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DAMPING OF GENERALIZED THERMO ELASTIC WAVES IN A HOMOGENEOUS ISOTROPIC PLATE

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Abstract. In this paper, the damping of generalized thermo elastic waves in a homogeneous isotropic plate is studied based on generalized two dimensional theory of thermo elasticity. Two displacement potential functions are introduced to uncouple the equations of motion. The frequency equations are obtained by the traction free boundary conditions using the Bessel function solutions. The numerical calculations are carried out for the material Zinc and the computed non-dimensional thermo elastic damping factor is plotted as the dispersion curves for the plate with thermally insulated and isothermal boundaries.

1. Introduction

Cylindrical thin plate plays a vital role in many engineering fields such as aerospace, civil, chemical, mechanical, naval and nuclear engineering. The analysis of thermally induced wave propagation of a cylindrical plate is a problem that may be encountered in the design of structures such as atomic reactors, steam turbines, submarine structures subjected to wave loadings, jets and other devices operating at elevated temperatures. Moreover, it is recognized that the thermal effects on the elastic wave propagation supported may have implications related to many seismological applications. This study can be potentially used in applications involving nondestructive testing (NDT), and qualitative nondestructive evaluation (QNDE).

The generalized theory of thermo elasticity was developed by Lord and Schulman [1], which involves one relaxation time for isotropic homogeneous media, and is called the first generalization to the coupled theory of elasticity. Their equations determine the finite speed of wave propagation of heat and the displacement distributions. The corresponding equations for an isotropic case were obtained by Dhaliwal and Sherief [2]. The second generalization to the coupled theory of elasticity is known as the theory of thermo elasticity with two relaxation times, or as the theory of temperature-dependent thermoelectricity. A generalization of this inequality was proposed by Green and Laws [3]. Green and Lindsay [4] obtained an explicit version of the constitutive equations. These equations were also obtained independently by Suhubi [5]. This theory contains two constants that act as the relaxation times and modifies not only the heat equations, but also all the equations of the coupled theory. The classical Fourier's law of heat conduction is not violated if the medium under consideration has a center of symmetry. Erbay and Suhubi [6] studied the longitudinal wave propagation in a generalized thermoplastic infinite cylinder and obtained the dispersion relation for the cylinder with a constant surface temperature. Ponnusamy [7] has studied wave propagations in a generalized thermo elastic solid cylinder of arbitrary cross sections using the Fourier expansion collocation method. Later, Ponnusamy and Selvamani [8] obtained mathematical modeling and analysis for a thermo elastic cylindrical panel using the wave propagation approach. Sharma and Pathania [9] investigated the generalized wave propagation in circumferential curved plates. Modeling of circumferential waves in a cylindrical thermo elastic plate with voids was discussed by Sharma and Kaur [10]. Ashida and Tauchert [11] presented the temperature and stress analysis of an elastic circular cylinder in contact with heated rigid stamps. Later, Ashida [12] analyzed the thermally induced wave propagation in a piezoelectric plate.

In this paper, the damping of generalized thermo elastic thin plate composed of homogeneous isotropic material is studied. The solutions to the equations of motion for an isotropic medium is obtained by using the two dimensional theory of thermo elasticity and Bessel function solutions. The numerical calculations are carried out for the material Zinc. The computed non-dimensional damping factor is plotted as dispersion curves for the plate with thermally insulated and isothermal boundaries.

2. Formulation of the problem

We consider a thin homogeneous, isotropic, thermally conducting elastic plate of radius R with uniform thickness d and temperature T_0 in the undisturbed state initially. The system displacements and stresses are defined in the polar coordinates r and θ for an arbitrary point inside the plate, with u denoting the displacement in the radial direction of r and v the displacement in the tangential direction of θ. The in-plane vibration and displacements of the plate embedded on an elastic medium is obtained by assuming that there is no vibration and a displacement along the z axis in the cylindrical coordinate system (r, θ, φ).

