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EFFECT OF ROTATION IN AN AXISYMMETRIC VIBRATION OF A TRANSVERSELY ISOTROPIC SOLID BAR IMMERSED IN AN INVISCID FLUID

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Abstract. Effect of rotation in a axis symmetric vibration of a finite, homogeneous transversely isotropic solid bar immersed in a fluid is studied using the linearized, three-dimensional theory of elasticity. The equations of motion of solid and fluid are respectively formulated using the constitutive equations of a transversely isotropic solid bar and the constitutive equations of an inviscid fluid. Two displacement potentional functions are introduced to uncouple the equations of motion. The computed non-dimensional frequency, phase velocity and attenuation are presented in the form of dispersion curves for the material Zinc.

1. Introduction

The axisymmetric waves of a rotating bar have taking interest in many structural applications because of high tensile strength and high corrosion resistance properties. The axisymmetric modes often used to evaluate the material properties of thin metal wires, reinforcement filament in ultrasonic transducers and resonators. A thorough knowledge of various wave propagation characteristics, as a function of material and geometrical parameters is necessary for a wide range of applications, from geophysical prospecting in cased holes, non-destructive evaluation of oil and gas pipelines, to the insulated fiber optic cables for data transmission. The most general form of harmonic waves in a hollow cylinder of circular cross section of infinite length has been analyzed by Gazis [1]. Mirsky [2] investigated analyzed the wave propagation in transversely isotropic circular cylinders of infinite length and presented the frequency equation in Part I and numerical results in Part II. A method, for solving wave propagation in arbitrary cross-sectional cylinders and plates and to find out the phase velocities in different modes of vibrations namely longitudinal, torsional and flexural, by constructing frequency equations was devised by Nagaya [3-5]. He formulated the Fourier expansion collocation method for this purpose. Following Nagaya, Paul and Venkatesan [6] studied the wave propagation in an infinite piezoelectric solid cylinder of arbitrary cross section using Fourier expansion collocation method. The longitudinal waves inhomogeneous anisotropic cylindrical bars immersed in a fluid are studied by Dayal [7]. Guided waves in a transversely isotropic cylinder immersed in a fluid are analyzed by Ahmad [8]. Following Ahmad, Nagay [9] have studied the longitudinal guided wave propagation in a transversely isotropic rod immersed in fluid, later, Nagy with Nayfeh [10] discussed the viscosity-induced attenuation of longitudinal guided waves in fluid-loaded rods. Easwaran and Munjal [11] reported a note on the effect of wall compliance on lowest-order mode propagation in fluidfilled/submerged impedance tubes. Sinha et. al. [12] have discussed the axisymmetric wave propagation in circular cylindrical shell immersed in fluid, in two parts. In Part I, the theoretical analysis of the propagating modes are discussed and in Part II, the axisymmetric modes excluding torsional modes are obtained theoretically and experimentally and are compared. Berlinear and Solecki [13] have studied the wave propagation in fluid loaded transversely isotropic cylinder. In that paper, Part I consists of the analytical formulation of the frequency equation of the coupled system consisting of the cylinder with inner and outer fluid and Part II gives the numerical results. Venkatesan and Ponnusamy [14-15] have obtained the frequency equation of the free vibration of a solid cylinder of arbitrary cross section immersed in a fluid using Fourier expansion collocation method. The frequency equations are obtained for longitudinal and flexural vibrations and are studied numerically for elliptical and cardioidal cross-sectional cylinders. Selvamani and Ponnusamy [16] studied the damping of generalized thermo elastic waves in a homogeneous isotropic plate using the wave propagation approach and obtained the numerical result for Zinc plate. Since the speed of the disturbed waves depend upon rotation rate, this type of study is important in the design of high speed steam, gas turbine and rotation rate sensors. Loy and Lam [17] discussed the vibration of rotating thin cylindrical panel using Love first approximation theory. Zhang [18] investigated the parametric analysis of frequency of rotating laminated composite cylindrical shell using wave propagation approach. Body wave propagation in rotating thermo elastic media was investigated by Sharma and Grover [19]. The propagation of waves in conducting piezoelectric solid is studied for the case when the entire medium rotates with a uniform angular velocity by Wauer [20]. Roychoudhuri and Mukhopadhyay studied the effect of rotation and relaxation times on plane waves in generalized thermo-viscoelasticity [21]. Chen [22] has discussed the elastic-plastic deformation of the rotating solid disk. Lam [23] has studied the frequency characteristics of a thin rotating cylindrical shell using general differential quadrature method.

