Materials Physics and Mechanics 15 (2012) 78-95 Received: September 29, 2012

A STUDY OF GREEN'S FUNCTIONS FOR TWO-DIMENSIONAL PROBLEM IN ORTHOTROPIC MAGNETOTHERMOELASTIC MEDIA WITH MASS DIFFUSION

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Abstract. The present investigation deals with the study of Green's functions for two-dimensional problem in orthotropic magnetothermoelastic media with mass diffusion. After applying the dimensionless quantities and using the operator theory, two-dimensional general solution in orthotropic magnetothermoelastic diffusion media is derived. On the basis of general solution, the Green's functions for a steady line on the surface of a semi-infinite orthotropic magnetothermoelastic diffusion material are constructed by four newly introduced harmonic functions. The components of displacement, stress, temperature distribution and mass concentration are expressed in terms of elementary functions. From the present investigation, some special cases of interest are also deduced and compared with the previous results obtained. The resulting quantities are computed numerically for semi-infinite magneto thermoelastic material and presented graphically to depict the effect of magnetic.

1. Introduction

Fundamental solutions or Green's functions play an important role in both applied and theoretical studied on the physics of solids. Fundamental solutions can be used to construct many analytical solutions solving boundary value problems of practical problems when boundary conditions are imposed. They are essential in boundary element method (BEM) as well as the study of cracks, defects and inclusion. Many researchers have been investigated the Green's function for elastic solid in isotropic and anisotropic elastic media, notable among them are Freedholm [1], Lifshitz and Rezentsveig [2], Elliott [3], Kröner [4], Synge [5] , Lejcek [6], Pan and Chou [7], and Pan and Yuan [8].

When thermal effects are considered, Sharma [9] investigated the fundamental solution for transversely isotropic thermoelastic material in an integral form. Chen et al. [10] derived the three dimensional general solution for transversely isotropic thermoelastic materials. Hou et al. [11, 12] investigated the Green's function for two and three-dimensional problem for a steady Point heat source in the interior of a semi-infinite thermoelastic materials. Also, Hou et.al [13] investigated the two dimensional general solutions and fundamental solutions for orthotropic thermoelastic materials.

The theory of magnetothermoelasticity is concerned with the interacting effects of the applied magnetic field on the elastic and thermoelastic deformation of a solid body. This theory has drawn the attention of many researchers because of its extensive uses in diverse fields, such as geophysics for understanding the effect of Earth's magnetic field on seismic waves, damping of acoustic waves in a magnetic field. Kolaski and Nowacki [14] studied the magnetothermoelastic disturbance in a perfectly conducting elastic half-space in contact with vacuum due to applied thermal disturbance on the plane boundary. Othman and Song [15] investigated Reflection of magnetothermoelastic waves with two relaxation times. Hou et al. [16] investigated the general solution and fundamental solution for orthotropic magnetothermoelastic materials.

Diffusion is defined as the spontaneous movement of the particles from a high concentration region to the low concentration region and it occurs in response to a concentration region and it occurs in response to a concentration gradient expressed as the change in the concentration due to change in position. Thermal diffusion utilizes the transfer of heat across a thin liquid or gas to accomplish isotope separation. Today, thermal remains a practical process to separate isotopes of noble gases (e.g. xexon) and other light isotopes (e.g. carbon) for research purpose.

Nowacki [17-20] developed the theory of thermoelastic diffusion by using coupled thermoelastic model. Sherief et al. [21] developed the generalized theory of thermoelastic diffusion with one relaxation time which allows finite speeds of propagation of waves. When diffusion effects are considered, Kumar and Chawla [22] investigated the fundamental solution in orthotropic thermoelastic diffusion material. Kumar and Chawla [23] studied the Green's functions for two-dimensional problem in orthotropic thermoelastic diffusion material. Kumar and Chawla [24] derived the three-dimensional fundamental solution in transversely isotropic thermoelastic diffusion media. However, the important fundamental solution for two-dimensional problem in magnetothermoelastic material with mass diffusion has not been discussed so far in the literature.

The Green's functions for two-dimensional problem in orthotropic magnetothermoelastic diffusion medium are investigated in this paper. Based on the two-dimensional general solution of orthotropic magnetothermoelastic diffusion media, the Green's functions for a steady line heat source on the surface of a semi-infinite magnetothermoelastic diffusion material are obtained by four newly introduced harmonic functions**.** From the present investigation, some special cases of interest are also deduced.

2. Basic equations

Following Ezzat [25], the simplified linear equations of electrodynamics of slowly moving medium for a homogeneous and perfectly conducting elastic solid are given by

$$curl\,\mathbf{h} = \mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}, \tag{1}$$

$$curl\,\mathbf{E} = -\mu_0 \frac{\partial \mathbf{h}}{\partial t}, \tag{2}$$

$$\mathbf{E} = -\mu_0 \left(\frac{\partial \mathbf{u}}{\partial t} \times \mathbf{H}_0 \right), \tag{3}$$

$$div\,\mathbf{h} = 0, \tag{4}$$

where $\mathbf{H}_0$ is the external applied magnetic field intensity vector, $\mathbf{h}$ is the induced magnetic field vector, $\mathbf{E}$ is the induced electric field vector, $\mathbf{J}$ is the current density vector, $\mathbf{u}$ is the displacement vector, μ_0 and ε_0 are the magnetic and electric permeabilities respectively.

The above equations (1)-(4) are supplemented by equations of motion and constitutive relations in the theory of generalized thermoelastic diffusion, taking into account the Lorentz force (Eringen [26]).

(i) Constitutive relations:

$$\sigma_{ij} = c_{ijkm}\varepsilon_{km} + a_{ij}\mathrm{T} + b_{ij}C, \tag{5}$$

(ii) Equations of motion:

$$c_{ijkm}e_{km,j} + a_{ij}T_{,j} + b_{ij}C_{,j} + F_i = \rho\ddot{u}_i, \tag{6}$$

(iii) Equation of heat conduction:

$$\rho C_E\dot{\mathrm{T}} + a\mathrm{T}_0\dot{C} - a_{ij}\mathrm{T}_0\dot{\varepsilon}_{ij} = K_{ij}\mathrm{T}_{,ij}, \tag{7}$$

(iv) Equation of mass diffusion:

$$-\alpha_{ij}{}^{*}b_{km}\varepsilon_{km,ij} - \alpha_{ij}{}^{*}bC_{,ij} + \alpha_{ij}{}^{*}a\mathrm{T}_{,ij} = -\dot{C}. \tag{8}$$

Here, $c_{ijkm}(=c_{kmij}=c_{jikm}=c_{ijmk})$ are elastic parameters, $a_{ij}(=a_{ji}), b_{ij}(=b_{ji})$ are, respectively, the tensors of thermal and diffusion modules, ρ is the density and C_E is the specific heat at constant strain, a,b are, respectively, coefficients describing the measure of thermoelastic diffusion effects and diffusion effects, T_0 is the reference temperature assumed to be such that $\left|\frac{T}{T_0}\right| << 1$, $K_{ij}(=K_{ji}), \sigma_{ij}(=\sigma_{ji})$ and $\varepsilon_{ij} = \frac{u_{i,j}+u_{j,i}}{2}$ denote the components of thermal conductivity, stress and strain tensor respectively, $T(x,y,z,t)$ is the temperature change from the reference temperature T_0 and C is the mass concentration, u_i are components of displacement vector, $\alpha_{ij}^{*}(=\alpha_{ji}^{*})$ are diffusion parameters, F_i are components of Lorentz force.

