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MECHANICAL ASPECTS OF THE BEHAVIOR OF THE SURFACE OF COATED BODIES WHEN EXPOSED TO CORROSIVE LIQUID

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Abstract. The block element method is used to study the behavior of a coated material under the assumption that the surface is exposed to a corrosive liquid medium that can destroy the coating. It is assumed that the destruction begins with the formation of vertical local cracks in the coating, which then grow and lead to the exposure of the unprotected surface. Assuming the possibility of modeling a liquid layer by shallow water equations, the block structure including a body in the form of a deformable layer, a defective coating modeled by Kirchhoff plates and a heavy liquid layer is investigated. The distribution of stress concentration in such a block structure is studied. The analysis showed that even in the presence of a liquid, the features of the stress-strain state of the material inherent in the case of its absence are preserved. The conditions both allowing further use of such object and excluding this possibility are revealed.

Keywords: block element, plates, topology, differential factorization methods, exterior forms, block structures, boundary problems, starting earthquakes, hidden defects

1. Introduction

The paper studies the behavior model of a material having a thin coating that reacts to the influence of aggressive liquids. It is assumed that the result of the surface reaction is the formation of vertical cavities in the coating-cracks. The appearance of the latter can contribute to the loading of the material by external influences. The theory of cracks, including those located parallel to the boundaries of the areas occupied by the material, as well as on the border between the coating and the material, has been developed in numerous works, references to which are available in publications [1-11]. Defects in coatings described above are classified as hidden [12], since their vertical plane is not easily detectable by ultrasound or x-ray methods scanning in the vertical direction. The presence of a layer of liquid complicates the detection of defects. The result of the work shows the importance of their detection, since already single defects can lead to irreversible consequences for such structures. The study is based on the block element method, which has successfully proved itself in problems with hidden defects.

2. Basic equations

The case of a single defect in the coating is considered. The result of this study can be transferred to the study of multiple parallel defects using the method proposed in [13] The described problem is considered in a four-block structure consisting of a deformable layer modeling a body, two semi-infinite Kirchhoff plates, between the ends of which there may or may not be some distance, and a layer of liquid. In each block of such a structure, corresponding boundary problems are set. By the block element method the study is reduced to the study of a system of functional equations, allowing to identify stress concentration in

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the block structure. We consider that external harmonic in time forces are acting vertically on a layer of liquid and a plate. In the local coordinate system $x_1x_2x_3$ with the beginning in the plane x_1x_2, coinciding with the median plane of the plate, axis ox_3 , directed upwards along the normal to the plate, axis ox_1, directed tangentially to the boundary of the plate end, axis ox_2 directed along the normal to its boundary. The domain occupied by the left plate is denoted by λ , and described by relations $|x_1|\le\infty, \quad x_2\le-\theta$, the domain occupied by the right plate is denoted by index r and coordinates $|x_1|\le\infty, \quad \theta\le x_2$. For the plates, the Kirchhoff equation for fragments $b=\lambda,r$, occupying domains Ω_b with boundaries $\partial\Omega_b$, under vertical harmonic effects by stress $t_{3b}e^{-i\omega t}$ from above and $g_{3b}e^{-i\omega t}$ from below, after eliminating the time parameter, has the form:

$$\mathbf{R}_b(\partial x_1,\partial x_2)u_{3b}+\boldsymbol{\varepsilon}_{53b}(t_{3b}-g_{3b})\equiv(\frac{\partial^4}{\partial x_1^4}+2\frac{\partial^2}{\partial x_1^2}\frac{\partial^2}{\partial x_2^2}+\frac{\partial^4}{\partial x_2^4}-\boldsymbol{\varepsilon}_{43b})u_{3b}+\boldsymbol{\varepsilon}_{53b}(g_{3b}-t_{3b})=0,$$

$$\mathbf{R}_b(-i\boldsymbol{\alpha}_1,-i\boldsymbol{\alpha}_2)U_{3b}=\left[\ (\boldsymbol{\alpha}_1^2+\boldsymbol{\alpha}_2^2)^2-\boldsymbol{\varepsilon}_{43b}\ \right]U_{3b},$$

$$U_{3b}=\mathbf{F}_2u_{3b},\quad G_{3b}=\mathbf{F}_2g_{3b},\quad T_{3b}=\mathbf{F}_2t_{3b},\quad b=\boldsymbol{\lambda},r,$$