In this paper, the axisymmetric vibration in an finite, homogeneous transversely isotropic rotating bar immersed in a fluid is studied using the linearized, three-dimensional theory of elasticity**.** Two displacement potentional functions are introduced to uncouple the equations of motion. The computed non-dimensional phase velocity and attenuation are presented in the form of dispersion curves for the material Zinc.

2. Formulation of the problem

We consider a transversely isotropic bar of length L and radius “a” immersed in inviscid fluid. The system is assumed to be linear so that the linearized three-dimensional stress equations of motion are used for both the cylinder and the fluid. The system displacements and stresses are defined by the cylindrical coordinates r, θ, and z.

In cylindrical coordinates, the three-dimensional axisymmetric stress equations of motion and strain-displacement relations in the absence of body for rotating medium are given as

$$\sigma_{rr,r} + \sigma_{rz,z} + r^{-1}\left(\sigma_{rr}\right) + \rho\,(\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{u}) + 2\,\mathbf{\Omega}\,\mathbf{u}_{,t}) = \rho\, u_{,tt}\,, \tag{1a}$$

$$\sigma_{rz,r} + \sigma_{zz,z} + r^{-1}\sigma_{rz} + \rho(\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{u}) + 2\,\mathbf{\Omega}\,\mathbf{u}_{,t}) = \rho\, w_{,tt}\,, \tag{1b}$$

where

$$\sigma_{rr} = c_{11}e_{rr} + c_{12}e_{\theta\theta} + c_{13}e_{zz}\,, \tag{2a}$$

$$\sigma_{zz} = c_{13}e_{rr} + c_{13}e_{\theta\theta} + c_{33}e_{zz}, \tag{2b}$$

$$\sigma_{rz} = 2c_{44}e_{rz}. \tag{2c}$$

Here $\sigma_{rr,}\ \sigma_{zz,}\ \sigma_{rz}$ are the stress components, $e_{rr,}\ e_{\theta\theta,}\ e_{zz,}\ e_{r\theta,}\ e_{\theta z,}\ e_{rz}$ are the strain components, $c_{11}, c_{12}, c_{13}, c_{33}$, c_{44} and $c_{66} = (c_{11} - c_{12})/2$ are the five independent elastic constants, ρ is the mass density of the material. The displacement equation of motion has the additional terms with a time dependent centripetal acceleration $\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{u})$ and $2\,\mathbf{\Omega}\times\mathbf{u}_{,t}$, where, $\mathbf{u} = (u, 0, w)$ is the displacement vector and $\mathbf{\Omega} = (0, \Omega, 0)$ is a constant, the comma notation used in the subscript denotes the partial differentiation with respect to the variables.

The strain e_{ij} are related to the displacements by

$$e_{rr} = u_{,r}, \quad e_{\theta\theta} = r^{-1}(u), \qquad e_{zz} = w_{,z}, \tag{3a}$$

$$2e_{zr} = (u_{,z} + w_{,r}), \tag{3b}$$

in which u, and w are the displacement components along radial and axial directions respectively. The comma in the subscripts denotes the partial differentiation with respect to the variables.