In the above equations symbol (",") followed by a suffix denotes differentiation with respect to spatial coordinate and a superposed dot (".") denotes the derivative with respect to time respectively.

3. Formulation of the problem

We consider homogenous orthotropic magnetothermoelastic diffusion medium. Let us take Oxyz as the frame of reference in Cartesian coordinates, the origin O being any point on the plane boundary.

For two-dimensional problem, we assume the displacement vector, temperature change and mass concentration are, respectively, of the form

$$\mathbf{u} = (u,0,w), \qquad \mathrm{T}(x,z,t), \qquad C(x,z,t), \tag{9}$$

and Lorentz force is taken in the form (for two dimensional problem):

$$F_1 = \mu_0 H_0^2\left(\frac{\partial e}{\partial x} - \varepsilon_0\mu_0\frac{\partial^2 u}{\partial t^2}\right), \tag{10a}$$

$$F_3 = \mu_0 H_0^2\left(\frac{\partial e}{\partial z} - \varepsilon_0\mu_0\frac{\partial^2 w}{\partial t^2}\right), \tag{10b}$$

where

$$e=\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}.$$

Moreover, we are discussing static problem

$$\frac{\partial u}{\partial t}=\frac{\partial w}{\partial t}=\frac{\partial C}{\partial t}=\frac{\partial T}{\partial t}=0. \tag{11}$$

We define the dimensionless quantities as:

$$(x',z',u'w')=\frac{\omega_1^*}{v_1}(x,z,u,w),\quad (T',C')=\frac{1}{c_{11}}(a_1T,b_1C),$$

$$\sigma_{ij}'=\frac{\sigma_{ij}}{a_1\mathrm{T}_0},\quad H'=\frac{a_1v_1}{c_{11}K_1\omega_1^*}H,$$

where

$$v_1^2=b_1,\ \omega_1^*=\frac{ac_{11}}{K_1}, \tag{12}$$

and b_1 is the tensor of diffusion modules and K_1 is the component of thermal conductivity.

Equations (5)-(8) for orthotropic materials, with the aid of Eqs. (9)-(12), after suppressing the primes, yields:

$$\left(\delta_1\frac{\partial^2}{\partial x^2}+\delta_2\frac{\partial^2}{\partial z^2}\right)u+\left(\delta_3\frac{\partial^2}{\partial x\partial z}\right)w-\left(\frac{\partial}{\partial x}\right)C-\left(\frac{\partial}{\partial x}\right)T=0, \tag{13}$$

$$\left(\delta_3\frac{\partial^2}{\partial x\partial z}\right)u+\left(\delta_2\frac{\partial^2}{\partial x^2}+\delta_4\frac{\partial^2}{\partial z^2}\right)w-\varepsilon_1\left(\frac{\partial}{\partial z}\right)C-\varepsilon_2\left(\frac{\partial}{\partial z}\right)T=0\,, \tag{14}$$

$$\left(\frac{\partial^2}{\partial x^2}\right)T+\varepsilon_3\left(\frac{\partial^2}{\partial z^2}\right)T=0, \tag{15}$$

$$\frac{\partial}{\partial x}\left(q_1^*\frac{\partial^2}{\partial x^2}+q_3^*\frac{\partial^2}{\partial z^2}\right)u+\frac{\partial}{\partial z}\left(q_2^*\frac{\partial^2}{\partial x^2}+q_4^*\frac{\partial^2}{\partial z^2}\right)w-\left(q_5^*\frac{\partial^2}{\partial x^2}+q_6^*\frac{\partial^2}{\partial z^2}\right)C+\left(q_7^*\frac{\partial^2}{\partial x^2}+q_8^*\frac{\partial^2}{\partial z^2}\right)T=0, \tag{16}$$

where

$$\delta_1=1+\frac{\mu_0H_0^2}{c_{11}},\qquad (\delta_2,\delta_3,\delta_4)=\frac{1}{c_{11}}\left(c_{55},\ c_{13}+c_{55}+\mu_0H_0^2\omega_1^*,\ c_{33}+\mu_0H_0^2\right),\qquad \varepsilon_1=\frac{b_3}{b_1},$$

$$\varepsilon_2 = \frac{a_3}{a_1}, \qquad \varepsilon_3 = \frac{K_3}{K_1}, \quad (q_1^*, q_2^*) = \frac{\alpha_1^* \omega_1^*}{c_{11}}\left(b_1, b_3\right), \quad (q_3^* \; q_4^*) = \frac{\alpha_3^* \omega_1^*}{c_{11}}\left(b_1, b_3\right),$$

$$(q_5^*, q_6^*) = \frac{\omega_1^* b}{b_1}\left(\alpha_1^*, \alpha_3^*\right), \quad (q_7^*, q_8^*) = \frac{a\omega_1^*}{a_1}\left(\alpha_1^*, \alpha_3^*\right).$$

The equations (13)-(16) can be written as

$$D\{u, w, C, T\}^t = 0. \tag{17}$$

where D is differential operator matrix given by

$$\begin{bmatrix} \delta_1 \frac{\partial^2}{\partial x^2} + \delta_2 \frac{\partial^2}{\partial z^2} & \delta_3 \frac{\partial^2}{\partial x \partial z} & -\frac{\partial}{\partial x} & -\frac{\partial}{\partial x} \\ \delta_3 \frac{\partial^2}{\partial x \partial z} & \delta_2 \frac{\partial^2}{\partial x^2} + \delta_4 \frac{\partial^2}{\partial z^2} & -\varepsilon_1 \frac{\partial}{\partial z} & -\varepsilon_2 \frac{\partial}{\partial z} \\ \frac{\partial}{\partial x}\left(q_1^* \frac{\partial^2}{\partial x^2} + q_3^* \frac{\partial^2}{\partial z^2}\right) & \frac{\partial}{\partial z}\left(q_2^* \frac{\partial^2}{\partial x^2} + q_4^* \frac{\partial^2}{\partial z^2}\right) & -\left(q_5^* \frac{\partial^2}{\partial x^2} + q_6^* \frac{\partial^2}{\partial x^2}\right) & \left(q_7^* \frac{\partial^2}{\partial x^2} + q_8^* \frac{\partial^2}{\partial x^2}\right) \\ 0 & 0 & 0 & \left(\frac{\partial^2}{\partial x^2} + \varepsilon_3 \frac{\partial^2}{\partial z^2}\right) \end{bmatrix}. \tag{18}$$