The two dimensional stress equations of motion and heat conduction equation in the absence of body force for a linearly elastic medium are

$$\sigma_{rr,r} + r^{-1}\sigma_{r\theta,r} + r^{-1}(\sigma_{rr} - \sigma_{\theta\theta}) = \rho u_{r,tt} \ ,$$

$$\sigma_{r\theta,r} + r^{-1}\sigma_{\theta\theta,\theta} + 2r^{-1}\sigma_{r\theta} = \rho u_{\theta,tt} \ ,$$

$$k\left(T_{,rr} + r^{-1}T_{,r} + r^{-2}T_{,\theta\theta}\right) - \rho c_v\left(T + \tau_0 T_{,tt}\right) = \beta T_0\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\left[e_{rr} + e_{\theta\theta}\right] , \tag{1}$$

where ρ is the mass density, c_v is the specific heat capacity, $\kappa = k / \rho c_v$ is the diffusivity, k is the thermal conductivity, τ_0 is a thermal relaxation time, and T_0 is the reference temperature. The strain-displacement relations for the plate are

$$\sigma_{rr} = \lambda\left(e_{rr} + e_{\theta\theta}\right) + 2\mu e_{rr} - \beta\left(T + \delta_{2k}\tau_1 T_{,t}\right) \ ,$$

$$\sigma_{\theta\theta} = \lambda\left(e_{rr} + e_{\theta\theta}\right) + 2\mu e_{\theta\theta} - \beta\left(T + \delta_{2k}\tau_1 T_{,t}\right),$$

$$\sigma_{r\theta} = 2\mu e_{r\theta} , \tag{2}$$

where e_{ij} are the strain components, $\beta = (3\lambda + 2\mu)\alpha_T$ is the thermal stress coefficients, α_T is the coefficient of linear thermal expansion, T is the temperature, t is time, λ and μ are Lame' constants, τ_1 is a thermal relaxation time, and the comma in the subscripts denotes the partial differentiation with respect to the variable following. Here δ_{ij} is the Kronecker delta function. In addition, we can replace $k=1$ for the L-S theory and $k=2$ for the G-L theory. The thermal relaxation times τ_0 and τ_1 satisfies the inequalities $\tau_0 \ge \tau_1 \ge 0$ for the G-L theory only.

The strain e_{ij} are related to the displacements as given by

$$e_{rr} = u_{,r}\,, \qquad e_{\theta\theta} = r^{-1}(u + v_{,\theta})\,, \qquad e_{r\theta} = v_{,r} - r^{-1}(u - v_{,\theta})\,, \tag{3}$$

in which u and v are the displacement components along the radial and circumferential directions, respectively. σ_{rr}, $\sigma_{\theta\theta}$ are the normal stress components and $\sigma_{r\theta}, \sigma_{\theta z}, \sigma_{zr}$ the shear stress components, e_{rr}, $e_{\theta\theta}$, e_{zz} the normal strain components, and $e_{r\theta}, e_{\theta z}, e_{zr}$ the shear strain components.

By substituting Eqs. (3) and (2) into Eqs. (1), the following displacement equations of motions are obtained

$$\left(\lambda + 2\mu\right)\left(u_{,rr} + r^{-1}u_{,r} - r^{-2}u\right) + \mu r^{-2}u_{,\theta\theta} + r^{-1}\left(\lambda + \mu\right)v_{,r\theta}$$
$$+ r^{-2}\left(\lambda + 3\mu\right)v_{,\theta} - \beta\left(T_{,r} + T\delta_{2k}\tau_1 T_{,rt}\right) = \rho u_{,tt} \quad ,$$

$$\mu\left(v_{,rr} + r^{-1}v_{,r} - r^{-2}v\right) + r^{-2}\left(\lambda + 2\mu\right)v_{,\theta\theta} + r^{-2}\left(\lambda + 3\mu\right)u_{,\theta}$$
$$+ r^{-1}\left(\lambda + \mu\right)u_{,r\theta} - \beta\left(T_{\theta} + \eta T_{,\theta t}\right) = \rho v_{,tt} \quad ,$$

$$k\left(T_{,rr} + r^{-1}T_{,r} + r^{-2}T_{,\theta\theta}\right) - \rho c_v\left(T + \tau_0 T_{,tt}\right) = \beta T_0\left(\frac{\partial}{\partial t} + \tau_0\delta_{1k}\frac{\partial^2}{\partial t^2}\right)\left[u_{,r} + r^{-1}(u + v_{,\theta})\right]. \tag{4}$$