$$m_b=-D_{b1}(\frac{\partial^2u_{3b}}{\partial x_2^2}+\nu_b\frac{\partial^2u_{3b}}{\partial x_1^2})=f_{3b}(\partial\Omega_b),\quad D_{b1}=\frac{D_b}{H^2},\quad D_{b2}=\frac{D_b}{H^3},\quad x_{k0}=Hx_k,\quad k=1,2,$$

$$q_b=-D_{b2}(\frac{\partial^3u_{3b}}{\partial x_2^3}+(2-\nu_b)\frac{\partial^3u_{3b}}{\partial x_1^2\partial x_2})=f_{4b}(\partial\Omega_b),$$

$$u_{3b}=f_{1b}(\partial\Omega_b),\quad \frac{\partial u_{3b}}{H\partial x_2}=f_{2b}(\partial\Omega_b),\ D_b=\frac{E_bh_b^3}{12(1-\nu_b^2)},$$

$$\boldsymbol{\varepsilon}_{43b}=\boldsymbol{\omega}^2\boldsymbol{\rho}_b\frac{(1-\boldsymbol{\nu}_b^2)12H^4}{Eh_b^2},\quad \boldsymbol{\varepsilon}_{53b}=\frac{(1-\boldsymbol{\nu}_b^2)12H^4}{E_bh_b^3},\quad \boldsymbol{\varepsilon}_6^{-1}=\frac{(1-\boldsymbol{\nu})H}{\boldsymbol{\mu}}.$$

The notation for the plates is: $\boldsymbol{\nu}_b$ - Poisson's ratio, E_b - Young's modulus, h_b - plate thickness, $\boldsymbol{\rho}_b$ - density, $\boldsymbol{\omega}$ - oscillation frequency. g_{3b}, t_{3b} are the values of contact stresses from the base and pressures on the plates of the liquid layer on top, acting along the axis x_3 in the domain Ω_b. $\mathbf{F}_2\equiv\mathbf{F}_2(\alpha_1,\alpha_2)$ and $\mathbf{F}_1\equiv\mathbf{F}_1(\alpha_1)$ are two-dimensional and one-dimensional Fourier transform operators, respectively, m_b and q_b - bending moment and shear force, $f_1(\partial\Omega_b)$ vertical displacement at the boundary $f_2(\partial\Omega_b)$; the angle of rotation of the median plane around the axis x_1, in the coordinate system x_1ox_2; h_b - plate thickness, H - dimensional parameter of the substrate, for example, thickness of the deformable layer of material.

The behavior of the block element, which is the layer of thickness H of incompressible liquid on the surface, describes by the following shallow water equations [14]:

$$p=(i\omega\rho\varphi+\rho g\frac{ih_b}{\omega H_1^2}\Delta\varphi)e^{-i\omega t}-we^{-i\omega t}.$$

Here p is pressure in the liquid layer, ρ - liquid density, g - acceleration of gravity, φ - velocity potential in the liquid, w - external action on the layer. Considering that at the upper boundary of the lithospheric plate a pressure of a layer of liquid is applied to it, and taking into account the model taken, it is necessary to take:

$$t_{3b} = p\,,\ u_{3b} = \frac{h_b}{i\omega H_1{}^2}\Delta\varphi_b\,.$$

As a result, the differential equation for the velocity potential has the following form:

$$\Delta^3\boldsymbol{\varphi}_b + (\,\boldsymbol{\varepsilon}_{53b}\boldsymbol{\rho} g\text{-}\boldsymbol{\varepsilon}_{43b}\,)\Delta\boldsymbol{\varphi}_b + \boldsymbol{\varepsilon}_{53b}\boldsymbol{\rho}\frac{\boldsymbol{\omega}^2 H_1{}^2}{h_b}\boldsymbol{\varphi}_b - i\boldsymbol{\varepsilon}_{53b}\frac{\boldsymbol{\omega} H_1{}^2}{h_b}(\,g_{3b} - w_b\,)=0\,.$$