Equation (17) is a homogeneous set of differential equations in u, w, C, T. The general solution by the operator theory as follows

$$u = A_{i1}F, \qquad w = A_{i2}F, \qquad C = A_{i3}F, \qquad T = A_{i4}F, \qquad (i = 1, 2, 3, 4) \tag{19}$$

where A_{ij} are algebraic cofactors of the matrix D, of which the determinant is

$$|D| = \left(a^* \frac{\partial^6}{\partial z^6} + b^* \frac{\partial^6}{\partial x^2 \partial z^4} + c^* \frac{\partial^6}{\partial x^4 \partial z^2} + d^* \frac{\partial^6}{\partial x^6}\right) \times \left(\frac{\partial^2}{\partial x^2} + \varepsilon_3 \frac{\partial^2}{\partial z^2}\right), \tag{20}$$

where

$$a^* = \delta_2(\varepsilon_1 q_4^* - \delta_4 q_6^*), \;\; b^* = \delta_1(\varepsilon_1 q_4^* - \delta_4 q_6^*) - \delta_2(\delta_2 q_6^* + \delta_4 q_5^*) + \delta_2(\varepsilon_1 q_2^* + \delta_3 q_6^*) - q_7^*(\delta_4 + \delta_2 \varepsilon_1) + \delta_3 q_4^*,$$

$$c^* = \delta_1(\varepsilon_1 q_2^* - \delta_4 q_5^*) - \delta_2(\delta_1 q_6^* + \delta_2 q_5^*) + \delta_2(\delta_3 q_5^* - \varepsilon_1 q_1^*) + \delta_3 q_2^* - \delta_4 q_1^* - \delta_2 q_7^*, \quad d = -\delta_2(\delta_1 q_5^* + q_1^*).$$

The function F in equation (19) satisfies the following homogeneous equation:

$$|D|F = 0. \tag{21}$$

It can be seen that if $i = 1, 2, 3$ are taken in equation (19), three general solution are obtained in which $T = 0$. These solutions are identical to those without thermal fact and are not discussed here. Therefore if $i = 4$ should be taken in equation (19), the following solution is obtained

$$u = \left(p_1 \frac{\partial^2}{\partial x^4} + q_1 \frac{\partial^2}{\partial z^2 \partial x^2} + r_1 \frac{\partial^4}{\partial z^4} \right) \frac{\partial F}{\partial x},$$

$$w = \left(p_2 \frac{\partial^2}{\partial x^4} + q_2 \frac{\partial^2}{\partial z^2 \partial x^2} + r_2 \frac{\partial^4}{\partial z^4} \right) \frac{\partial F}{\partial z},$$

$$C = \left(p_3 \frac{\partial^6}{\partial z^6} + q_3 \frac{\partial^6}{\partial z^4 \partial x^2} + r_3 \frac{\partial^6}{\partial z^2 \partial x^4} + l_3 \frac{\partial^6}{\partial x^6} \right) F,$$

$$T = \left(a^* \frac{\partial^6}{\partial z^6} + b^* \frac{\partial^6}{\partial z^4 \partial x^2} + c^* \frac{\partial^6}{\partial z^2 \partial x^4} + d^* \frac{\partial^6}{\partial x^6} \right) F, \tag{22}$$

$$p_1 = (q_7^* - q_5^*)\delta_2, \quad q_1 = -\delta_2(\varepsilon_1 q_7^* + q_5^* \varepsilon_2) + \delta_2(q_6^* + q_8^*) + \delta_4(q_7^* + q_8^*)q_8^* - \varepsilon_1 q_2^*,$$

$$r_1 = -\delta_2(\varepsilon_1 q_8^* + \varepsilon_2 q_6^*) + \delta_2 q_8^* + \delta_4(q_6^* + q_8^*) + (q_4^* + q_6^*)\delta_4 + \varepsilon_1 q_4^*,$$

$$p_2 = \delta_3(q_5^* + q_7^*) + q_1^*(\varepsilon_2 - \varepsilon_1) - \delta_1(\varepsilon_1 q_7^* + \varepsilon_2 q_5^*), \quad r_2 = -\delta_1(\varepsilon_1 q_8^* + \varepsilon_2 q_6^*),$$

$$q_2 = -\delta_1(\varepsilon_1 q_8^* + q_6^* \varepsilon_2) - \delta_2(\varepsilon_1 q_7^* + \varepsilon_2 q_5^*) + \delta_3(q_8^* + q_6^*) + q_7^*(\varepsilon_2 - \varepsilon_1), \quad p_3 = (\varepsilon_2 q_4^* + \delta_4 q_8^*)\delta_2,$$

$$q_3 = \delta_1(\delta_4 q_8^* + \varepsilon_2 q_4^*) + \delta_2(\delta_2 q_8^* + \delta_4 q_7^*) + \delta_2(\varepsilon_2 q_2^* - \delta_3 q_8^*) - q_7^*(\delta_2 \varepsilon_2 + \delta_4) + \delta_3 q_4^*,$$

$$r_3 = \delta_1(\delta_2 q_8^* + \delta_4 q_7^*) + \delta_2(\delta_2 q_7^* - \delta_3 q_7^*) + q_2^*(\varepsilon_2 \delta_1 + \delta_3) - \delta_2(q_1^* \varepsilon_2 + q_7^*) - \delta_4 q_1^*, \quad l_3 = \delta_2(\delta_1 q_7^* - q_1^*).$$

Equations (21) can be rewritten as

$$\prod_{j=1}^{4} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z_j^2} \right) F = 0, \tag{23}$$

where

$$z_j = s_j z, \quad s_4 = \sqrt{\frac{K_1}{K_3}}$$

and s_j $(j = 1, 2, 3)$ are three roots (with positive real part) of the following algebraic equation:

$$a^* s^6 - b^* s^4 + c^* s^2 - d^* = 0. \tag{24}$$

As known from the generalized Almansi theorem (Ding et al. [10]), the function F can be expressed in terms of four harmonic functions:

(i) $F = F_1 + F_2 + F_3 + F_4$ for distinct s_j $(j = 1, 2, 3, 4)$; (25a)

(ii) $F = F_1 + F_2 + F_3 + zF_4$ for $s_1 \neq s_2 \neq s_3 = s_4$; (25b)

(iii) $F = F_1 + F_2 + zF_3 + z^2 F_4$ for $s_1 \neq s_2 = s_3 = s_4$; (25c)

(iv) $F = F_1 + zF_2 + z^2 F_3 + z^3 F_4$ for $s_1 = s_2 = s_3 = s_4$. (25d)

Here F_j $(j = 1,2,3,4)$ satisfies the following harmonic equation:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z_j^2}\right) F_j = 0 \qquad (j = 1,2,3,4). \tag{26}$$

The general solution for the case of distinct roots, can be derived as follows

$$u = \sum_{j=1}^{4} p_{1j} \frac{\partial^5 F_j}{\partial x \partial z_j^4}, \qquad w = \sum_{j=1}^{4} s_j p_{2j} \frac{\partial^5 F_j}{\partial z_j^5}, \qquad C = \sum_{j=1}^{4} p_{3j} \frac{\partial 6 F_j}{\partial z_j^6}, \qquad T = \sum_{j=1}^{4} p_{44} \frac{\partial^6 F_4}{\partial z_4^6}. \tag{27}$$

