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NONLINEAR DYNAMICS OF SERRATED DEFORMATION OF METALS AT LOW TEMPERATURES

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Abstract. The mode of the serrated deformation is investigated mathematically in framework of the thermoactivation model of unstable plastic deformation in metals at low temperatures. The obtained solutions in the framework of known models, however, do not fully correspond to experimental situation, as the stress oscillations in reality are usually irregular and inhomogeneous. By this reason, additionally, the elastic correlations of neighboring deformation zones were taken into account in the model. Numerical analysis model showed that time dependence of the stress, temperature and plastic deformation rate acquires an irregular stochastic character and is represented as an attractive set of trajectories in the phase space of these variables – "strange attractor".

Keywords: thermomechanical instability, serrated plastic deformation, low temperatures, relaxation self-oscillations

1. Introduction

Nowadays unstable character of plastic deformation is considered as a universal property of solid bodies, which is able to manifest itself in a wide range of temperatures [1]. In crystalline materials it is caused by self-consistent collective movement of dislocations under the action of external and internal stresses [2,3]. At helium temperatures, the instability of plastic deformation (the serrated deformation) has been found in a large number of materials and it is typical at the very low temperature region [4,5,6]. Its character is determined by numerous parameters related to both the deformation conditions (deformation rate, temperature) and the properties of the material itself (lattice type, grain size, etc.).

Several hypotheses [7,8] have been proposed to explain the physical nature of the serrated deformation. To date, the hypothesis explaining the most number of experimental facts is the hypothesis of the thermal nature of jumps [7]. For example, it was established [9] that in the range of helium temperatures and the plastic deformation rates $\dot{\varepsilon} = 10^{-2} - 10^{-5}$ each stress jump was accompanied by an almost delta-like temperature rise to a value of about 50 K, and the greater the amplitude of the jump, the higher the temperature.

The most rigorous criterion of thermomechanical instability of deformation at low temperatures was formulated in [8]. Despite the strictness of the criterion in [8] a description of serrated deformation itself and dynamics of its origin were not completely investigated in this work and a number of others [10]. These problems require further investigation. In the present paper, the instability of the plastic flow will be considered on the basis of the analysis of the nonlinear dynamics of the serrated deformation within framework of the autowave model.

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2. Traditional model

First we examine a model, frequently used to establish the instability criterion of plastic deformation under thermoactive slip of dislocations [8,11,12]. In thus case, for sufficiently thin metal samples ($R << L$, where R is the radius of the cylindrical sample, L – the length of the sample), the deformation and thermal conductivity processes (inhomogeneous along the cylinder axis) can be described by the following system of equations [11,12]

$$c\frac{\partial T}{\partial t} = \kappa\frac{\partial^2 T}{\partial x^2} - \frac{2h}{R}(T - T_0) + \sigma\dot{\varepsilon}. \tag{1}$$

$$\frac{\partial \varepsilon(t)}{\partial t} = \dot{\varepsilon}, \qquad \dot{\varepsilon} = \nu \exp[-W/k_B T], \tag{2}$$

$$\frac{\partial \sigma}{\partial t} = G^*\left(\dot{\varepsilon}_0 - \frac{1}{L}\int_0^L \dot{\varepsilon}(x,t)dx\right). \tag{3}$$

Here, the equation (19) is the equation of heat conduction where T is temperature of the metal sample, κ is the coefficient of thermal conductivity, T_0 is the temperature of environment, h is the heat transfer coefficient, σ is the external stress, c is the specific heat of the sample, which, as in [8], for simplicity of analysis is considered as a constant.

The thermoactive mode of plastic deformation is characterized by the equation (20), where $\dot{\varepsilon}$ is the local plastic deformation rate in the deformation zones, ε is the plastic deformation value, W is the activation energy, ν is pre-exponential factor, k_B is the Boltzmann constant.

The equation (21) determines the dynamics of the system "sample – loading device" at a given constant plastic deformation rate $\dot{\varepsilon}_0$, where $G^* = k_m L/S$ is an effective module system "sample–machine", k_m is the rigidity of the machine, the S is the cross-sectional area of the sample.