3. Solution Method

To use the block element method, it is necessary to apply its algorithm, which includes the steps of exterior algebra, external analysis and factor-topology. At the stage of exterior algebra, the boundary problem is reduced to a functional equation of the following form:

$$N_b(\alpha_1,\alpha_2)\Phi_b(\alpha_1,\alpha_2) = \int\limits_{\partial\Omega_b} \omega_b(\alpha_1,\alpha_2) + S_b(\alpha_1,\alpha_2)$$

$$N_b(\alpha_1,\alpha_2) = (\alpha_1^2+\alpha_2^2)^3 + (\alpha_1^2+\alpha_2^2)(\varepsilon_{53b}\rho g - \varepsilon_{43b}) - \varepsilon_{53b}R_b \qquad (1)$$

$$S_b(\alpha_1,\alpha_2) = i\varepsilon_{53b}\frac{\omega H_1{}^2}{h_b}\mathbf{F}_2(\alpha_1,\alpha_2)(g_{3b}-w_b), \quad \Phi_b(\alpha_1,\alpha_2) = \mathbf{F}_2(\alpha_1,\alpha_2)\varphi_b, \quad R_b = \rho\frac{\omega^2 H_1{}^2}{h_b}$$

Here $\omega_b(\alpha_1,\alpha_2),\ b=\lambda,r$ are exterior forms corresponding to the boundary problem under consideration, which are enough simple to construct. The relationship between the boundary stresses and displacements on the surface of the elastic medium on which the plates are located has the form:

$$u_{3m}(x_1,x_2) = \varepsilon_6^{-1}\sum_{n=1}^{2}\iint\limits_{\Omega_n} k(x_1-\xi_1,x_2-\xi_2)g_{3n}(\xi_1,\xi_2)d\xi_1 d\xi_2, \quad x_1,x_2\in\Omega_m,\, m=1,2,3,$$

$$u_{31}=u_{3\lambda},\quad u_{32}=g_{3r},\quad u_{33}=u_{3\theta},\quad g_{31}=g_{3\lambda},\quad g_{32}=g_{3r},$$

$$\Omega_1 \equiv \Omega_\lambda(|x_1|\le\infty;\quad x_2\le-\theta),\quad \Omega_2\equiv\Omega_r(|x_1|\le\infty;\quad \theta\le x_2),\quad \Omega_3\equiv\Omega_\theta(|x_1|\le\infty;\quad -\theta\le x_2\le\theta)$$

or

$$u_{3m}(\,x_1,x_2\,) = \frac{1}{4\boldsymbol{\pi}^2\boldsymbol{\varepsilon}_6}\sum_{n=1}^{2}\int\limits_{-\infty}^{\infty}\int K(\,\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2\,)G_{3n}(\,\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2\,)e^{-i\langle\boldsymbol{\alpha},x\rangle}d\boldsymbol{\alpha}_1 d\boldsymbol{\alpha}_2\,,$$

$$K(\,\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2\,) = \mathbf{F}_2(\,\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2\,)k(\,x_1,x_2\,)$$

$$\langle\boldsymbol{\alpha},x\rangle = \boldsymbol{\alpha}_1 x_1 + \boldsymbol{\alpha}_2 x_2\,,\quad K(\,\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2,0\,) = O(\,A^{-1}\,),\quad A=\sqrt{\boldsymbol{\alpha}_1^2+\boldsymbol{\alpha}_2^2}\to\infty.$$

Here $K(\alpha_1,\alpha_2)$ is an analytic function of two complex variables $\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2$, meromorphic in particular, its various examples are given in numerous publications [14].

Let us apply to the study of the functional equations the stage of external analysis, called so since the differential operations are performed on exterior forms. For this purpose we represent the functional equation in the following form:

$$U_{3b}(\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2) = N_b^{-1}(\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2)[\ \ \boldsymbol{\omega}_b(\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2) + \boldsymbol{\varepsilon}_{53b}S_{3b}(\,\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2\,)\ \ ]. \qquad (2)$$