In the similar way general solution for the other three cases can be derived.
Equation (23) can be further simplified by taking

$$p_{1j} \frac{\partial^4 F_j}{\partial z_j^4} = \psi_j. \tag{28}$$

Using the formula (23) in equation (22) gives

$$u = \sum_{j=1}^{4} \frac{\partial \psi_j}{\partial x}, \qquad w = \sum_{j=1}^{4} s_j P_{1j} \frac{\partial \psi_j}{\partial z_j}, \qquad C = \sum_{j=1}^{4} P_{2j} \frac{\partial^2 \psi_j}{\partial z_j^2}, \qquad T = \sum_{j=1}^{4} P_{34} \frac{\partial^2 \psi_4}{\partial z_4^2}, \tag{29}$$

where

$$P_{1j} = p_{2j} / p_{1j}, \qquad P_{2j} = p_{3j} / p_{1j}, \qquad P_{34} = p_{44} / p_{14}. \tag{30}$$

The function ψ_j satisfies the harmonic equations:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z_j^2}\right) \psi_j = 0 \qquad j = 1, 2, 3, 4. \tag{31}$$

Making use of Eqs. (9), (11) and (12) in equation (1) and after suppressing the primes, with the aid of Eq. (29), we obtain:

$$\sigma_{xx} = \sum_{j=1}^{4} \left(-f_1 + f_1 s_j^2 P_{1j} - f_1 P_{3j} - f_1 P_{2j} \right) \frac{\partial^2 \psi_j}{\partial z_j^2}, \tag{32a}$$

$$\sigma_{zz} = \sum_{j=1}^{4} \left(-f_2 + h_1 s_j^2 P_{1j} - h_2 P_{3j} - h_3 P_{2j} \right) \frac{\partial^2 \psi_j}{\partial z_j^2}, \tag{32b}$$

$$\sigma_{zx} = \sum_{j=1}^{4} h_4 (1 + P_{1j}) s_j \frac{\partial^2 \psi_j}{\partial x \partial z_j}, \tag{32c}$$

where

$$P_{31} = P_{32} = P_{33} = 0, \tag{33a}$$

and

$$(f_1, f_2, h_1, h_2, h_3, h_4) = \frac{1}{a_1 T_0} \left(c_{11}, c_{13}, c_{33}, \frac{a_3 c_{11}}{a_1}, \frac{b_3 c_{11}}{b_1}, c_{55} \right). \tag{33b}$$

Substituting the values of σ_{xx}, σ_{zz} and σ_{zx} from Eq. (32) in equations (6)-(7), with the aid of Eqs. (9), (11) and (12), gives

$$f_1 - f_2 s_j^2 P_{1j} + f_1 P_{3j} + f_1 P_{2j} = h_4 (1 + P_{1j}) s_j^2,$$

$$-f_2 + h_1 s_j^2 P_{1j} - h_2 P_{3j} - h_3 P_{2j} = h_4 (1 + P_{1j}),$$

$$\left(1 - \varepsilon_3 s_j^2\right) P_{3j} = 0. \tag{34}$$

The general solution (32) with the help of Eq. (34) can be simplified as

$$\sigma_{xx} = -\sum_{j=1}^{4} s_j^2 w_{1j} \frac{\partial^2 \psi_j}{\partial z_j^2}, \quad \sigma_{zz} = \sum_{j=1}^{4} w_{1j} \frac{\partial^2 \psi_j}{\partial z_j^2}, \quad \sigma_{zx} = \sum_{j=1}^{4} s_j w_{1j} \frac{\partial^2 \psi_j}{\partial x \partial z_j}, \tag{35}$$

where

$$w_{1j} = \frac{f_1 - P_{1j} s_j^2 f_2 + P_{3j} f_1 + P_{2j} f_1}{s_j^2} = h_4 (1 + P_{1j}) = -f_2 + P_{1j} h_1 s_j^2 - P_{3j} h_2 - P_{3j} h_3. \tag{36}$$

4. Green's functions for a steady line heat source in a semi infinite orthotropic magnetothermoelastic diffusion material

As shown in Fig. 1 we consider a semi-infinite orthotropic magnetothermoelastic diffusion material $z \geq 0$. A linear heat source H is applied at the line $(0, h)$ in two dimensional

Cartesian coordinate (x,z) and the surface $z=0$ is free, impermeable boundary and thermally insulated. The general solution given by equations (29) and (35) is derived in this section.

For future reference, following notations are introduced:

$$z_j = s_j z, \qquad h_k = s_k h, \qquad z_{jk} = z_j + h_k,$$

$$r_{jk} = \sqrt{x^2 + z_{jk}^2}, \qquad \bar{z}_{jk} = z_j - h_k, \qquad \bar{r}_{jk} = \sqrt{x^2 + \bar{z}_{jk}^2}, \qquad (j,k = 1,2,3,4). \tag{37}$$

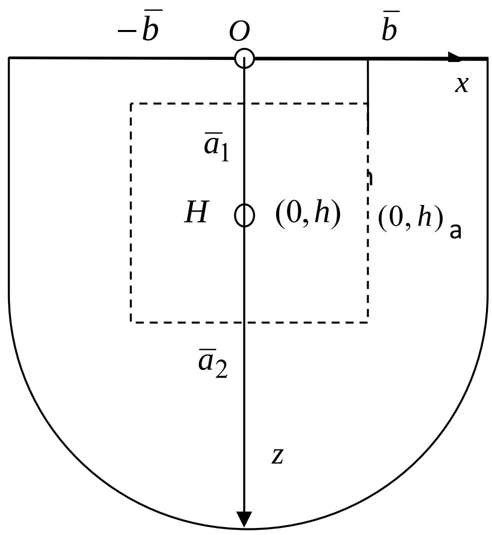

Fig. 1. Geometry of the problem.

Green's functions in the semi-infinite plane are assumed of the following form:

$$\psi_j = A_j \left[\frac{1}{2}(\bar{z}_{jj}^2 - x^2)\left(\log \bar{r}_{jj} - \frac{3}{2} \right) - x\bar{z}_{jj} \tan^{-1}(\frac{x}{\bar{z}_{jj}}) \right] +$$

$$+ \sum_{k=1}^{4} A_{jk} \left[\frac{1}{2}(z_{jk}^2 - x^2)\left(\log r_{jk} - \frac{3}{2} \right) - xz_{jk} \tan^{-1}(\frac{x}{z_{jk}}) \right], \tag{38}$$

where A_j and A_{jk} $(j,k = 1,2,3,4)$ are twenty constant to be determined.