Substituting the Eqs. (3) and (2) in the Eq. (1), results in the following three-dimensional displacement equations of motion:

$$c_{11}\left(u_{,rr} + r^{-1}u_{,r} - r^{-2}u\right) + c_{44}u_{,zz} + (c_{44} + c_{13})w_{,rz} + \rho\left(\Omega^2 u + 2\Omega w_{,t}\right) = \rho u_{,tt}, \tag{4a}$$

$$c_{44}\left(w_{,rr} + r^{-1}w_{,r}\right) + r^{-1}(c_{44} + c_{13})(u_{,z}) + (c_{44} + c_{13})u_{,rz} + c_{33}w_{,zz} + \rho\left(\Omega^2 w + 2\Omega u_{,t}\right) = \rho w_{,tt}, \tag{4b}$$

In an inviscid fluid-solid interface, the perfect-slip boundary condition allows discontinuity in planar displacement components. That is, the radial component of displacement of the fluid and solid must be equal and the longitudinal components are discontinuous at the interface. The above coupled partial differential equations are also subjected to the following non-dimensional boundary conditions at the surfaces $r = a$:

$$\left(\sigma_{rr} + p^f\right) = (\sigma_{rz}) = \left(u - u^f\right) = 0. \tag{5}$$

3. Solution to solid medium

The Equation (4) is coupled partial differential equations of the three displacement components. This system of equations can be uncoupled by eliminating two of the three displacement components through two of the three equations, but this results in a partial differential equations of fourth order. To uncouple the Eq. (4), we follow Mirsky [2] and assuming the solution of Eqs. (4) as follows:

$$u(r, z, t) = \left[(\phi_{,r})\right]e^{i(kz+\omega t)}, \tag{6a}$$

$$w(r, z, t) = (i/a)\left[W_n\right]e^{i(kz+\omega t)}, \tag{6b}$$

where $i=\sqrt{-1}$, k is the wave number, ω is the angular frequency, $\phi(r,z,t)$, $W(r,z,t)$, are the displacement potentials and a is the geometrical parameter of the solid bar.

By introducing the dimensionless quantities such as $\varsigma = ka$, $\varpi^2 = \rho\omega^2 a^2 / c_{44}$, $\bar{c}_{11} = c_{11}/c_{44}$, $\bar{c}_{13} = c_{13}/c_{44}$, $\bar{c}_{33} = c_{33}/c_{44}$, $T = t\sqrt{\dfrac{c_{44}}{(\rho/a)}}$, $\Gamma = \dfrac{\rho\,\Omega^2 R^2}{2+\bar{c}_{11}}$ and $x = r/a$ and substituting Eq. (6) in Eq. (4), we obtain

$$\left(\bar{c}_{11}\nabla^2 + \left(\varpi^2 - \varsigma^2 + \Gamma\right)\right)\phi - \left(1+\bar{c}_{13}\right)W = 0, \tag{7a}$$

$$\varsigma\left(1+c_{13}\right)\phi + \left(\nabla^2 + \left(\varpi^2 - \bar{c}_{33}\,\varsigma^2 + \Gamma\right)\right)W = 0, \tag{7b}$$

where $\nabla^2 \equiv \partial^2/\partial x^2 + x^{-1}\,\partial/\partial x$.

A non-trivial solution of the algebraic system Eqs. (7) exists only when the determinant of Eqs. (7) are equal to zero.

$$\begin{vmatrix} \left(\bar{c}_{11}\nabla^2 + \left(\varpi^2 - \zeta^2 + \Gamma\right)\right) & -\left(1+\bar{c}_{13}\right) \\ \zeta\left(1+\bar{c}_{13}\right) & \left(\nabla^2 + \left(\varpi^2 - \bar{c}_{33}\,\zeta^2 + \Gamma\right)\right) \end{vmatrix} (\phi, W) = 0 \ . \tag{8}$$

Equation (8) on simplification reduces to the following differential equation:

$$\left(A\nabla^4 + B\nabla^2 + C\right)\phi = 0, \tag{9}$$

where

$$A = \bar{c}_{11}, \qquad B = \left[\left(1+\bar{c}_{11}\right)\varpi^2 - \left(\varsigma^2 + \Gamma\right)\left(1+\bar{c}_{11}\bar{c}_{33}\right)\right],$$

$$C = \left(\varpi^2 - \varsigma^2 + \Gamma\right)\left(\varpi^2 - \bar{c}_{33}\,\varsigma^2 + \Gamma\right). \tag{10}$$

