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DAMPING OF GENERALIZED THERMO ELASTIC WAVES IN A HOMOGENEOUS ISOTROPIC PLATE

R. Selvamani[1*], P. Ponnusamy[2]

[1]Department of Mathematics, Karunya University, Coimbatore, Tamil Nadu, India

[2]Department of Mathematics, Government Arts College (Autonomous), Coimbatore, Tamil Nadu, India

*e-mail: selvam1729@gmail.com

Abstract. In this paper, the damping of generalized thermo elastic waves in a homogeneous isotropic plate is studied based on generalized two dimensional theory of thermo elasticity. Two displacement potential functions are introduced to uncouple the equations of motion. The frequency equations are obtained by the traction free boundary conditions using the Bessel function solutions. The numerical calculations are carried out for the material Zinc and the computed non-dimensional thermo elastic damping factor is plotted as the dispersion curves for the plate with thermally insulated and isothermal boundaries.

1. Introduction

Cylindrical thin plate plays a vital role in many engineering fields such as aerospace, civil, chemical, mechanical, naval and nuclear engineering. The analysis of thermally induced wave propagation of a cylindrical plate is a problem that may be encountered in the design of structures such as atomic reactors, steam turbines, submarine structures subjected to wave loadings, jets and other devices operating at elevated temperatures. Moreover, it is recognized that the thermal effects on the elastic wave propagation supported may have implications related to many seismological applications. This study can be potentially used in applications involving nondestructive testing (NDT), and qualitative nondestructive evaluation (QNDE).

The generalized theory of thermo elasticity was developed by Lord and Schulman [1], which involves one relaxation time for isotropic homogeneous media, and is called the first generalization to the coupled theory of elasticity. Their equations determine the finite speed of wave propagation of heat and the displacement distributions. The corresponding equations for an isotropic case were obtained by Dhaliwal and Sherief [2]. The second generalization to the coupled theory of elasticity is known as the theory of thermo elasticity with two relaxation times, or as the theory of temperature-dependent thermoelectricity. A generalization of this inequality was proposed by Green and Laws [3]. Green and Lindsay [4] obtained an explicit version of the constitutive equations. These equations were also obtained independently by Suhubi [5]. This theory contains two constants that act as the relaxation times and modifies not only the heat equations, but also all the equations of the coupled theory. The classical Fourier's law of heat conduction is not violated if the medium under consideration has a center of symmetry. Erbay and Suhubi [6] studied the longitudinal wave propagation in a generalized thermoplastic infinite cylinder and obtained the dispersion relation for the cylinder with a constant surface temperature. Ponnusamy [7] has studied wave propagations in a generalized thermo elastic solid cylinder of arbitrary cross sections using the Fourier expansion collocation method. Later, Ponnusamy and Selvamani [8] obtained mathematical modeling and analysis for a thermo elastic cylindrical panel using the wave propagation

function. In addition, we can replace $k=1$ for the L-S theory and $k=2$ for the G-L theory. The thermal relaxation times τ_0 and τ_1 satisfies the inequalities $\tau_0 \geq \tau_1 \geq 0$ for the G-L theory only.

The strain e_{ij} are related to the displacements as given by

$$e_{rr} = u_{,r}\,, \qquad e_{\theta\theta} = r^{-1}(u+v_{,\theta})\,, \qquad e_{r\theta} = v_{,r} - r^{-1}(u-v_{,\theta})\,, \tag{3}$$

in which u and v are the displacement components along the radial and circumferential directions, respectively. σ_{rr}, $\sigma_{\theta\theta}$ are the normal stress components and $\sigma_{r\theta}, \sigma_{\theta z}, \sigma_{zr}$ the shear stress components, e_{rr}, $e_{\theta\theta}$, e_{zz} the normal strain components, and $e_{r\theta}, e_{\theta z}, e_{zr}$ the shear strain components.

By substituting Eqs. (3) and (2) into Eqs. (1), the following displacement equations of motions are obtained

$$\begin{aligned}&(\lambda+2\mu)\left(u_{,rr}+r^{-1}u_{,r}-r^{-2}u\right)+\mu r^{-2}u_{,\theta\theta}+r^{-1}(\lambda+\mu)v_{,r\theta}\\&+r^{-2}(\lambda+3\mu)v_{,\theta}-\beta\left(T_{,r}+T\delta_{2k}\tau_1 T_{,rt}\right)=\rho u_{,tt}\quad,\end{aligned}$$

$$\begin{aligned}&\mu\left(v_{,rr}+r^{-1}v_{,r}-r^{-2}v\right)+r^{-2}(\lambda+2\mu)v_{,\theta\theta}+r^{-2}(\lambda+3\mu)u_{,\theta}\\&+r^{-1}(\lambda+\mu)u_{,r\theta}-\beta\left(T_{\theta}+\eta T_{,\theta t}\right)=\rho v_{,tt}\quad,\end{aligned}$$

$$k\left(T_{,rr}+r^{-1}T_{,r}+r^{-2}T_{,\theta\theta}\right)-\rho c_v\left(T+\tau_0 T_{,tt}\right)=\beta T_0\left(\frac{\partial}{\partial t}+\tau_0\delta_{1k}\frac{\partial^2}{\partial t^2}\right)\left[u_{,r}+r^{-1}(u+v_{,\theta})\right]. \tag{4}$$