The boundary conditions on the surface $(z=0)$ are in the form of

$$\sigma_{zz} = \sigma_{zx} = 0, \qquad \frac{\partial C}{\partial z} = 0, \qquad \frac{\partial T}{\partial z} = 0. \tag{39}$$

Substituting the equation (38) in equations (29) and (35) gives the expressions for components of displacement, mass concentration, temperature distribution and stress components as follows:

$$u = -\sum_{j=1}^{4} A_j \left[x(\log \bar{r}_{jj} - 1) + \bar{z}_{jj} \tan^{-1} \frac{x}{\bar{z}_{jj}} \right] - \sum_{j=1}^{4}\sum_{k=1}^{4} A_{jk} \left[x(\log r_{jk} - 1) + z_{jk} \tan^{-1} \frac{x}{z_{jk}} \right], \tag{40a}$$

$$w=-\sum_{j=1}^{4}s_jP_{1j}A_j\left[\bar{z}_{jj}(\log\bar{r}_{jj}-1)-x\tan^{-1}\frac{x}{\bar{z}_{jj}}\right]+\sum_{j=1}^{4}\sum_{k=1}^{4}s_jP_{1j}A_{jk}\left[z_{jk}(\log r_{jk}-1)-x\tan^{-1}\frac{x}{z_{jk}}\right],\tag{40b}$$

$$C=\sum_{j=1}^{4}P_{2j}A_j\log\bar{r}_{jj}-\sum_{j=1}^{4}\sum_{k=1}^{4}P_{2j}A_{jk}\log r_{jk},\tag{40c}$$

$$T=P_{34}A_4\log\bar{r}_{44}+P_{34}\sum_{k=1}^{4}A_{4k}\log r_{4k},\tag{40d}$$

$$\sigma_{xx}=-\sum_{j=1}^{4}s_j^2w_{1j}A_j\log\bar{r}_{jj}-\sum_{j=1}^{4}\sum_{k=1}^{4}s_j^2w_{1j}A_{jk}\log r_{jk},\tag{40e}$$

$$\sigma_{zz}=\sum_{j=1}^{4}w_{1j}A_j\log\bar{r}_{jj}+\sum_{j=1}^{4}\sum_{k=1}^{4}w_{1j}A_{jk}\log r_{jk},\tag{40f}$$

$$\sigma_{zx}=-\sum_{j=1}^{4}s_jw_{1j}A_j\tan^{-1}\frac{x}{\bar{z}_{jj}}-\sum_{j=1}^{4}\sum_{k=1}^{4}s_jw_{1j}A_{jk}\tan^{-1}\frac{x}{z_{jk}}.\tag{40g}$$

Considering the continuity on plane $z=h$ for w and σ_{zx} gives the following expressions

$$\sum_{j=1}^{4}s_jP_{1j}A_j=0,\tag{41}$$

$$\sum_{j=1}^{4}s_jw_{1j}A_j=0.\tag{42}$$

Substituting w_{1j} from equation (36) in equation (42) yields

$$\sum_{j=1}^{4}s_jh_4(1+P_{1j})A_j=0.\tag{43}$$

By virtue of equation (41), equation (43) can be simplified to

$$\sum_{j=1}^{4}s_jA_j=0.\tag{44}$$

When the mechanical, concentration and thermal equilibrium for a rectangle of $\bar{a}_1 \le z \le \bar{a}_2$ $(0 < \bar{a}_1 < h < \bar{a}_2)$ and $-\bar{b} \le x \le \bar{b}$ are considered (Fig. 1), three equations can be obtained:

$$\int_{-\bar{b}}^{\bar{b}} [\sigma_{zz}(x,\bar{a}_2) - \sigma_{zz}(x,\bar{a}_1)] dx + \int_{\bar{a}_1}^{\bar{a}_2} [\sigma_{zx}(\bar{b},z) - \sigma_{zx}(-\bar{b},z)] dz = 0, \tag{45a}$$

$$-\varepsilon_3 \int_{-\bar{b}}^{\bar{b}} \left[\frac{\partial T}{\partial z}(x,\bar{a}_2) - \frac{\partial T}{\partial z}(x,\bar{a}_1) \right] dx - \int_{\bar{a}_1}^{\bar{a}_2} \left[\frac{\partial T}{\partial x}(\bar{b},z) - \frac{\partial T}{\partial x}(-\bar{b},z) \right] dz = H, \tag{45b}$$

$$\int_{-\bar{b}}^{\bar{b}} \left[\frac{\partial C}{\partial z}(x,\bar{a}_2) - \frac{\partial C}{\partial z}(x,\bar{a}_1) \right] dx + \int_{\bar{a}_1}^{\bar{a}_2} \left[\frac{\partial C}{\partial x}(\bar{b},z) - \frac{\partial C}{\partial x}(-\bar{b},z) \right] dz = 0. \tag{45c}$$

Some useful integrals are given as follows:

$$\int \log \bar{r}_{jj} dx = x(\log \bar{r}_{jj} - 1) + \bar{z}_{jj} \tan^{-1}\left(\frac{x}{\bar{z}_{jj}}\right), \tag{46a}$$

$$\int \log r_{jk} dx = x(\log r_{jk} - 1) + z_{jk} \tan^{-1}\left(\frac{x}{z_{jk}}\right), \tag{46b}$$

$$\int \frac{\partial T}{\partial z} dx = s_4 P_{34} \left(A_4 \tan^{-1} \frac{x}{\bar{z}_{44}} + \sum_{k=1}^{4} A_{4k} \tan^{-1} \frac{x}{z_{4k}} \right), \tag{46c}$$

$$\int \frac{\partial T}{\partial x} dz = -\frac{P_{34}}{s_4} \left(A_4 \tan^{-1} \frac{x}{\bar{z}_{44}} + \sum_{k=1}^{4} A_4 \tan^{-1} \frac{x}{z_{4k}} \right), \tag{46d}$$

$$\int \frac{\partial C}{\partial z} dx = A_j s_j P_{2j} \tan^{-1} \frac{x}{\bar{z}_{jj}} + \sum_{k=1}^{4} A_{jk} s_j P_{2j} \tan^{-1} \frac{x}{z_{jk}}, \tag{46e}$$

$$\int \frac{\partial C}{\partial x} dz = -\frac{A_j}{s_j} P_{2j} \tan^{-1} \frac{x}{\bar{z}_{jj}} - \sum_{k=1}^{4} \frac{A_{jk}}{s_j} P_{2j} \tan^{-1} \frac{x}{z_{jk}}. \tag{46f}$$

It is noticed that the integrals (46d) and (46f) are not continuous at $z = h$, thus following expression should be used

$$\int_{\bar{a}_1}^{\bar{a}_2} \frac{\partial T}{\partial x} dz = \int_{\bar{a}_1}^{h^-} \frac{\partial T}{\partial x} dz + \int_{h^+}^{\bar{a}_2} \frac{\partial T}{\partial x} dz, \tag{47a}$$

$$\int_{\bar{a}_1}^{\bar{a}_2} \frac{\partial C}{\partial x} dz = \int_{\bar{a}_1}^{h^-} \frac{\partial C}{\partial x} dz + \int_{h^+}^{\bar{a}_2} \frac{\partial C}{\partial x} dz. \tag{47b}$$