Solving the Eq. (9), the solutions for the axisymmetric modes are obtained as

$$\phi = \sum_{i=1}^{2}\left[A_i\, J_n\left(\alpha_i\, a\, x\right)\right], \tag{11a}$$

$$W = \sum_{i=1}^{2} d_i\left[A_i\, J_n\left(\alpha_i\, a\, x\right)\right], \tag{11b}$$

where J_n and Y_n are Bessel functions of the first and second kind of order n.
Here $\left(\alpha_i a\right)^2 > 0$, $\left(i = 1,2\right)$ are the roots of the algebraic equation:

$$A\left(\alpha\, a\right)^4 - B\left(\alpha\, a\right)^2 + C = 0. \tag{12}$$

The Bessel functions J_n is used when the roots $(\alpha_i\, a)^2, (i=1,2)$ are real or complex and the modified Bessel function I_n is used when the roots are imaginary.
The constants d_i defined in the Eq. (11b) can be calculated from the equation

$$d_i = (1+\bar{c}_{13})(\alpha_i\, a)^2 \Big/ \left((\alpha_i\, a)^2 + \varpi^2 - \bar{c}_{33}^2 \varsigma^2 + \Gamma\right), \quad i=1,2, \tag{13}$$

where $(\alpha_3\, a)^2 = (\varpi^2 - \varsigma^2 + \Gamma)\big/\bar{c}_{66}$. If $(\alpha_3\, a)^2 < 0$, the Bessel function J_n is replaced by the modified Bessel function I_n.

4. Solution of fluid medium

In cylindrical polar coordinates r, θ, and z, the acoustic pressure and radial displacement equations of motion for an invicid fluid are of the form [16]:

$$p^f = -B^f\left(u_{r,r}^f + r^{-1}(u_r^f) + u_{z,z}^f\right), \tag{14}$$

and

$$c_f^{-2} u_{r,tt}^f = \Delta,_r \tag{15}$$

respectively,
where B^f is the adiabatic bulk modulus, ρ^f is the density, $c^f = \sqrt{B^f/\rho^f}$ is the acoustic phase velocity in the fluid, $\left(u_r^f, u_z^f\right)$ is the displacement vector, and

$$\Delta = \left(u_{r,r}^f + r^{-1}(u_r^f) + u_{z,z}^f\right). \tag{16}$$

Substituting

$$u_r^f = \phi_{,r}^f \text{ and } u_z^f = \phi_{,z}^f, \tag{17}$$

and seeking the solution of Eq. (15) in the form:

$$\phi^f(r,\theta,z,t) = \left[\phi^f(r)\right] e^{i(kz+\omega t)}, \tag{18}$$

the fluid represents the oscillatory waves propagating away is given by

$$\phi^f = A_3 H^{(1)}(\delta a x), \tag{19}$$

where $(\delta a)^2 = \varpi^2/(\bar{\rho}^f \bar{B}^f) - \varsigma^2$, in which $\bar{\rho}^f = \rho/\rho^f$, $\bar{B}^f = B^f/c_{44}$, $H_n^{(1)}$ is the Hankel function of the first kind. If $(\delta_2 a)^2 < 0$, then the Hankel function of first kind is to be replaced by K_n, where K_n is the modified Bessel function of the second kind. By substituting Eq. (19) in (14) along with (19) and (20), the acoustic pressure for the fluid can be expressed as:

$$p^f = A_3\varpi^2 \,\overline{\rho}\, H^{(1)}(\delta ax)e^{i(\varsigma\overline{z}+\Omega T_a)}. \qquad (20)$$