The low-temperature serrated deformation is similar to mechanical relaxation self-oscillations in its external manifestations. A necessary condition for the occurrence of relaxation oscillations in a mechanical system is the presence of an interval of negative rate sensitivity of the friction force. In the case of plastic deformation, this is equivalent to the negative dependence of the deformation stress on the temperature or the deformation rate. Experimental investigations of the dependence of the deforming stress on the deformation rate for many metals and alloys have shown that the rate sensitivity decreases, when temperature decreases,and at helium temperatures becomes negative [7].

A dynamical system described by the equations (1)–(3) refers to the class of self-oscillating systems in the case when the dependence

$$\sigma = \left(\frac{2h}{\nu R}\right)(T - T_0)\exp\left[\frac{W}{kT}\right], \tag{4}$$

which is obtained if one equates the righthand part of equation (19) to zero, has a descending part, i.e. the inequality $\partial\sigma/\partial T < 0$ holds in some temperature range and, consequently, $\partial\sigma/\partial\dot{\varepsilon} = (\partial\sigma/\partial T)(\partial T/\partial\dot{\varepsilon}) < 0$ (as it follows from (20) always $\partial\dot{\varepsilon}/\partial T > 0$).

From the condition $\partial\sigma/\partial T = 0$, we have

$$\left(\frac{T}{T_0}\right)^2 - \frac{W}{kT_0}\left(\frac{T}{T_0}\right) = 0, \tag{5}$$

where we find the roots of equation (5)

$$T_{1,2} = T_0\left[\frac{\alpha}{2} \pm \sqrt{\left(\frac{\alpha}{2} - 1\right)^2 - 1}\right], \tag{6}$$

where $\alpha = W/k_B T_0$ — parameter normalized activation energy.

The coincidence of roots in (6) occurs at $\alpha = 4$, which corresponds to the inflection point on the curve $\sigma = \sigma(T)$.

Thus, if

$$\alpha > \alpha_c = 4 \tag{7}$$

then N – shaped dependence $\sigma(T)$ takes place with the negative dependence of the deforming stress on temperature (Fig. 1a), furthermore the critical stress at the inflection point is related to the temperature by the equality

$$\sigma_c = \sigma(2T_0). \tag{8}$$

Since the rate of plastic deformation $\dot{\varepsilon}$ monotonically depends on the temperature T, the same N – shaped dependence takes place for $\sigma = \sigma(\dot{\varepsilon})$ (Fig. 1b).

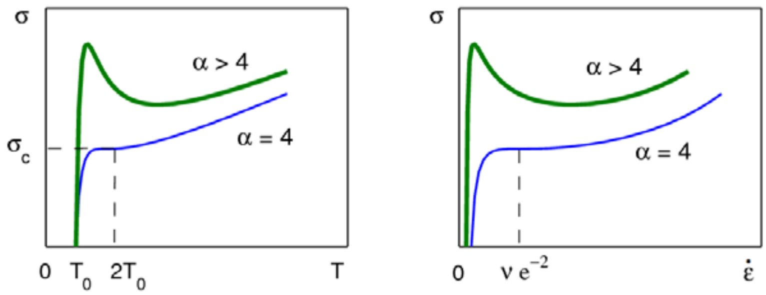

Fig. 1. Dependence of $\sigma = \sigma(T)$ and $\sigma = \sigma(\dot{\varepsilon})$ for different values of parameter $\alpha = W/k_B T$

For further analysis we write system (1)–(3) in a dimensionless form, introducing the dimensionless variables

$$u = \frac{\dot{\varepsilon}}{\nu}, \qquad \theta = \frac{T}{T_0}, \qquad \tau = \frac{\sigma}{G^*}, \tag{9}$$

and time $t' = t\nu$. As a result we have

$$\frac{d\theta}{dt'} = K\frac{\partial^2\theta}{\partial x'^2} + \mu u\tau - \beta(\theta - 1), \tag{10}$$

$$\frac{d\varepsilon}{dt'} = u, \quad u = \exp\left[-\frac{\alpha}{\theta}\right], \tag{11}$$

$$\frac{d\tau}{dt'} = u_0 - \int_0^1 u\,dx', \tag{12}$$

where we introduced the dimensionless parameters $\mu = G^*/cT_0$, $\beta = 2h/c\nu R$ $u_0 = \dot{\varepsilon}_0/\nu$, $K = \kappa/cL^2\nu$.