Substituting equations (40f) and (40g) into equation (45a) and using the integrals (46a) and (46b), we obtain

$$\sum_{j=1}^{4} w_{1j} A_j I_1 + \sum_{j=1}^{4} w_{1j} \sum_{k=1}^{4} A_{jk} I_2 = 0, \tag{48}$$

where

$$I_1 = \left[\left(x(\log \bar{r}_{jj} - 1) + \bar{z}_{jj} \tan^{-1}\left(\frac{x}{\bar{z}_{jj}}\right) \right)_{z=\bar{a}_1}^{z=\bar{a}_2} \right]_{x=-\bar{b}}^{x=\bar{b}} - \left[\left(x \log \bar{r}_{jj} + \bar{z}_{jj} \tan^{-1}\left(\frac{x}{\bar{z}_{jj}}\right) \right)_{x=-\bar{b}}^{x=\bar{b}} \right]_{z=\bar{a}_1}^{z=\bar{a}_2} = 0, \tag{49a}$$

$$I_2 = \left[\left(x(\log r_{jk} - 1) + z_{jk} \tan^{-1}\left(\frac{x}{z_{jk}}\right) \right)_{z=\bar{a}_1}^{z=\bar{a}_2} \right]_{x=-\bar{b}}^{x=\bar{b}} - \left[\left(x \log r_{jk} + z_{jk} \tan^{-1}\left(\frac{x}{z_{jk}}\right) \right)_{x=-\bar{b}}^{x=\bar{b}} \right]_{z=\bar{a}_1}^{z=\bar{a}_2} = 0. \tag{49b}$$

Equations (49a) and (49b) show that the equations (45a) and (48) are satisfied automatically. Substituting the value of C from equation (40c) into equation (45c) and using the integrals (46e), (46f) and (47b), we obtain

$$\sum_{j=1}^{4} P_{2j} A_j \bar{r}_j + \sum_{j=1}^{4} r_j^* \sum_{k=1}^{4} P_{2j} A_{jk} = 0, \tag{50}$$

where

$$\bar{r}_j = \left[\left(s_j^2 \tan^{-1}\left(\frac{x}{\bar{z}_{jj}}\right) \right)_{z=\bar{a}_1}^{z=\bar{a}_2} \right]_{x=-\bar{b}}^{x=\bar{b}} - \left[\left(\tan^{-1}\left(\frac{x}{\bar{z}_{jj}}\right) \right)_{x=-\bar{b}}^{x=\bar{b}} \right]_{z=\bar{a}_1}^{z=h^-} + \left[\left(\tan^{-1}\left(\frac{x}{\bar{z}_{jj}}\right) \right)_{x=-\bar{b}}^{x=\bar{b}} \right]_{z=h^+}^{z=\bar{a}_2} =$$

$$= 2(s_j^2 - 1) \left[\tan^{-1} \frac{\bar{b}}{s_j \bar{a}_2 - s_j h} - \tan^{-1} \frac{\bar{b}}{s_j \bar{a}_1 - s_j h} \right] + 2\pi$$

and

$$r_j^* = \left[\left(\tan^{-1}\left(\frac{x}{z_{jk}}\right) \right)_{z=\bar{a}_1}^{z=\bar{a}_2} \right]_{x=-\bar{b}}^{x=\bar{b}} - \left[\left(\tan^{-1}\left(\frac{x}{z_{jk}}\right) \right)_{x=-\bar{b}}^{x=\bar{b}} \right]_{z=\bar{a}_1}^{z=\bar{a}_2} =$$

$$= 2(s_j^2 - 1)\left[\tan^{-1}\frac{\bar{b}}{s_j\bar{a}_2 + s_j h} - \tan^{-1}\frac{\bar{b}}{s_j\bar{a}_1 + s_j h}\right].$$

Substituting equation (40d) into equation (45b) with the aid of $s_4 = \sqrt{K_1 / K_3}$ and integrals (46c) and (46d) and (47a), yields

$$A_4 I_5 + \sum_{k=1}^{4} A_{4k} I_6 = \frac{H}{P_{34}\sqrt{K_3/K_1}}, \tag{51}$$

where

$$I_5 = -\left[\left(\tan^{-1}(\frac{x}{\bar{z}_{44}})\right)_{z=\bar{a}_1}^{z=\bar{a}_2}\right]_{x=-\bar{b}}^{x=\bar{b}} - \left[\left(\tan^{-1}(\frac{x}{\bar{z}_{44}})\right)_{x=-\bar{b}}^{x=\bar{b}}\right]_{z=\bar{a}_1}^{z=h^-} + \left[\left(\tan^{-1}(\frac{x}{\bar{z}_{44}})\right)_{x=-\bar{b}}^{x=\bar{b}}\right]_{z=h^+}^{z=\bar{a}_2} = -2\pi, \tag{52a}$$

$$I_6 = \left[\left(\tan^{-1}(\frac{x}{z_{4k}})\right)_{x=-\bar{b}}^{x=\bar{b}}\right]_{z=\bar{a}_1}^{z=\bar{a}_2} - \left[\left(\tan^{-1}(\frac{x}{z_{4k}})\right)_{z=\bar{a}_1}^{z=\bar{a}_2}\right]_{x=-\bar{b}}^{x=\bar{b}} = 0. \tag{52b}$$

From equations (51) and (52), we obtain

$$A_4 = -\frac{H}{2\pi P_{34}\sqrt{K_3/K_1}}. \tag{53}$$

Equation (37) at the surface $z = 0$ gives

$$z_j = 0, \qquad h_k = s_k h, \qquad z_{jk} = h_k,$$

$$r_{jk} = \sqrt{x^2 + h_k^2}, \qquad \bar{z}_{jk} = -h_k, \qquad \bar{r}_{jk} = \sqrt{x^2 + h_k^2}. \tag{54}$$

Substituting equations (40c), (40d), (40f) and (40g) into equation (39) with the aid of $s_4 = \sqrt{K_1 / K_3}$ and equation (54), yields

$$-s_j w_{1j} A_j + \sum_{k=1}^{4} s_k w_{1k} A_{kj} = 0, \tag{55}$$

$$w_{1j} A_j + \sum_{k=1}^{4} w_{1k} A_{kj} = 0, \tag{56}$$

$$A_4 - A_{44} = 0, \qquad A_{4k} = 0, \tag{57}$$

$$P_{2j}A_j + \sum_{k=1}^{4} s_k^2 P_{2k} A_{kj} = 0, \tag{58}$$

where $j = 1, 2, 3, 4, \quad k = 1, 2, 3.$

We have determined the twenty constants A_j and A_{jk} $(j, k = 1, 2, 3, 4)$ from twenty equations including equations (41), (44), (50), (53), (55), (56), (57) and (58) by the method of Crammer rule.