The above coupled partial differential equations are also subjected to the following non-dimensional boundary conditions at the surfaces $r = a,\ b$

(i) Stress free boundary (Free edge)

$$\sigma_{rr} = \sigma_{r\theta} = 0 \quad , \tag{5a}$$

(ii) Rigidly fixed boundary (Clamped edge)

$$u = v = 0 \quad , \tag{5b}$$

(iii) Thermal boundary

$$T_{,r} + hT = 0\,, \tag{5c}$$

where h is the surface heat transfer coefficient. Here $h \to 0$ corresponds to a thermally insulated surface and $h \to \infty$ refers to an isothermal one.

2.1. Lord-Schulman (L-S) theory

Based on the Lord-Schulman theory of thermo elasticity, the three dimensional rate dependent temperature with one relaxation time is obtained by replacing k=1 in the heat conduction equation of Eq. (1), namely,

$$k\left(T_{,rr}+\frac{1}{r}T_{,r}+\frac{1}{r^2}T_{,\theta\theta}\right)=\rho C_v\left[T+\tau_0 T_{,tt}\right]+\beta T_0\left[\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right]\left(e_{rr}+e_{\theta\theta}\right). \tag{6a}$$

The stress-strain relation is replaced by

$$\sigma_{rr}=\lambda\left(e_{rr}+e_{\theta\theta}\right)+2\mu e_{\theta\theta}-\beta\left(T\right),$$

$$\sigma_{\theta\theta}=\lambda\left(e_{rr}+e_{\theta\theta}\right)+2\mu e_{\theta\theta}-\beta\left(T\right), \tag{6b}$$

$$\sigma_{r\theta}=2\mu e_{r\theta}.$$

By substituting the preceding stress-strain relations into Eq. (1), we can get the following displacement equation

$$\left(\lambda+2\mu\right)\left(u_{,rr}+r^{-1}u_{,r}-r^{-2}u\right)+r^{-2}\mu u_{,\theta\theta}+r^{-1}\left(\lambda+\mu\right)v_{,r\theta}+r^{-2}\left(\lambda+3\mu\right)v_{,\theta}-\beta\left(T\right)=\rho u_{,tt},$$

$$\begin{aligned}&\left(\mu\right)\left(v_{,rr}+r^{-1}v_{,r}-r^{-2}v\right)+r^{-1}\left(\lambda+\mu\right)u_{,r\theta}+r^{-2}\left(\lambda+3\mu\right)u_{,\theta}+r^{-2}\left(\lambda+3\mu\right)v_{,\theta}\\&+r^{-2}\left(\lambda+2\mu\right)v_{,\theta\theta}-\beta\left(T\right)=\rho v_{,tt}\end{aligned} \tag{6c}$$

The symbols and notations involved have the same meanings as defined in earlier sections. Since the heat conduction equation of this theory is of the hyperbolic wave type, it can automatically ensure the finite speeds of propagation for heat and elastic waves.

2.2. Green-Lindsay (G-L) theory

The second generalization to the coupled thermo elasticity with two relaxation times called the Green-Lindsay theory of thermo elasticity is obtained by setting k=2 in the heat conduction equation of Eq. (1), namely,

$$k\left(T_{,rr}+\frac{1}{r}T_{,r}+\frac{1}{r^2}T_{,\theta\theta}\right)=\rho C_v\left[T+\tau_0 T_{,tt}\right]+\beta T_0\frac{\partial}{\partial t}\left(e_{rr}+e_{\theta\theta}\right). \tag{7a}$$