Then let us require implementation of the automorphism, one of the ways to accomplish which is to turn into zero Leray residue forms only in those zeros $\boldsymbol{\alpha}_{2n\pm} = \boldsymbol{\alpha}_{2n\pm}(\,\boldsymbol{\alpha}_1\,)$ of the function $N_b^{-1}(\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2)$, which provide as carriers for each of the boundary problems only their own plates. Pseudo-differential equations degenerate into algebraic ones. Taking into account the accepted notation, the equation for the left plate can represent in the form:

$$-e^{-i\alpha_{2-}\theta)}\{\ B_{1\lambda}(\alpha_1,\alpha_{2n-})Q_\lambda(\alpha_1,-\theta)+B_{2\lambda}(\alpha_1,\alpha_{2n-})M_\lambda(\alpha_1,-\theta)+B_{3\lambda}(\alpha_1,\alpha_{2n-})U_{3\lambda\partial x_2}(\alpha_1,-\theta)+$$
$$+B_{4\lambda}(\alpha_1,\alpha_{2n-})U_{3\lambda}(\alpha_1,-\theta)+B_{5\lambda}(\alpha_1,\alpha_{2n-})P(\alpha_1,-\theta)+B_{6\lambda}(\alpha_1,\alpha_{2n-})V_{x_2}(\alpha_1,-\theta)\ \}+$$
$$+S_\lambda(\alpha_1,\alpha_{2n-})=0\quad n=1,2,3$$

A similar form has the second pseudo-differential equation.

The unknowns in the functional equations are external actions defined at the ends of the plates, which are defective, that is when $x_2=\pm\theta$ external influence. Their boundary values in the form of Fourier transforms, represented by groups for the left and right plates, have the form:

$$Q_\lambda(\alpha_1,-\theta),\ M_\lambda(\alpha_1,-\theta),\ U_{3\lambda\partial x_2}(\alpha_1,-\theta),\ U_{3\lambda}(\alpha_1,-\theta),\ P_\lambda(\alpha_1,-\theta),\ V_{\lambda x_2}(\alpha_1,-\theta),$$
$$Q_r(\alpha_1,\theta),\ M_r(\alpha_1,\theta), U_{3r\partial x_2}(\alpha_1,\theta), U_{3r}(\alpha_1,\theta), P_r(\alpha_1,\theta),\ V_{rx_2}(\alpha_1,\theta)\ \}.$$

In each of the two groups of pseudo-differential equations, in accordance with the formulation of a particular boundary problem, three boundary conditions can be set at the ends of the plates and on the sections of the water layer at the ends of the plates. The remaining unknowns are determined by solving pseudo-differential equations. After introducing the found unknowns into the exterior forms in (2), we obtain packed block elements for the plates and the liquid layers above them. Thus, the external analysis stage for the block structure under consideration is completed, since, by its construction, the base block element having an unlimited carrier is always packed. The same could be also assumed for the layer of liquid. However, taking into account that the left and right plates can have different thicknesses, a more complex boundary problem is considered. The factor-topology stage involves the conjugation of block elements with each other as topological manifolds with a boundary. Equivalence relations are the dictated by the interests of the study boundary conditions adopted in the boundary problems under consideration. For conjugation of the block elements with the base, boundary displacements and contact stresses of the plates and base are conjugated. For displacements we have:

$$U_{3\lambda}+U_{3r}+U_{3\theta}=U_3.$$

Here $U_{3\theta}$ is the volume of liquid in the zone between the ends of the plates and the upper boundary of the liquid surface. When the ends of the plates converge the function $U_{3\theta}$ disappears. The latter relation can be, by selecting the contact stresses $G_{3\lambda}(\alpha_1,\alpha_2)$, $G_{3r}(\alpha_1,\alpha_2)$, $G_3(\alpha_1,\alpha_2)$, represented as:

$$N_\lambda^{-1}(\alpha_1,\alpha_2)\langle\ \omega_\lambda(\alpha_1,\alpha_2)+\varepsilon_{53\lambda}iR_\lambda[\ G_{3\lambda}(\alpha_1,\alpha_2)-W_\lambda\ ]\ \rangle+U_{3\theta}+$$
$$+N_r^{-1}(\alpha_1,\alpha_2)\langle\ \omega_r(\alpha_1,\alpha_2)+\varepsilon_{53r}iR_r[\ G_{3r}(\alpha_1,\alpha_2)-W_r\ ]\ \rangle=\varepsilon_6^{-1}K(\alpha_1,\alpha_2)G_3(\alpha_1,\alpha_2).$$

Taking into account that $G_3(\alpha_1,\alpha_2)=-G_{3\lambda}(\alpha_1,\alpha_2)-G_{3r}(\alpha_1,\alpha_2)$, and introducing the notation