5. Special cases

(I) In case of negligible magnetic effect

Eqs. (40a)-(40g) are reduced to

$$u = -\sum_{j=1}^{4} A_j \left[x(\log \bar{r}_{jj} - 1) + \bar{z}_{jj} \tan^{-1} \frac{x}{\bar{z}_{jj}} \right] - \sum_{j=1}^{4}\sum_{k=1}^{4} A_{jk} \left[x(\log r_{jk} - 1) + z_{jk} \tan^{-1} \frac{x}{z_{jk}} \right], \tag{59a}$$

$$w = -\sum_{j=1}^{4} s_j P_{1j} A_j \left[\bar{z}_{jj} (\log \bar{r}_{jj} - 1) - x \tan^{-1} \frac{x}{\bar{z}_{jj}} \right] + \sum_{j=1}^{4}\sum_{k=1}^{4} s_j P_{1j} A_{jk} \left[z_{jk} (\log r_{jk} - 1) - x \tan^{-1} \frac{x}{z_{jk}} \right], \tag{59b}$$

$$C = \sum_{j=1}^{4} P_{2j} A_j \log \bar{r}_{jj} - \sum_{j=1}^{4}\sum_{k=1}^{4} P_{2j} A_{jk} \log r_{jk}, \tag{59c}$$

$$T = P_{34} A_4 \log \bar{r}_{44} + P_{34} \sum_{k=1}^{4} A_{4k} \log r_{4k}, \tag{59d}$$

$$\sigma_{xx} = -\sum_{j=1}^{4} s_j^2 w_{1j} A_j \log \bar{r}_{jj} - \sum_{j=1}^{4}\sum_{k=1}^{4} s_j^2 w_{1j} A_{jk} \log r_{jk}, \tag{59e}$$

$$\sigma_{zz} = \sum_{j=1}^{4} w_{1j} A_j \log \bar{r}_{jj} + \sum_{j=1}^{4}\sum_{k=1}^{4} w_{1j} A_{jk} \log r_{jk}, \tag{59f}$$

$$\sigma_{zx} = -\sum_{j=1}^{4} s_j w_{1j} A_j \tan^{-1} \frac{x}{\bar{z}_{jj}} - \sum_{j=1}^{4}\sum_{k=1}^{4} s_j w_{1j} A_{jk} \tan^{-1} \frac{x}{z_{jk}}, \tag{59g}$$

which are similar to the results as those obtained by Kumar and Chawla [22].

(II) In the absence of magnetic and diffusion effect

Eqs. (40a)-(40g) are reduced to

$$u = -\sum_{j=1}^{3} A_j \left[x(\log r_j - 1) + z_j \tan^{-1} \frac{x}{z_j} \right], \tag{60a}$$

$$w = \sum_{j=1}^{3} s_j P_{1j} A_j \left[z_j (\log r_j - 1) - x \tan^{-1} \frac{x}{z_j} \right], \tag{60b}$$

$$T = A_4 P_{34} \log r_4, \tag{60c}$$

$$\sigma_{xx} = -\sum_{j=1}^{3} s_j^2 w_{1j} A_j \log r_j, \tag{60d}$$

$$\sigma_{zz} = \sum_{j=1}^{3} w_{1j} A_j \log r_j, \tag{60e}$$

$$\sigma_{zx} = -\sum_{j=1}^{3} s_j w_{1j} A_j \tan^{-1} \frac{x}{z_j}. \tag{60f}$$

The above results are similar to the results obtained by Hou et al. [13].

6. Numerical results and discussion

For the purpose of numerical computation, we take the following values of the relevant parameters as:

$c_{11} = 18.78 \times 10^{10}$ Kg m^{-1}s^{-2}, $c_{13} = 8.0 \times 10^{10}$ Kg m^{-1}s^{-2}, $c_{33} = 10.2 \times 10^{10}$ Kg m^{-1}s^{-2}, $c_{55} = 10.06 \times 10^{10}$ Kg m^{-1}s^{-2}, $T_0 = 0.293 \times 10^3\ K$, $\alpha_1 = 2.98 \times 10^{-5} K^{-1}$, $\alpha_3 = 2.4 \times 10^{-5} K^{-1}$, $\alpha_{1c} = 1.1 \times 10^{-4}\ m^3 Kg^{-1}$, $K_1 = 0.12 \times 10^3\ W\ m^{-1} K^{-1}$, $K_3 = 0.33 \times 10^3\ W\ m^{-1} K^{-1}$, $a = 1.4 \times 10^4\ m^2 s^{-2} K^{-1}$, $b = 9 \times 10^5\ Kg^{-1} m^5 s^{-2}$, $\alpha_1^* = 0.95 \times 10^{-8}\ m^{-3} s\ Kg$, $\alpha_3^* = 0.90 \times 10^{-8}\ m^{-3} s\ Kg$, $H_0 = 0.38$, $\mu_0 = 1$

$a_1 = c_{11}\alpha_1 + c_{13}\alpha_3$, $a_3 = c_{13}\alpha_1 + c_{33}\alpha_3$, $b_1 = c_{11}\alpha_{1c} + c_{13}\alpha_{3c}$, $b_3 = c_{13}\alpha_{1c} + c_{33}\alpha_{3c}$.

Figures 2-5 depict the variation of horizontal displacement (u), vertical displacement (w), temperature distribution (T) and mass concentration (C) w.r.t x. The solid line and dotted line correspond to thermoelastic diffusion (TD) and centre symbol on these lines correspond to magnetothermoelastic diffusion (MTD).

Figure 2 depicts the variation of horizontal displacement u with x and it indicates that the values of u increases for all values of x in both cases TD and MTD. It is noticed that the values of u in case of TD remain more (in comparison with MTD). Figure 3 shows that the values of vertical displacement w slightly decrease for smaller values of x, whereas for higher values of x the values of w oscillates. It is evident that the values of w in case of MTD remain more for smaller values of x although for higher values of x reverse behavior occurs. Figure 4 exhibits the variation of temperature distribution T with x and it indicates that the values of T slightly increases for smaller values of x whereas for higher values of x, the values of T increases monotonically. It is noticed that the values of T in case of MTD remain more (in comparison with MTD) for higher values of x. Figure 5 shows that the values of mass concentration C slightly decrease for smaller values of x, although for higher values of x the values of C increase. It is evident that the values of C in case of TD remain more (in comparison with MTD) for higher values of x.

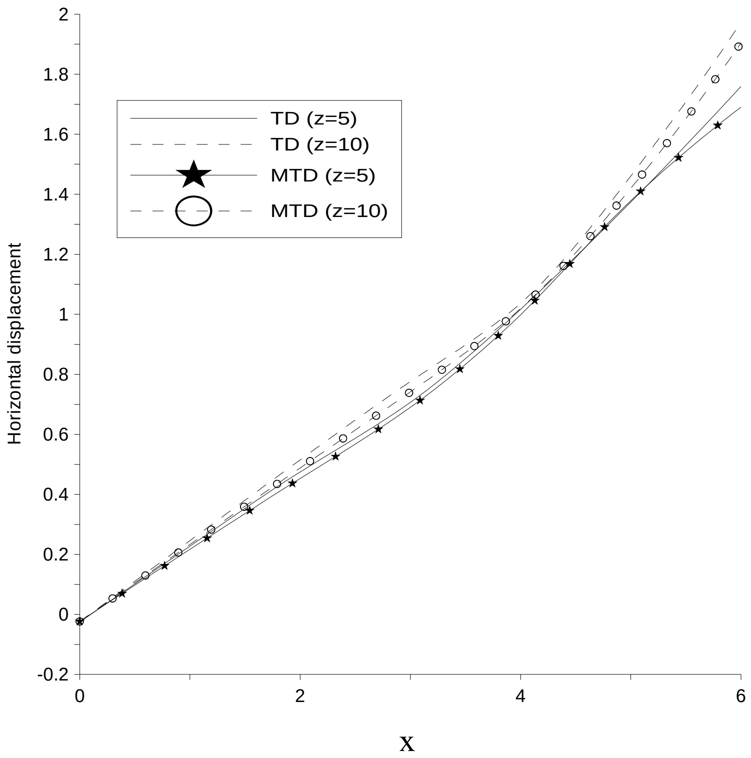

Fig. 2. Variation of horizontal displacement w.r.t. *x*.