5. Frequency equations

In this section we shall derive the frequency equation for the three dimensional vibration of the solid bar immersed in fluid subjected to perfect slip boundary conditions at $r = a$. Substituting the expressions in Esq. (1)- (3) into Eqs. (5), we can get the frequency equation for free vibration as follows:

$$\left|E_{ij}\right| = 0, \qquad i, j = 1,2,3, \qquad (21)$$

$$E_{11} = 2\overline{c}_{66}(nJ_n(\alpha_1 ax) + (\alpha_1 ax)J_{n+1}(\alpha_1 ax)) - \overline{c}_{11}((\alpha_1 ax)^2(ax)^2 + \overline{c}_{13}x^2\zeta d_1 J_n(\alpha_1 ax),$$

$$E_{12} = 2\overline{c}_{66}(nJ_n(\alpha_2 ax) + (\alpha_2 ax)J_{n+1}(\alpha_2 ax)) - \overline{c}_{11}(\alpha_2 ax)^2(ax)^2 + \overline{c}_{13}x^2\zeta d_2 J_n(\alpha_2 ax),$$

$$E_{13} = \varpi^2\overline{\rho}(ax)^2\left(nH_n(\delta ax) - (\delta ax)H_{n+1}(\delta ax)\right),$$

$$E_{21} = 2\zeta\left(nJ_n(\alpha_1 ax) - (\alpha_1 ax)J_{n+1}(\alpha_1 ax)\right),$$

$$E_{22} = 2\zeta\left(nJ_n(\alpha_2 ax) - (\alpha_2 ax)J_{n+1}(\alpha_2 ax)\right),$$

$$E_{23} = 0,$$

$$E_{31} = nJ_n(\alpha_1 ax) - (\alpha_1 ax)J_{n+1}(\alpha_1 ax),$$

$$E_{32} = nJ_n(\alpha_2 ax) - (\alpha_2 ax)J_{n+1}(\alpha_2 ax),$$

$$E_{33} = nJ_n(\delta ax) - (\delta ax)J_{n+1}(\delta ax).$$

6. Numerical results and discussion

The coupled free wave propagation in a homogenous transversely isotropic solid bar immersed in water is numerically solved for the Zinc material. The material properties of Zinc are given as follows and for the purpose of numerical computation the liquid is taken as water. For the solid the elastic constants are $c_{11} = 1.628\times10^{11}\,N\,m^{-2}$, $c_{12} = 0.362\times10^{11}\,N\,m^{-2}$, $c_{13} = 0.508\times10^{11}\,N\,m^{-2}$, $c_{33} = 0.627\times10^{11}\,N\,m^{-2}$, $c_{44} = 0.385\times10^{11}\,N\,m^{-2}$ and density $\rho = 7.14\times10^3\,kg\,m^{-3}$ and for the fluid the density $\rho^f = 1000\,kg\,m^{-3}$ and phase velocity $c^f = 1500\,m\,s^{-1}$.

Because the algebraic equation (8) contains all the information about the wave speed and angular frequency, and the roots are complex for all considered values of wave number, therefore the waves are attenuated in space. We can write $c^{-1} = v^{-1} + i\omega^{-1}q$, so that $p = R + iq$, where $R = \omega/v$, v and q are real numbers. Upon using the above relation in Eq. (21), the values of the wave speed (v) and the attenuation coefficient (q) for different modes of wave propagation can be obtained. When a solid medium such as the solid bar is surrounded by fluid medium, guided waves are transmitted across the interface. Thus bulk waves are excited in the embedding medium, radiating away from the solid medium.

Table 1. The non-dimensional frequencies for different axisymmetric modes (S1, S2, S3) for the solid bar in space and immersed in fluid with $\Omega = 0.2$.

Wave number	Bar in space						Bar immersed in fluid				
	S1		S2		S3		S1		S2		S3
0.1	0.0435		0.1236		0.1681		0.0359		0.1139		0.1284
0.2	0.1801		0.2036		0.2747		0.1702		0.1213		0.2530
0.4	0.2263		0.3928		0.4492		0.1950		0.3563		0.4220
0.6	0.3967		0.4727		0.5391		0.2737		0.4702		0.5295
0.8	0.5310		0.6036		0.7853		0.5029		0.5031		0.6752
1.0	0.6400		0.7308		1.5288		0.7727		0.6092		0.8349
1.2	0.9025		1.7015		1.5824		0.8081		0.9231		0.9142

Table 2. The non-dimensional frequencies for different axisymmetric modes (S1, S2, and S3) for the solid bar in space and immersed in fluid with $\Omega = 0.4$.

