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DISPERSION AND THE EQUATIONS OF MECHANICS

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Abstract. This work is devoted to studying the influence of angular momentum variation in an elementary volume and cross flux on its sides on some equations of continuum mechanics. The self- diffusion effect is investigated for fluctuations of shock waves for the Mach number M≈1. As another example, a beam and a plate are considered.

1. Introduction

Many experimental facts tell us about the importance of gradients of physical values (density, linear momentum, energy). In the previous studies, the problem of dispersion influence on models and equations of continuum mechanics was considered carefully for various applications [1-6]. In those papers one can find also historical facts concerning different approaches to this problem, as well as some examples; in particular, modified Navier-Stokes equations, connection to kinetic theory, boundary layer, shock waves, numerical solutions, asymptotical methods, etc., so there is no need to repeat all the details and to say again on the importance of this question.

In the present investigation, some new examples are given.

2. Equations

Our previous results can be summarized for continuum mechanics as follows: in the phenomenological theory we have four equilibrium equations, but if we choose only the equilibrium conditions for forces, we obtain only three equilibrium equations and symmetric stress tensor; as a consequence, the following interpretation being based on the traditional theory. The degree of asymmetric stress tensor can be received from the momentum equation (in projections). The corresponding formulas for the nonsymmetric tensor are given in [5]. The equations are classic for gas, fluid and solid. As a result, we have another interpretation instead of force equilibrium interpretation. From the modified Boltzmann equation, we received the modified Navier-Stokes equations as well as the equation for angular momentum, which can not be obtained from the classic Boltzmann equation [4-6].

3. Examples

Shock wave. The formulas describing the classical structure of a normal shock wave, as well as the influence of self-diffusion, were given in [7]. It was shown that density diffusion leads to new equations, in particular, for Mach number M=1. Remember that Mach number is the ratio of the shock wave velocity to the sound velocity. Consider the influence of self-diffusion (only of density). Usually in a shock wave we have growth of density. For M≈1 velocity can change and the shock wave can modify to a rarefaction wave. So we can tell about unstable shock waves for some special parameters of flows. After density decrease in the shock wave the process goes into opposite direction. For pressure we have similar process.

If gas is a mixture and contains light components, we can have tongues of light components at two side of the shock wave.

Infinite plate. The Blasius problem was considered in [8] by numerically and analytically. Some results for an infinite plate are formulated here. For this case the equation is (the second term is new and therefore allocated)

$$\frac{d}{dy}\left(\mu\frac{du}{dy}\right)+\frac{d}{dy}\left(\mu\, y\frac{d^2u}{dy^2}\right)=0.$$

The boundary conditions are

$$y=0 \quad u=0,\quad \mu\frac{du}{dy}=\tau_w,\qquad y\to\infty,\quad u=U_\infty$$

Here u is velocity, μ is viscosity, index "w" relates to a surface, and index"∞" indicates an outer boundary; τ_w being the surface friction.
Rewrite the equation as

$$y\frac{d^2u}{dy^2}+\frac{du}{dy}=C_1 \qquad \text{or} \qquad \frac{d}{dy}\left(y\frac{du}{dy}\right)=C_1.$$

Integrating gives

$$y\frac{du}{dy}=C_1y+C_2 \qquad \text{and} \qquad u=C_1y+C_2\ln y+C_3.$$

From the boundary condition $\frac{du}{dy}=\tau_w/\mu$ we have $C_1=\tau_w/\mu$, therefore

$$u=(\tau_w/\mu)y+C_2\ln y+C_3$$

As a result, the task is reduced to the problem of boundary layer [3,8]. In this case it is possible to satisfy the boundary condition, i.e. to have the logarithmic term equal to zero, if to take

$$y=\frac{v}{v_w},\quad v=\frac{\mu}{\rho},\quad v_w=\left(\frac{\tau_w}{\rho_w}\right)^{1/2},$$

where ρ is the density.