It can be shown that the instability condition reduced to the condition $\alpha > 4$ and is a necessary condition, but not sufficient one. A sufficient condition is that values of the quantity $\theta = \alpha/\ln(\nu/\dot{\varepsilon}_0)$ belong to the interval where $\partial\sigma/\partial\theta < 0$, or values of the deformation rate $\dot{\varepsilon}_0$ belong to the interval where $\partial\sigma/\partial\dot{\varepsilon} < 0$.

We analyze the stability of the stationary homogeneous solution

$$u = u_0, \quad \theta_0 = \alpha/\ln u_o, \quad \tau = \left(\frac{\beta}{\mu}\right)(\theta_0 - 1)/u_0 \tag{13}$$

to the system of equations (10)–(12). Let us linearize this system with respect to (13) and assuming $u, \theta, \tau \propto \exp(\lambda t + ikx)$, we obtain the dispersion equation

$$\lambda^2+(k^2K+\beta-\mu\tau_0 u_{\theta'})\lambda+\mu u_{\theta'}\delta(k)=0, \tag{14}$$

where $u_{\theta'}=\alpha u_0/\theta_0^2$, $\delta(k)$ is the Dirac delta-function.

The instability in the system of equations (10)–(12) occurs if at least one of the roots of the dispersion equation satisfies the condition $Re\{\lambda(k)\}>0$, which is achieved when

$$\tau>\tau_c=\frac{\beta\theta_0^2}{\mu\alpha u_0}\left(1+\frac{k^2K}{\beta}\right). \tag{15}$$

Let us now consider the properties of solutions of the system of equations (10)–(12). Investigating this system numerically, we find that at $\alpha<4$ the plastic flow regime is monotonous, and at $\alpha>4$ homogeneous regular oscillations of the stress, temperature and deformation appear corresponding to the limit cycle on the phase plane $\sigma-T$ (Fig. 2a). Also, if the deforming stress has a sawtooth-like dependence on time (Fig. 2b) then the dependence of temperature looks like regular set of peaks (Fig. 2c) and deformation as step-like curve (Fig. 2d).

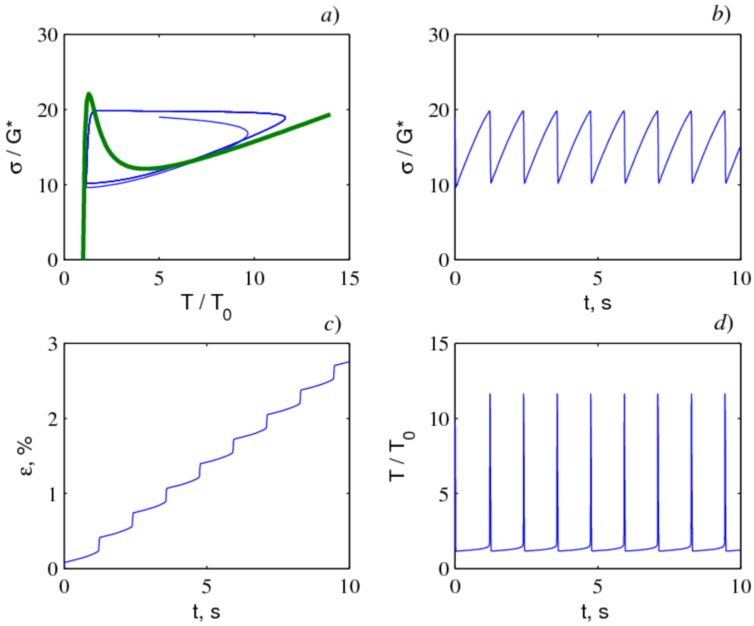

Fig. 2. Phase portrait of (a) system (10)–(12) with $\alpha=5.8$, when isocline $T=T(\dot{\varepsilon}_0)$ is located on the descending part of the branch of isocline $\sigma=\sigma(T)$ (it corresponds to equation (4)) and the dependence of the applied stress σ, temperature T and deformation ε on time t (b,c,d). The graphs correspond to the following values of parameters: $\mu=0.5$, $\beta=1$, $u_0=0.015$, $K=0.01$.