The stress-strain relation is replaced by

$$\sigma_{rr}=\lambda\left(e_{rr}+e_{\theta\theta}\right)+2\mu e_{\theta\theta}-\beta\left(T+\tau_1 T_{,t}\right),$$

$$\sigma_{\theta\theta}=\lambda\left(e_{rr}+e_{\theta\theta}\right)+2\mu e_{\theta\theta}-\beta\left(T+\tau_1 T_{,t}\right). \tag{7b}$$

By substituting the preceding relations into Eq. (1), the displacement equation can be reduced as

$$\left(\lambda+2\mu\right)\left(u_{,rr}+r^{-1}u_{,r}-r^{-2}u\right)+r^{-2}\mu u_{,\theta\theta}+r^{-1}\left(\lambda+\mu\right)v_{,r\theta}+r^{-2}\left(\lambda+3\mu\right)v_{,\theta}-\beta\left(T_{,r}+\tau_1 T_{,rt}\right)=\rho u_{,tt}$$

$$(\mu)\left(v_{,rr}+r^{-1}v_{,r}-r^{-2}v\right)+r^{-1}(\lambda+\mu)u_{,r\theta}+r^{-2}(\lambda+3\mu)u_{,\theta}+r^{-2}(\lambda+3\mu)v_{,\theta}$$
$$+r^{-2}(\lambda+2\mu)v_{,\theta\theta}-\beta\left(T_{,\theta}+\tau_1 T_{,\theta t}\right)=\rho v_{,tt} \quad , \tag{7c}$$

where the symbols and notations have been defined in the previous sections. In view of available experimental evidence in favor of the finiteness of heat propagation speeds, the generalized thermo elasticity theories are considered to be more realistic than the conventional theory in dealing with practical problems involving very large heat fluxes and/or short time intervals, such as those occurring in laser units and energy channels.

To uncouple Eqs. (7), the mechanical displacement u, v along the radial and circumferential directions given by Sharma [9] are adopted as follows:

$$u=\phi_{,r}+r^{-1}\psi_{,\theta}, \qquad v=r^{-1}\phi_{,\theta}-\psi_{,r} \tag{8}$$

Substituting Eqs. (8) into Eqs. (7) yields the following second order partial differential equation with constant coefficients:

$$\left\{(\lambda+2\mu)\nabla^2+\rho\omega^2\right\}\phi-\beta\left(T+\delta_{2k}\tau_1 T_{,t}\right)=0\,, \tag{9a}$$

$$\left\{k\nabla^2-\rho C_\upsilon i\omega\eta_0\right\}T+\beta T_0\left(i\omega\eta_1\right)\nabla^2\phi=0\,, \tag{9b}$$

$$\left(\nabla^2+\frac{\rho}{\mu}\omega^2\right)\psi=0\,, \tag{9c}$$

where $\quad \nabla^2 \equiv \partial^2/\partial x^2 + x^{-1}\,\partial/\partial x + x^{-2}\,\partial^2/\partial\theta^2$.

3. Solutions to the Problem

The Eqs. (9) are coupled partial differential equations with two displacements and heat conduction components. To uncouple these equations, we assume the vibration and displacements along the axial direction z to be zero. Hence, the solutions of Eqs. (9) can be presented in the following form:

$$u(r,\theta,t)=\overline{\phi}(r)\exp\left\{i(p\theta-\omega t)\right\}, \tag{10a}$$

$$v(r,\theta,t)=\overline{\psi}(r)\exp\left\{i(p\theta-\omega t)\right\}, \tag{10b}$$

$$T(r,\theta,t)=\left(\lambda+2\mu/\beta a^2\right)\overline{T}(r)\exp\left\{i(p\theta-\omega t)\right\}, \tag{10c}$$

where $i=\sqrt{-1}$, ω is the angular frequency, p is the angular wave number, $\phi(r,\theta), \psi(r,\theta)$, $T(r,\theta)$ are the displacement potentials. Substituting Eqs. (10) into Eqs. (9) and introducing the dimensionless quantities such as $x=r/a$, $c_1^2=(\lambda+2\mu)/\rho$, $\quad c_2^2=\mu/\rho$ $\Omega^2=\rho\omega^2a^2/\mu$, $\overline{\lambda}=\lambda/\mu \quad \overline{d}=\rho c_v\mu/\beta T_0$,we can get the following partial differential equation with constant coefficients:

$$\left\{\left(2+\overline{\lambda}\right)\nabla_1^2+\Omega^2\right\}\phi-\left(2+\overline{\lambda}\right)\eta_2 T=0 \quad , \tag{11a}$$

$$\left\{k_1\nabla_1^2-i\omega\overline{d}\eta_0\right\}T+\beta T_0\left(i\omega\eta_1\right)\nabla_1^2\phi=0 \quad , \tag{11b}$$

$$\left(\nabla_1^2+\Omega^2\right)\psi=0 \quad , \tag{11c}$$

where $\nabla_2^2=\dfrac{\partial^2}{\partial r^2}\dfrac{1}{r}\dfrac{\partial}{\partial r}-\dfrac{p^2}{r^2}$ and $\eta_0=1+i\omega\tau_0$ $\eta_0=1+i\omega\tau_0$, $\eta_1=1+i\omega\delta_{1k}\tau_0$, $\eta_2=1+i\omega\delta_{2k}\tau_1$. Eq. (11c) in terms of ψ gives a purely transverse wave. This wave is polarized in planes perpendicular to the z-axis. We assume that the disturbance is time harmonic through the factor $e^{i\omega t}$. Rewriting Eqs. (11) yields the following fourth order differential equation:

$$\left(A\nabla_2^4+B\nabla_2^2+C\right)\phi=0 \quad , \tag{12}$$

where $A=\left(2+\overline{\lambda}\right)k_1$, $B=\left\{k_1\Omega^2-i\omega\left(2+\overline{\lambda}\right)\overline{d}\eta_0+i\omega T_0\left(2+\overline{\lambda}\right)\beta\eta_1\eta_2\right\}$, $c=-\left(i\omega\Omega^2\overline{d}\eta_0\right)$.

By solving the partial differential equation (10), the solutions is obtained as

$$\overline{\phi}=\sum_{i=1}^{2}\left[A_i J_n\left(\alpha_i ax\right)+B_i Y_n\left(\alpha_i ax\right)\right] \quad , \tag{13a}$$

$$\overline{T}=\sum_{i=1}^{2}d_i\left[A_i J_n\left(\alpha_i ax\right)+B_i Y_n\left(\alpha_i ax\right)\right] \quad , \tag{13b}$$

where

$$d_i=\left\{k_1\left(\alpha_i ax\right)^4+\left(2+\overline{\lambda}\right)\beta T_0 i\omega\eta_1\eta_2\left(\alpha_i ax\right)^2-\left(2+\overline{\lambda}\right)i\omega\overline{d} \quad . \tag{14}$$

Equation (11c) is a Bessel equation with possible solutions given as

$$\overline{\psi}=\begin{cases}A_3J_n(\alpha_3 ax)+B_3Y_n(\alpha_3 ax) & \alpha_3 ax>0\\ A_3a^n+B_3a^{-n} & \alpha_3 ax=0\\ A_3I_n(\alpha_3 ax)+B_3K_n(\alpha_3 ax) & \alpha_3 ax<0\end{cases} \quad , \tag{15}$$

where J_n and Y_n are Bessel functions of the first and second kinds, respectively, while I_n and k_n are modified Bessel functions of first and second kinds, respectively. $\left(A_i, B_i\right)$ $i=1,2,3$ are arbitrary constants. Since $\alpha_3 ax\neq 0$, thus the condition $\alpha_3 ax\neq 0$ will not be discussed in the following. For convenience, we will pay attention only to the case of $\alpha_3 ax>0$ in what follows. The derivation for the case of $\alpha_3 ax<0$ is similar.

$$\overline{\psi}=\left[A_3J_n\left(\alpha_3ax\right)+B_3\left(\alpha_3ax\right)\right],\tag{16}$$

where $\left(\alpha_3a\right)^2=\Omega^2$.