On decreasing the velocity up to zero, the derivative can be very large but zero velocity observes between the surface and the layer y, i.e. the layer of rest liquid is formed; the thickness of this layer being 10^{-3} cm. However, we have no reliable measurements there. Probably for a laminar layer there is no layer with zero velocity. Near the edge the gradient of the velocity tends to work. It works near the rebuilding region too. Far from the edge, the friction tends to zero. It does not follow from the theory for a semi-infinite plate that the friction value is finite, but if we suggest zero friction in the first integral, we can get the

Karman formula for the mixture length. The equality $\tau_w = 0$ provides $u = 0$ at $y = 0$ and $u = U_\infty$ as $y \to \infty$ and leads to rebuilding of the flow. The velocity profile becomes more completed than near the edge. The region with $\tau_w = 0$ formulates an inertial layer (N.A. Kolmogorov). This case relies to a logarithmic profile for the boundary layer instead of a linear profile in the classic case. It is interesting that asymptotic friction for a half-infinite plate is not the same as for an infinite plate. In my opinion, we have similar situation for tubes.

Beam. Consider the simplest task of a beam. Usual equations which define equilibrium are as follows (Fig. 1)

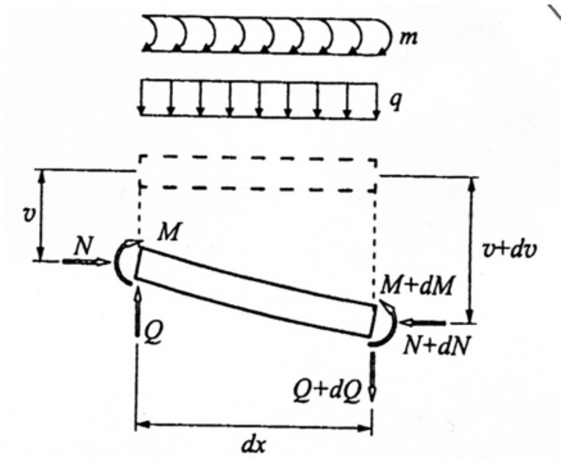

Fig. 1. Elementary unit of a beam considered.

$$Q' + q = 0.$$

$$M' - Q - m - Nv' = 0,$$

$$-M'' = q - m' - (Nv')'.$$

Taking into consideration the angular momentum it will be would

$$-M'' = q - m' - ((Nv)')'.$$

For this task, the solution is in general case and without N', i.e. for N=Const) the same. Consequently we can choose the beam of variable cross-section. For a beam on elastic foundation the results are different. In the case of transmission pressure on the beam by intermediate designs, an external force P (Fig. 2) passes through a hard lever of length a, which can twist around hinge.

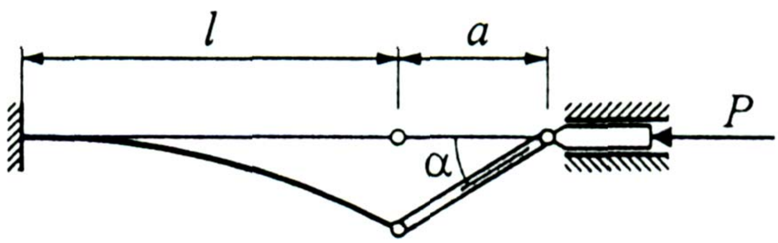

Fig. 2.

The normal force H in the hard hinge is

$$H = \frac{P}{\cos\alpha}.$$

At the end of elastic beam the force is $Q=H \sin \alpha$. As $\tan \alpha = v(l)/a$ at l, we have in classic case

$$Q = H \sin\alpha = N \tan\alpha$$

$$EIv'''(l) + N\left[v'(l) + \frac{v(l)}{a}\right] = 0.$$

So we have at the end

$$v(0) = 0 \Rightarrow C_1 + C_4 = 0$$

$$v'(0) = 0 \Rightarrow C_2 + kC_3 = 0$$

$$M(l) = 0 \Rightarrow v''(l) = 0 \Rightarrow C_3 \sin v + C_4 \cos v = 0.$$

$$v'''(l) + k^2 v'(l) + k^2 v(l)/a = 0 \Rightarrow C_1 + C_2(l+a) + C_3 \sin v + C_4 \cos v = 0.$$

Now we have the new equation

$$EIv'''(l) + (Nv)' + \frac{P}{\cos\alpha}\frac{v(l)}{Q} = 0.$$

4. Conclusion

We have discuss the problems that can be appearing when considering the angular momentum variation in an elementary volume near the surface, as well as its influence on the cross flows through the sides of an elementary volume for great gradients of the physical values. The unstable shock wave for M≈1 is also discussed.

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