The above coupled partial differential equations are also subjected to the following non-dimensional boundary conditions at the surfaces $r=a,\, b$

(i) Stress free boundary (Free edge)

$$\sigma_{rr} = \sigma_{r\theta} = 0 \quad , \tag{5a}$$

(ii) Rigidly fixed boundary (Clamped edge)

$$u = v = 0 \quad , \tag{5b}$$

(iii) Thermal boundary

$$T_{,r} + hT = 0\,, \tag{5c}$$

where h is the surface heat transfer coefficient. Here $h \to 0$ corresponds to a thermally insulated surface and $h \to \infty$ refers to an isothermal one.

2.1. Lord-Schulman (L-S) theory

Based on the Lord-Schulman theory of thermo elasticity, the three dimensional rate dependent temperature with one relaxation time is obtained by replacing k=1 in the heat conduction equation of Eq. (1), namely,

$$(\mu)\left(v_{,rr}+r^{-1}v_{,r}-r^{-2}v\right)+r^{-1}(\lambda+\mu)u_{,r\theta}+r^{-2}(\lambda+3\mu)u_{,\theta}+r^{-2}(\lambda+3\mu)v_{,\theta}$$
$$+r^{-2}(\lambda+2\mu)v_{,\theta\theta}-\beta\left(T_{,\theta}+\tau_1 T_{,\theta t}\right)=\rho v_{,tt} \quad , \tag{7c}$$

where the symbols and notations have been defined in the previous sections. In view of available experimental evidence in favor of the finiteness of heat propagation speeds, the generalized thermo elasticity theories are considered to be more realistic than the conventional theory in dealing with practical problems involving very large heat fluxes and/or short time intervals, such as those occurring in laser units and energy channels.

To uncouple Eqs. (7), the mechanical displacement u, v along the radial and circumferential directions given by Sharma [9] are adopted as follows:

$$u=\phi_{,r}+r^{-1}\psi_{,\theta}, \qquad v=r^{-1}\phi_{,\theta}-\psi_{,r} \tag{8}$$

Substituting Eqs. (8) into Eqs. (7) yields the following second order partial differential equation with constant coefficients:

$$\left\{(\lambda+2\mu)\nabla^2+\rho\omega^2\right\}\phi-\beta\left(T+\delta_{2k}\tau_1 T_{,t}\right)=0, \tag{9a}$$

$$\left\{k\nabla^2-\rho C_\upsilon i\omega\eta_0\right\}T+\beta T_0\left(i\omega\eta_1\right)\nabla^2\phi=0, \tag{9b}$$

$$\left(\nabla^2+\frac{\rho}{\mu}\omega^2\right)\psi=0, \tag{9c}$$

where $\nabla^2 \equiv \partial^2/\partial x^2+x^{-1}\,\partial/\partial x+x^{-2}\,\partial^2/\partial\theta^2$.

3. Solutions to the Problem

The Eqs. (9) are coupled partial differential equations with two displacements and heat conduction components. To uncouple these equations, we assume the vibration and displacements along the axial direction z to be zero. Hence, the solutions of Eqs. (9) can be presented in the following form:

$$u(r,\theta,t)=\overline{\phi}(r)\exp\left\{i(p\theta-\omega t)\right\}, \tag{10a}$$

$$v(r,\theta,t)=\overline{\psi}(r)\exp\left\{i(p\theta-\omega t)\right\}, \tag{10b}$$

$$T(r,\theta,t)=\left(\lambda+2\mu/\beta a^2\right)\overline{T}(r)\exp\left\{i(p\theta-\omega t)\right\}, \tag{10c}$$

where $i=\sqrt{-1}$, ω is the angular frequency, p is the angular wave number, $\phi(r,\theta), \psi(r,\theta)$, $T(r,\theta)$ are the displacement potentials. Substituting Eqs. (10) into Eqs. (9) and introducing the dimensionless quantities such as $x=r/a$, $c_1^2=(\lambda+2\mu)/\rho$, $c_2^2=\mu/\rho$ $\Omega^2=\rho\omega^2 a^2/\mu$, $\overline{\lambda}=\lambda/\mu$ $\overline{d}=\rho c_v \mu/\beta T_0$,we can get the following partial differential equation with constant coefficients:

$$\overline{\psi} = \left[A_3 J_n\left(\alpha_3 ax\right) + B_3\left(\alpha_3 ax\right)\right], \tag{16}$$

where $\left(\alpha_3 a\right)^2 = \Omega^2$.