4. Frequency equations

In this section we shall derive the frequency equation for the two dimensional thermo elastic damping of the cylindrical plate subjected to stress free boundary conditions at the upper and lower surfaces at $r=a,b$. Substituting the expressions in Eqs. (1)-(3) into Eqs. (5), we can get the frequency equation for free vibration as follows:

$$\left|E_{ij}^1\right|=0,\qquad i,j=1,2.....6\tag{17}$$

$$E_{11}^1=(2+\overline{\lambda})\Big((nJ_n(\alpha_1ax)+(\alpha_1ax)J_{n+1}(\alpha_1ax))-((\alpha_1ax)^2R^2-n^2)J_n(\alpha_1ax)\Big)$$
$$+\overline{\lambda}\Big(n(n-1)\big(Jn(\alpha_1ax)-\left(\alpha_1ax\right)J_{\delta+1}(\alpha_1ax)\big)\Big)-\beta T\left(i\omega\right)\eta_2d_1\left(\alpha_1ax\right)^2,$$

$$E_{13}^1=(2+\overline{\lambda})\Big((nJ_n(\alpha_2ax)+(\alpha_2ax)J_{n+1}(\alpha_2ax))-((\alpha_2ax)^2R^2-n^2)J_n(\alpha_2ax)\Big)$$
$$+\overline{\lambda}\Big(n(n-1)\big(Jn(\alpha_2ax)-\left(\alpha_2ax\right)J_{\delta+1}(\alpha_2ax)\big)\Big)-\beta T\left(i\omega\right)\eta_2d_2\left(\alpha_2ax\right)^2,$$

$$E_{15}^1=(2+\overline{\lambda})\big((n(n-1)J_n(\alpha_3ax)-(\alpha_3ax)J_{n+1}(\alpha_3ax)\big)+\overline{\lambda}\left(n(n-1)J_n(\alpha_3ax)-(\alpha_3ax)J_{n+1}(\alpha_3ax)\right),$$

$$E_{21}^1=2n(n-1)J_n(\alpha_1ax)-2n(\alpha_1ax)J_{n+1}(\alpha_1ax),$$

$$E_{23}^1=2n(n-1)J_n(\alpha_2ax)-2n(\alpha_2ax)J_{n+1}(\alpha_2ax),$$

$$E_{25}^1=2n(n-1)J_n(\alpha_3ax)-2\Big(\alpha_3ax)J_{\delta+1}(\alpha_3ax)+\left((\alpha_3ax)^2-n^2\right)J_n(\alpha_3ax)\Big),$$

$$E_{31}^1=d_1\left(nJ_n(\alpha_1ax)-(\alpha_1ax)J_{n+1}(\alpha_1ax)+hJ_n(\alpha_1ax)\right),$$

$$E_{33}^1=d_2\left(nJ_n(\alpha_2ax)-(\alpha_2ax)J_{n+1}(\alpha_2ax)+hJ_n(\alpha_2ax)\right),\qquad E_{35}^1=0.$$

Obviously

$E_{ij}\left(j=2,4,6\right)$ can be obtained by just replacing the Bessel functions of the first kind in $E_{ij}\left(i=1,3,5\right)$ with those of the second kind, respectively, while $E_{ij}\left(i=4,5,6\right)$ can be obtained by just replacing a in $E_{ij}\left(i=1,2,3\right)$ with b.

5. Numerical results and discussion

The damping of generalized thermo elastic waves in a simply supported homogenous isotropic cylindrical plate is numerically solved for the Zinc and the material properties of Zinc are given as follows:

$\rho=7.14\times10^3\,\mathrm{kgm^{-3}}$, $T_0=296\,\mathrm{K}$, $K=1.24\times10^2\,\mathrm{Wm^{-1}deg^{-1}}$, $\mu=0.508\times10^{11}\,\mathrm{Nm^{-2}}$,

$\beta = 5.75\times10^{6}$ Nm^{-2}deg^{-1}, $\in_1 = 0.0221$, $\lambda = 0.385\times10^{11}$ Nm^{-2}, and $C_V = 3.9\times10^{2}$ Jkg^{-1}deg^{-1}.