Wave number	Bar in space						Bar immersed in fluid				
	S1		S2		S3		S1		S2		S3
0.1	0.0239		0.0765		0.1572		0.0215		0.0442		0.1408
0.2	0.0751		0.2419		0.2345		0.0554		0.1969		0.2205
0.4	0.1674		0.4977		0.5337		0.1444		0.3888		0.4473
0.6	0.1894		0.5385		0.7292		0.2387		0.5023		0.5941
0.8	0.3964		0.6952		0.9408		0.5504		0.6050		0.7303
1.0	0.4051		0.9714		1.4579		0.5551		0.8512		0.8070
1.2	0.7478		1.1350		1.6707		0.6038		1.0230		1.2107

A comparison is made for the non-dimensional frequencies in case of $\Omega = 0.2$, $\Omega = 0.4$ with respect to the wave number for the free and fluid loaded cylinder in Tables 1 and 2, for first three axisymmetric modes of vibration respectively. From these tables, it is clear that as the sequential number of the wave number increases, the non dimensional frequencies also increases for both the free and fluid loaded cylinder. Also, it is clear that the non dimensional frequency exhibits higher amplitudes for $\Omega = 0.4$ compared with the $\Omega = 0.2$.

The dispersion of phase velocities with the wave number is discussed in Fig. 1 and Fig. 2 for both the rotational speed ($\Omega = 0.2$ and $\Omega = 0.4$) of the solid cylinder in different modes of vibration. In Fig. 1 the phase velocity is decreasing at small wave number between 0 and 4 and become steady for higher values of the wave number for remaining modes of vibration with $\Omega = 0.2$. For the case of $\Omega = 0.4$ there is a small deviation on the phase velocity in Fig. 2 due to the damping effect of fluid medium and rotational speed. The energy transmission occurs only on the surface of the bar because the cylinder acts as the semi infinite medium. The phase velocities of higher modes attain large values at vanishing wave number. From the Figs.1 and 2 it is observed that the non-dimensional phase velocity of the fundamental mode is non dispersive and decreases rapidly in the presence of liquid with increasing wave number.

In Figure 3, the dispersion of attenuation coefficients with respect to the wave number of the solid bar is presented for the rotational speed $\Omega = 0.2$. The magnitude of the attenuation coefficient increases monotonically, attaining the maximum in $0.1 \le k \le 0.5$ for first two modes of the solid bar both in space and immersed in fluid , and slashes down to become asymptotically linear in the remaining range of wave number. The dispersion of attenuation coefficients with respect to the wave number for $\Omega = 0.4$ of the solid bar is presented in Fig. 4, where the attenuation coefficient attains the maximum in $0.1 \le k \le 0.3$ with a small oscillation in the starting wave number, and decreases to become linear due to the fluid medium. From Figs. 3 and 4, it is clear that the effects of rotational speed in the solid bar are quite pertinent due to the combined effect of mechanical property and damping effect of the fluid medium. When the ratio of the densities of the fluid and elastic material is small (0.14), then the mode spectrum of fluid loaded bar is slightly different from that of free cylinder.

