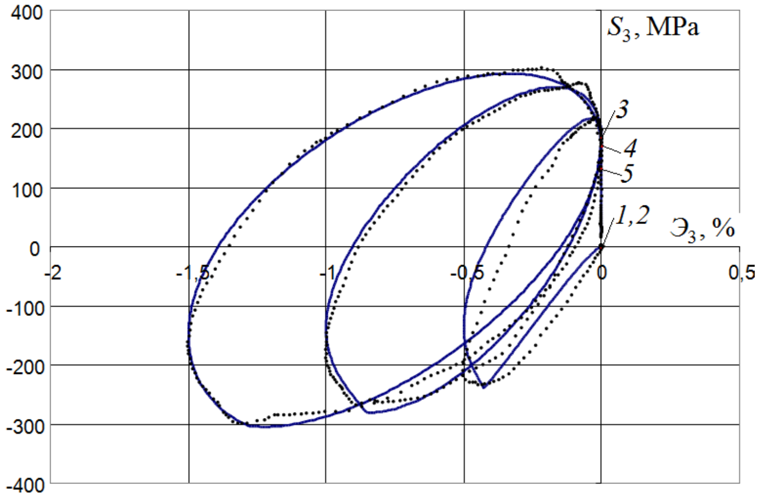

Fig. 6. Chart $S_3 - Э_3$

Figures 2-6 show a qualitative and acceptable quantitative agreement between the experimental data and numerical calculations according to the proposed mathematical model of the theory of elastoplastic processes for a plane curved strain trajectory (Fig. 1), which indicates the correct modeling of complex elastoplastic deformation of the material.

5. Conclusion

The verification of the mathematical model of the theory of elastoplastic processes by comparing numerical calculations with experimental data indicates the correct modeling of complex elastoplastic deformation of structural steels for a given class of plane curved strain trajectories.

Acknowledgements. *No external funding was received for this study.*

References

[1] Il'yushin AA. *Plasticity. Fundamentals of the General Mathematical Theory*. Moscow: Izd. Akad. Nauk SSSR; 1963. (In Russian)
[2] Il'yushin AA. *Continuum Mechanics*. Moscow: MSU Publishing House; 1990. (In Russian)
[3] Zubchaninov VG. *Mechanics of processes of plastic environments*. Moscow: Fizmatlit; 2010. (In Russian)
[4] Mitenkov FM, Volkov IA, Igumnov LA, Kaplienko AV, Korotkih YG, Panov VA. *Applied theory of plasticity*. Moscow: Fizmatlit; 2015. (In Russian)
[5] Annin BD, Zhigalkin VM. *Material Behavior under Conditions of Complex Loading*. Novosibirsk: Izdat. SO RAN; 1999. (In Russian)
[6] Bondar VS. *Inelasticity. Variants of the theory*. New York: Begell House; 2013.
[7] Temis YM, Alkhimov DA, Martynova AD. Application of the invariant plastic flow theory for mathematical modeling of the processes of testing specimens under complex elastic-plastic deformation. *Vestnik of Samara University. Aerospace and Mechanical Engineering.* 2015;14(3-1): 24-36. (In Russian)
[8] Krempl E, Lu H. The hardening and dependent behavior of fully annealed aisi type 304 stainless steel under biaxial in phase and out – of – phase strain cycling at room temperature. *ASME Journal of Engineering Materials and Technology*. 1984;106(4): 376-382.
[9] Krieg RD. A practical two surface plasticity theory. *Journal of Applied Mechanics.* 1975;42(3): 641-646.

[10] Chaboche JL. A review of some plasticity and viscoplasticity theories. *International Journal of Plasticity*. 2008;24: 1642-1693.
[11] Khutia N, Dey PP, Hassan T. An improved nonproportional cyclic plasticity model for multiaxial low-cycle fatique and ratcheting responses of 304 stainless steel. *Mechanics of Materials*. 2015;91: 12-25.
[12] Zubchaninov VG. The General Mathematical Theory of Plasticity and the Il'yushin Postulates of Macroscopic Definability and Isotropy. *Moscow University Mechanics Bulletin*. 2018;73(5): 101-116.
[13] Peleshko VA. Applied and engineering versions of the theory of elastoplastic processes of active complex loading. Part 2: Conditions of mathematical well-posedness and methods for solving boundary value problems. *Mechanics of Solids*. 2015;50(6): 650-656.
[14] Peleshko VA. Applied and engineering versions of the theory of elastoplastic processes of active complex loading. Part 2: Identification and verification. *Mechanics of Solids*. 2016;51(1): 91-113.
[15] Vasin RA. Some problems of the relationship between stresses and strains in multiaxial loading. *Elastisity and Inelasticity*. 1971;1: 59-126. (In Russian)
[16] Abirov RA. On the physical reliability and taking complex loading into account in plasticity. *Materials Science*. 2008;44(4): 512-516.
[17] Zubchaninov VG, Alekseeva EG. Modeling complex processes of elastoplastic deformation of materials. *Bulletin of the Yakovlev Chuvash State Pedagogical University. Series: Mechanics of Limit State*. 2010;2(8): 172-181. (In Russian)
[18] Zubchaninov VG, Alekseev AA, Gultiaev VI. Numerical simulation a processes of complex elastoplastic deformation steel on two-link broken trajectories. *Problems of strenght and plasticity*. 2014;76(1): 18-25. (In Russian)
[19] Zubchaninov VG, Alekseev AA, Gultiaev VI. Modeling of elastic-plastic deformation of work material along multielement piecewise zig-zag linear trajectories. *PNRPU Mechanics Bulletin*. 2017;3: 203-215.
[20] Zubchaninov VG, Alekseev AA, Alekseeva EG, Gultiaev VI. Experimental verification of postulate of isotropy and mathematical modeling of elastoplastic deformation processes following the complex angled nonanalytic trajectories. *Materials Physics and Mechanics.* 2017;32(3): 298-304.
[21] Zubchaninov VG, Alekseev AA, Alekseeva EG. Verification of the postulate of the isotropy and numerical simulation of the deformation of materials on a complex smooth trajectories. *Materials Physics and Mechanics.* 2016;29: 150-157. (In Russian)
[22] Zubchaninov VG, Alekseev AA, Alekseeva EG. Mathematical modeling of plastic deformation of materials on complex flat trajectories. *Materials Physics and Mechanics*. 2015;24(2): 107-118. (In Russian)
[23] Zubchaninov VG, Alekseev AA, Gultiaev VI. Modeling the processes of complex elastoplastic deformation of materials along flat curvilinear trajectories. *Problems of strength and plasticity*. 2015;77(2): 113-123. (In Russian)