The roots of the algebraic equation in Eq. (12) were calculated using a combination of the Birge-Vita method and Newton-Raphson method. For the present case, the simple Birge-Vita method does not work for finding the root of the algebraic equation. After obtaining the roots of the algebraic equation using the Birge-Vita method, the roots are corrected for the desired accuracy using the Newton-Raphson method. Such a combination can overcome the difficulties encountered in finding the roots of the algebraic equations of the governing equations. Here the values of the thermal relaxation times are calculated from Chandrasekharaiah [13] as $\tau_0 = 0.75\times10^{-13}$ sec and $\tau_1 = 0.5\times10^{-13}$ sec.

Due to the presence of dissipation term in the heat conduction equation, the frequency equation (12) in general complex transcendental equation provides us complex value of frequency. The thermo elastic damping factor is defined by $Q^{-1} = 2\left|\frac{\mathrm{Im}(\omega)}{\mathrm{Re}(\omega)}\right|$. A comparison is made for the non-dimensional frequencies among the Generalized Theory (GL), Lord-Schulman Theory (L-S) and Classical Theory (CT) of thermo-elasticity for the free and clamped boundaries of the thermally insulated and isothermal circular plate in Tables 1 and 2, respectively. From these tables, it is clear that as the sequential number of the vibration modes increases, the non dimensional frequencies also increase for both the clamped and unclamped cases. Also, it is clear that the non dimensional frequency exhibits higher amplitudes for the LS theory compared with the GL and CT due to the effect of thermal relaxation times

Table 1. Comparison of non-dimensional frequencies among the Generalized Theory (GL), Lord-Schulman Theory (L-S) and Classical Theory (CT) of thermo-elasticities for free and clamped boundaries of thermally insulated circular plate.

Mode	Free Edge			Clamped Edge		
	LS	GL	CT	LS	GL	CT
1	1.3937	1.3927	1.3295	1.2289	1.2278	1.5565
2	1.6542	1.6533	1.5886	1.4614	1.4604	1.8391
3	1.9176	1.9156	1.8529	1.7009	1.7019	2.1227
4	2.1832	2.1802	2.1204	1.9486	1.9475	2.4048
5	2.5840	2.5810	2.5245	2.3381	2.3375	2.8318

Table 2. Comparison of non-dimensional frequencies among the Generalized Theory (GL), Lord-Schulman Theory (L-S) and Classical Theory (CT) of thermo-elasticities for free and clamped boundaries of isothermal circular plate.

Mode	Free Edge			Clamped Edge		
	LS	GL	CT	LS	GL	CT
1	1.4558	1.4543	1.4153	14049	14037	1.3588
2	1.7260	1.7251	1.6827	1.6611	1.6602	1.6123
3	1.9967	1.9957	1.9511	1.9182	1.9176	1.8682
4	2.2678	2.2648	2.2213	2.1768	2.1753	2.1264
5	2.6754	2.6732	2.6303	2.5680	2.5670	2.5183