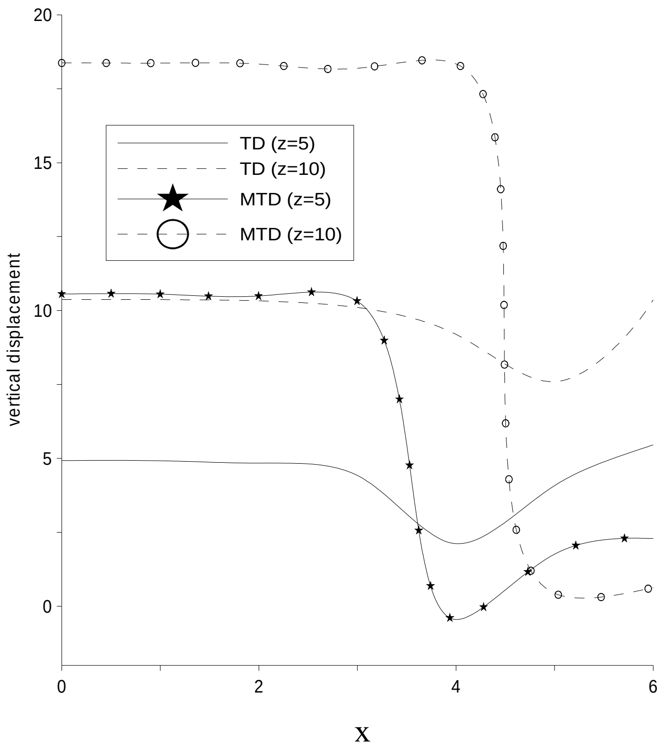

Fig. 3. Variation of vertical displacement w.r.t. *x*.

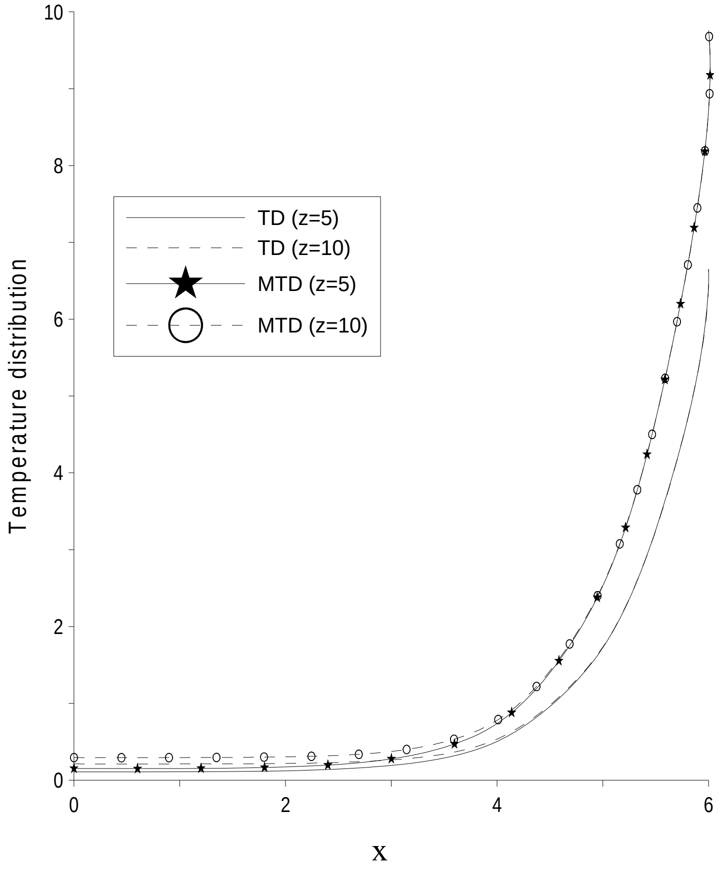

Fig. 4. Variation of temperature distribution w.r.t. *x*.

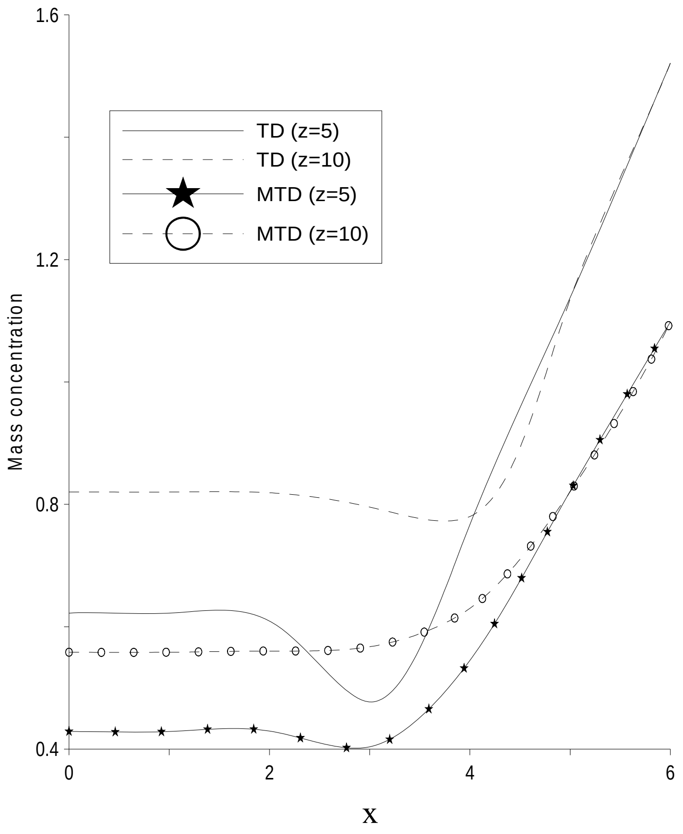

Fig. 5. Variation of mass concentration w.r.t. *x*.

7. Concluding remarks

The Green’s functions for two-dimensional in orthotropic magnetothermoelastic diffusion material have been derived. By virtue of the two-dimensional general solution of orthotropic magnetothermoelastic diffusion material, the Green functions for a steady line heat source on the surface of a semi-infinite plane are obtained by four newly introduced harmonic functions $\psi_j\ (j=1,2,3,4)$. The general expression for components of displacement, stress, mass concentration and temperature change are expressed in terms of elementary functions. Since all the components are expressed in terms of elementary functions, it is convenient to use them. From the present investigation, some special cases of interest are also deduced and compared with the previous results obtained. The components of displacement, mass concentration and temperature distribution are computed numerically and depicted graphically to depict the effect of magnetic.

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