Here we add two remarks. Firstly, because of variations of the system variables are uniform ($k=0$), then the condition (15) takes the form

$$\tau>\tau_c=\frac{\beta\theta_0^2}{\mu\alpha u_0}. \tag{16}$$

Although, at the inflection point $\tau=\tau_c$ it holds $\theta_0=2$ and, correspondingly, at this point we have $\tau_0=\beta/\mu u_0$. Then the condition (16) can also be written as

$$\alpha=W/k_BT>4, \tag{17}$$

which coincides with the above condition (7).

Secondly, it is not difficult to show that the instability condition (16) for the initial dimensional variables takes the form

$$\sigma > \sigma_c = \frac{2hk_B T_0^2}{R\dot{\varepsilon}_0 W}, \tag{18}$$

which coincides with the instability condition obtained in [8] under isothermal boundary conditions.

3. Effect of correlation of the elastic deformation zones

The obtained solutions in framework of the model (1)–(3), however, does not fully meet the experimental situation, since, in reality, fluctuations of the load are usually not regular, but quasistatical and non uniform character.

To describe this behavior, we need to consider the effect of elastic correlation of neighboring deformation regions (for example, slip bands) [13]. As a result of this effect, elastic perturbations $\sigma_i \sim \partial^2_{xx}\varepsilon$ appear, which create stress inhomogeneities along the sample axis.

Then the original system of equations for the correlation of deformation zones takes the form

$$c\frac{\partial T}{\partial t} = \kappa\frac{\partial^2 T}{\partial x^2} - \frac{2h}{R}(T - T_0) + (\sigma + \sigma_i)\dot{\varepsilon}. \tag{19}$$

$$\frac{\partial \varepsilon(t)}{\partial t} = \dot{\varepsilon}, \qquad \dot{\varepsilon} = \nu \exp[-W/k_B T], \tag{20}$$

$$\frac{\partial \sigma}{\partial t} = G^*\left(\dot{\varepsilon}_0 - \frac{1}{L}\int_0^L \dot{\varepsilon}(x,t)dx\right). \tag{21}$$

$$\frac{\partial \sigma_i}{\partial t} = -\frac{\sigma_i}{t_a} + \gamma_1\frac{\partial^2 \dot{\varepsilon}}{\partial x^2}, \tag{22}$$

where the parameter $\gamma_1 \approx Gd^2$ is a measure of the elastic correlation of adjacent deformation zones, d is their characteristic width, and t_a is a characteristic relaxation time of elastic perturbations.

Numerical study of the system (19)–(22) was carried out after reducing it to a dimensionless form. Analysis of the obtained solutions shows that the irregular dynamics of the system is mainly controlled by the elastic correlation parameter $S = Gd^2/G^*L^2$ and it occurs when the parameter exceeds a certain critical value $(S > S_c = 0.045)$. The length of the sample was chosen as L=3cm and the width of the deformation zones $d = 10\mu$m.

For small values of S a regular dynamics is implemented in the system, similar to (Fig. 2). As the elastic correlation parameter increases $(S > S_c = 0.045)$, the mode of changes of the stress, temperature, and plastic deformation rate becomes irregular (Fig. 3).

This mode of plastic deformation and temperature changes can be observed more distinctly in the phase portrait of the variables σ and T (Fig. 3*a*), where phase trajectories behave in an unstable manner in some bounded domain. In this case, the magnitude of the load drops-off distinctly correlates with the magnitude of temperature peaks in the deformation zones, which has been observed experimentally [6]. Irregular behavior of variables σ, T and $\dot{\varepsilon}$ represents in the phase space of these variables an attractive set of trajectories – "strange attractor" [15], on which all trajectories belonging to it are unstable (Fig. 4). This behavior is typical for stochastic systems[16]. When the plastic accommodation parameter increases ($\gamma > 0.01$), the oscillatory process again becomes regular.