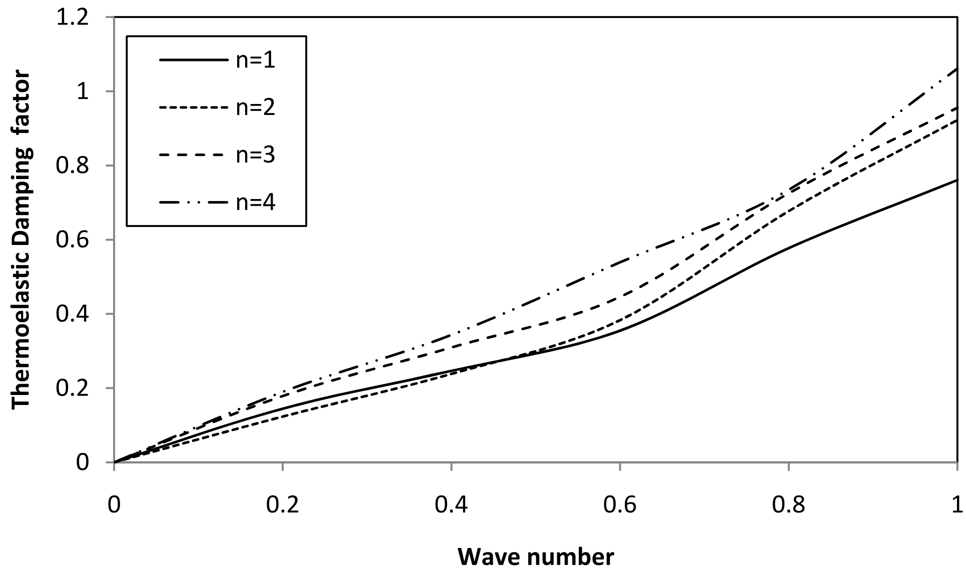

Fig. 1. Variation of thermo elastic damping factor of thermally insulated cylindrical plate with wave number.

In Figs. 1 and 2, the dispersion of thermo elastic damping factor with the wave number is studied for both the thermally insulated and isothermal boundaries of the cylindrical plate in different modes of vibration. From Fig. 2, it is observed that the damping factor increases exponentially with increasing wave number for thermally insulated modes of vibration. But smaller dispersion exist in the damping factor in the current range of wave numbers in Fig. 2 for the isothermal mode due to the combined effect of damping and insulation. . From Figs. 3 and 4, it is clear that the effects of stress free thermally insulated and isothermal boundaries of the plate are quite pertinent due to the combined effect of thermal relaxation times and mechanical field.

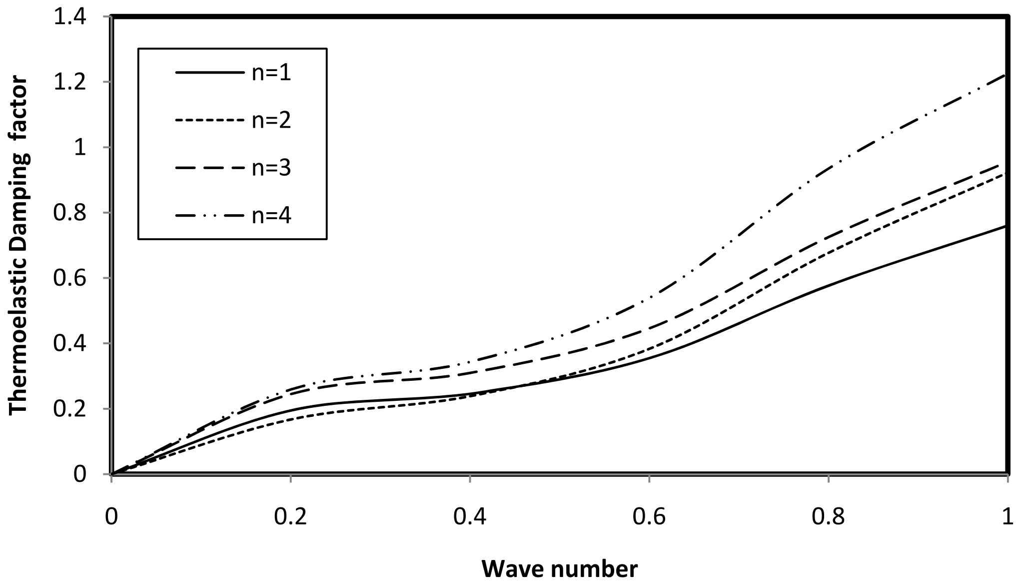

Fig. 2. Variation of thermo elastic damping factor of isothermal cylindrical plate with wave number.

Conclusion

The two dimensional damping of generalized thermo elastic waves in a homogeneous isotropic plate was investigated in this paper. For this problem, the governing equations of two dimensional linear theory of generalized thermo elasticity have been employed and solved by the Bessel function solutions with complex arguments. The effects of the thermo elastic damping factor with respect to the wave number of a Zinc cylindrical plate was investigated, with the results presented as the dispersion curves. In addition, a comparative study is made among the LS, GL and CT theories and the frequency change is observed to be highest for the LS theory, followed by the GL and CT theories due to the thermal relaxation effects and damping.

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