4. Frequency equations

In this section we shall derive the frequency equation for the two dimensional thermo elastic damping of the cylindrical plate subjected to stress free boundary conditions at the upper and lower surfaces at $r = a, b$. Substituting the expressions in Eqs. (1)-(3) into Eqs. (5), we can get the frequency equation for free vibration as follows:

$$\left|E_{ij}^1\right| = 0, \qquad i, j = 1, 2.....6 \tag{17}$$

$$E_{11}^1 = (2+\overline{\lambda})\Big((nJ_n(\alpha_1 ax) + (\alpha_1 ax)J_{n+1}(\alpha_1 ax)) - ((\alpha_1 ax)^2 R^2 - n^2)J_n(\alpha_1 ax)\Big)$$
$$+\overline{\lambda}\Big(n(n-1)\big(Jn(\alpha_1 ax) - (\alpha_1 ax)\, J_{\delta+1}(\alpha_1 ax)\big)\Big) - \beta T\,(i\omega)\eta_2 d_1\left(\alpha_1 ax\right)^2 ,$$

$$E_{13}^1 = (2+\overline{\lambda})\Big((nJ_n(\alpha_2 ax) + (\alpha_2 ax)J_{n+1}(\alpha_2 ax)) - ((\alpha_2 ax)^2 R^2 - n^2)J_n(\alpha_2 ax)\Big)$$
$$+\overline{\lambda}\Big(n(n-1)\big(Jn(\alpha_2 ax) - (\alpha_2 ax)\, J_{\delta+1}(\alpha_2 ax)\big)\Big) - \beta T\,(i\omega)\eta_2 d_2\left(\alpha_2 ax\right)^2 ,$$

$$E_{15}^1 = (2+\overline{\lambda})\big((n(n-1)J_n(\alpha_3 ax) - (\alpha_3 ax)J_{n+1}(\alpha_3 ax)\big) + \overline{\lambda}\left(n(n-1)J_n(\alpha_3 ax) - (\alpha_3 ax)J_{n+1}(\alpha_3 ax)\right),$$

$$E_{21}^1 = 2n(n-1)J_n(\alpha_1 ax) - 2n(\alpha_1 ax)J_{n+1}(\alpha_1 ax),$$

$$E_{23}^1 = 2n(n-1)J_n(\alpha_2 ax) - 2n(\alpha_2 ax)J_{n+1}(\alpha_2 ax),$$

$$E_{25}^1 = 2n(n-1)J_n(\alpha_3 ax) - 2\Big(\alpha_3 ax)J_{\delta+1}(\alpha_3 ax) + \Big((\alpha_3 ax)^2 - n^2\Big)J_n(\alpha_3 ax)\Big),$$

$$E_{31}^1 = d_1\left(nJ_n(\alpha_1 ax) - (\alpha_1 ax)J_{n+1}(\alpha_1 ax) + hJ_n(\alpha_1 ax)\right),$$

$$E_{33}^1 = d_2\left(nJ_n(\alpha_2 ax) - (\alpha_2 ax)J_{n+1}(\alpha_2 ax) + hJ_n(\alpha_2 ax)\right), \qquad E_{35}^1 = 0 .$$

Obviously

$E_{ij}\left(j = 2, 4, 6\right)$ can be obtained by just replacing the Bessel functions of the first kind in $E_{ij}\left(i = 1, 3, 5\right)$ with those of the second kind, respectively, while $E_{ij}\left(i = 4, 5, 6\right)$ can be obtained by just replacing a in $E_{ij}\left(i = 1, 2, 3\right)$ with b.

5. Numerical results and discussion

The damping of generalized thermo elastic waves in a simply supported homogenous isotropic cylindrical plate is numerically solved for the Zinc and the material properties of Zinc are given as follows:

$\rho = 7.14\times10^3\,\mathrm{kgm^{-3}}$, $T_0 = 296\,\mathrm{K}$, $K = 1.24\times10^2\,\mathrm{Wm^{-1}deg^{-1}}$, $\mu = 0.508\times10^{11}\,\mathrm{Nm^{-2}}$,