$G_{3\lambda}(\alpha_1,\alpha_2)=G^-(\alpha_1,\alpha_2)$, $G_{3r}(\alpha_1,\alpha_2)=G^+(\alpha_1,\alpha_2)$, we obtain the following functional Wiener-Hopf type equations for identifying contact stresses for two cases: $\theta>0$; $\theta=0$ when $U_{3\theta\lambda}=0$ in the form:

$$M_{r\lambda}(\alpha_1,\alpha_2)G^+(\alpha_1,\alpha_2)=G^-(\alpha_1,\alpha_2)+T_{r\lambda}(\alpha_1,\alpha_2)+U_{3\theta\lambda}.$$

Functions $G^+(\alpha_1,\alpha_2)$, $G^-(\alpha_1,\alpha_2)$ are regular in the parameter α_2 in the upper and lower half-planes, respectively. Here the notation is as follows:

$$-[\ N_\lambda^{-1}(\alpha_1,\alpha_2)\varepsilon_{53\lambda} iR_\lambda + \varepsilon_6^{-1} K(\alpha_1,\alpha_2)\ ] = M_\lambda(\alpha_1,\alpha_2),$$

$$[\ N_r^{-1}(\alpha_1,\alpha_2)\varepsilon_{53r} iR_r + \varepsilon_6^{-1} K(\alpha_1,\alpha_2)\ ] = M_r(\alpha_1,\alpha_2)$$

$$N_\lambda^{-1}(\alpha_1,\alpha_2)\omega_\lambda(\alpha_1,\alpha_2) - N_\lambda^{-1}(\alpha_1,\alpha_2)\varepsilon_{53\lambda} iR_\lambda W_\lambda = T_\lambda(\alpha_1,\alpha_2)$$

$$N_r^{-1}(\alpha_1,\alpha_2)\omega_r(\alpha_1,\alpha_2) - N_r^{-1}(\alpha_1,\alpha_2)\varepsilon_{53r} iR_r W_r = T_r(\alpha_1,\alpha_2)$$

$$M_{r\lambda}(\alpha_1,\alpha_2) = M_r(\alpha_1,\alpha_2) M_\lambda^{-1}(\alpha_1,\alpha_2),\quad U_{3\theta\lambda} = U_{3\theta}$$

$$T_{r\lambda}(\alpha_1,\alpha_2) = [\ T_\lambda(\alpha_1,\alpha_2) - T_r(\alpha_1,\alpha_2)\ ] M_\lambda^{-1}(\alpha_1,\alpha_2.$$

Further analysis of the obtained functional equations and their solution is described in detail in [12].

The analysis of the obtained functional equations, which are more complex than in the case of the absence of a liquid layer, showed that, nevertheless, the features of the stress-strain state of the material are preserved in the presence of a liquid. The behavior of the contact stresses in the zone of convergence of the plates is described by the functions given below. When studying the solution of the first equation, $\theta > 0$, there are the following properties of contact stresses between the plates and the base:

$$g_{3\lambda}(x_1,x_2) = \sigma_{1\lambda}(x_1,x_2)(-x_2-\theta)^{-1/2},\quad x_2 < -\theta,$$

$$g_{3r}(x_1,x_2) = \sigma_{1r}(x_1,x_2)(x_2-\theta)^{-1/2},\quad x_2 > \theta.$$

When $\theta = 0$ - there are also solution properties similar to those found when there is no liquid layer.

$$g_{3\lambda}(x_1,x_2) \to \sigma_{2\lambda}(x_1,x_2) x_2^{-1},$$

$$g_{3r}(x_1,x_2) \to \sigma_{2r}(x_1,x_2) x_2^{-1}.$$

Conclusions

Thus, the presence of corrosive liquids capable of destroying the coating can lead to disastrous consequences if this destruction leads to the formation of hidden defects. Their appearance can be facilitated by bending deformations of the surface of a coated structure in combination with the effect of corrosive liquid. At the same time, preliminary studies have shown that, depending on the type of action on the liquid layer, conditions may be formed that reduce the value of coefficients of singularities describing concentration of contact stresses. However, these studies are quite complex and are scheduled to be performed in the future.

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