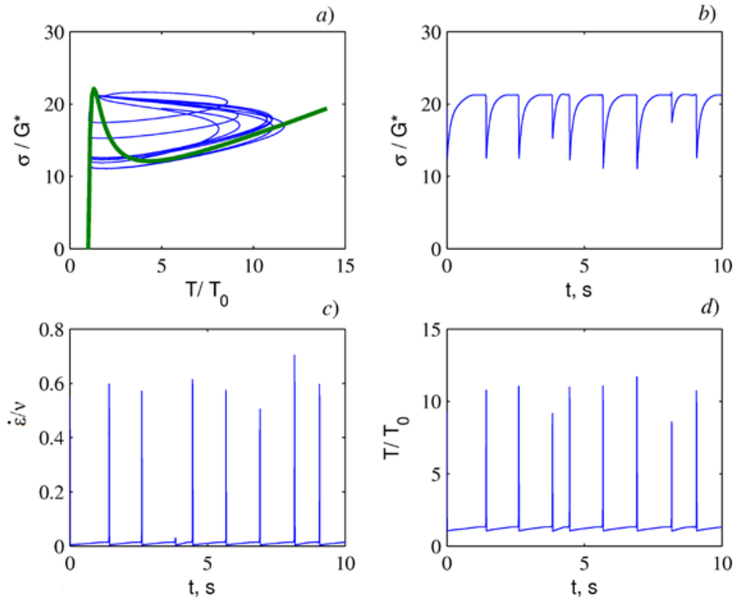

Fig. 3. Phase portrait of the system (19)–(22) for $\alpha = 5.8$ on the plane of variables $\sigma - T$ (*a*) and the dependence of the applied stress σ, strain rate $\dot{\varepsilon}$, and temperature T on time t (*b*,*c*,*d*). The graphics are obtained for the parameter values: $\mu = 0.5$, $\beta = 1$, $u_0 = 0.015$, $K = 0.01$, $S = 0.05$, $\gamma = \nu/t_a = 0.01$.

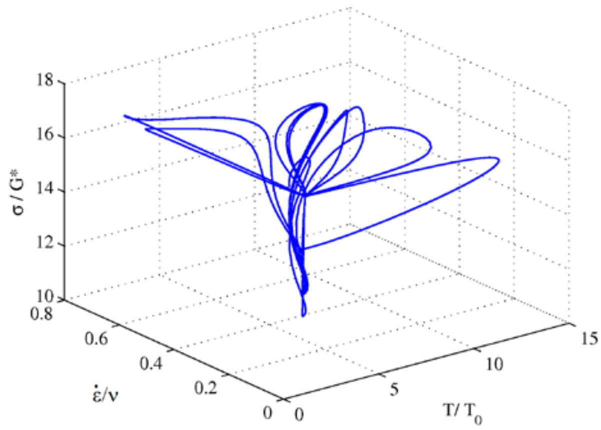

Fig. 4. The phase space trajectories of a system described by equations (19)–(22) form a "strange attractor"

4. Conclusion

1. The mode of the serrated deformation is investigated mathematically in framework of the known thermoactivation model of unstable plastic deformation in metals at low temperatures. Using analytical and numerical analysis of the model, it is found that under certain conditions the plastic flow mode becomes unstable and uniform regular oscillations of the stress, temperature and deformation appear, forming a limiting cycle on the phase plane $\sigma - T$. If deforming stress has sawtooth-like dependency on time, then the temperature behaves as regular sequences of peaks and deformation as step-like curve.

2. The obtained the previous model solutions however, do not fully correspond to experimental situation, as the load fluctuations in reality are usually irregular and non uniform. By this reason, a modified model was proposed, in which, additionally, the elastic correlations of neighboring deformation zones were taken into account. As a consequence, the elastic perturbations $\sigma_i \sim \partial^2_{xx}\varepsilon$ appear generating the stress inhomogeneities along the axis of

the sample. Numerical study of the generalized model showed that the irregular dynamics of the system is controlled by the elastic correlation parameter $S = Gd^2/G^*L^2$ and it occurs when the parameter exceeds a certain critical value $(S > S_c)$. In this case, the behavior in time of the stress, temperature, and plastic deformation rate becomes an irregular stochastic and it is represented in the phase space of these variables as an attractive set of trajectories – "strange attractor". Although, magnitude of the sress drops-off distinctly correlates with magnitude of the temperature peaks in deformation zones, which has been observed experimentally several times.

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