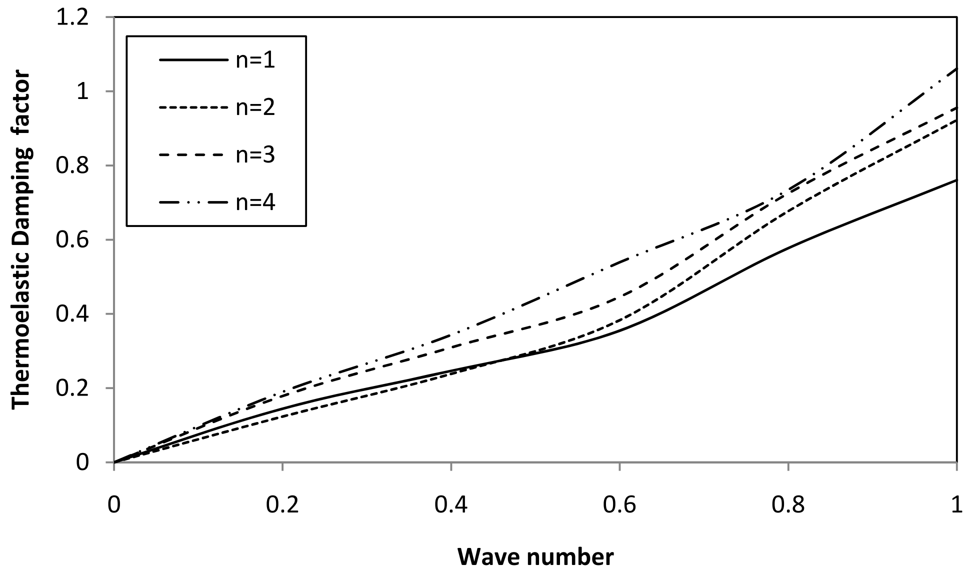

Fig. 1. Variation of thermo elastic damping factor of thermally insulated cylindrical plate with wave number.

In Figs. 1 and 2, the dispersion of thermo elastic damping factor with the wave number is studied for both the thermally insulated and isothermal boundaries of the cylindrical plate in different modes of vibration. From Fig. 2, it is observed that the damping factor increases exponentially with increasing wave number for thermally insulated modes of vibration. But smaller dispersion exist in the damping factor in the current range of wave numbers in Fig. 2 for the isothermal mode due to the combined effect of damping and insulation. . From Figs. 3 and 4, it is clear that the effects of stress free thermally insulated and isothermal boundaries of the plate are quite pertinent due to the combined effect of thermal relaxation times and mechanical field.

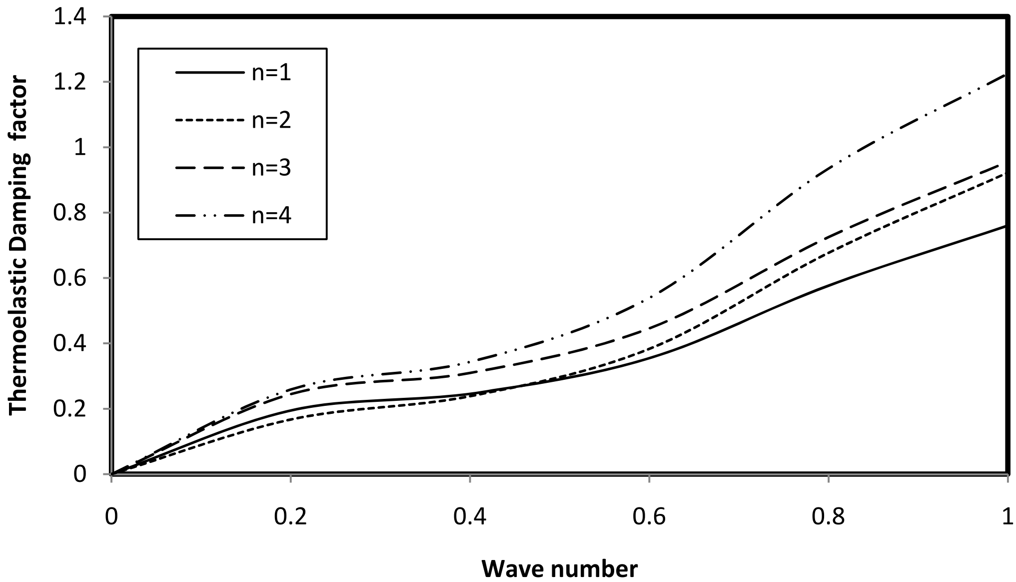

Fig. 2. Variation of thermo elastic damping factor of isothermal cylindrical plate with wave number.

Conclusion

The two dimensional damping of generalized thermo elastic waves in a homogeneous isotropic plate was investigated in this paper. For this problem, the governing equations of two dimensional linear theory of generalized thermo elasticity have been employed and solved by the Bessel function solutions with complex arguments. The effects of the thermo elastic damping factor with respect to the wave number of a Zinc cylindrical plate was investigated, with the results presented as the dispersion curves. In addition, a comparative study is made among the LS, GL and CT theories and the frequency change is observed to be highest for the LS theory, followed by the GL and CT theories due to the thermal relaxation effects and damping.

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