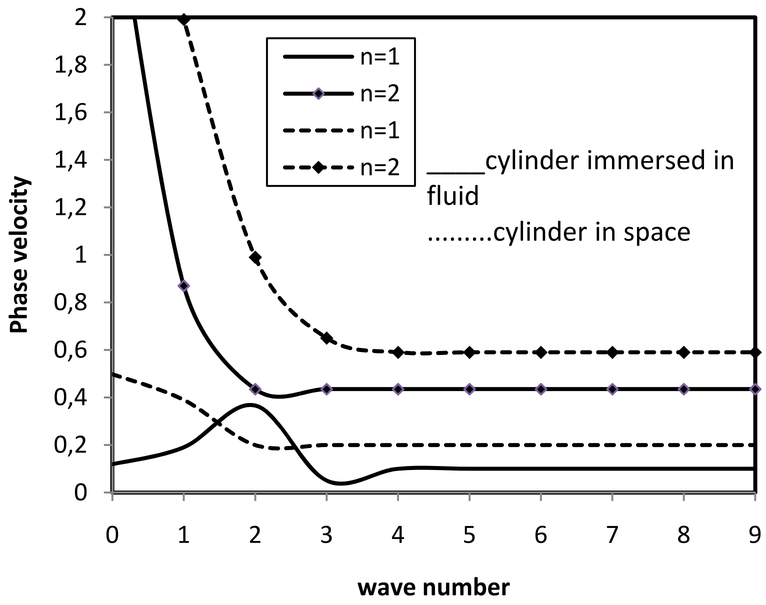

Fig. 1. Dispersion of phase velocity with wave number of Zinc solid bar with $\Omega = 0.2$.

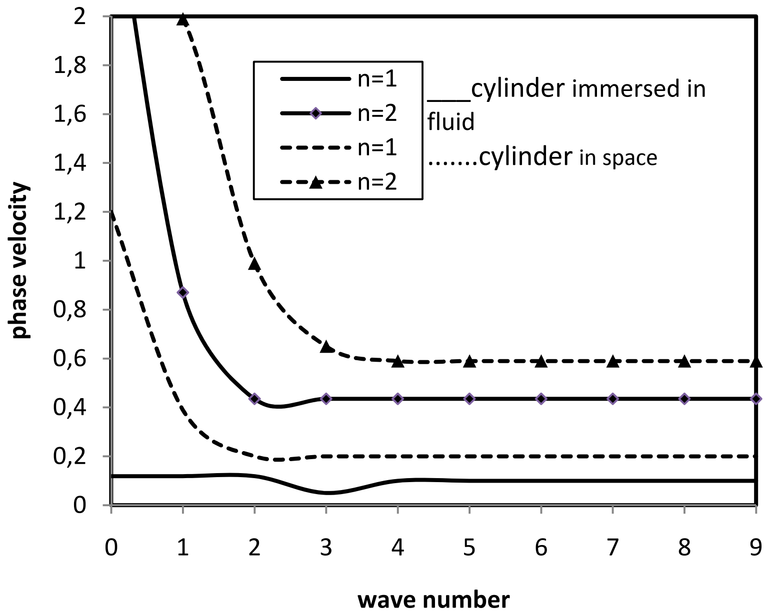

Fig. 2. Dispersion of phase velocity with wave number of Zinc solid bar with $\Omega = 0.4$.

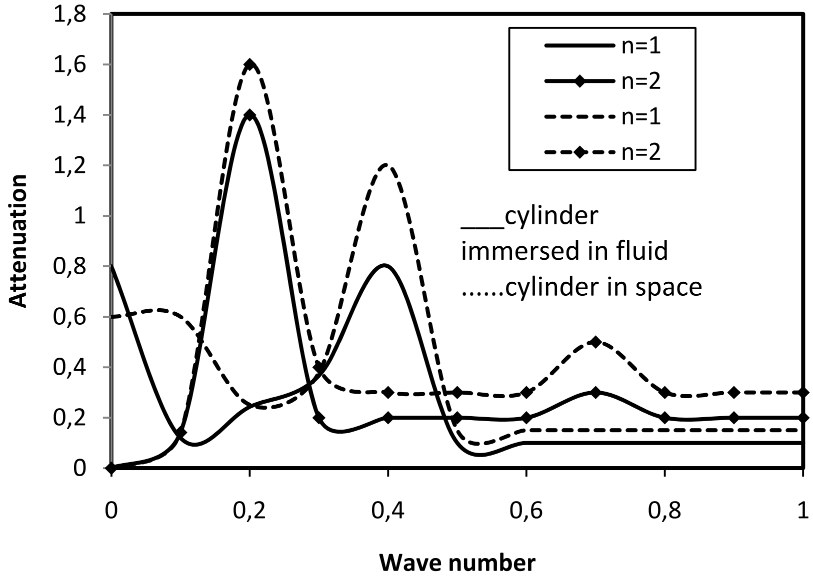

Fig. 3. Dispersion of attenuation with wave number of Zinc solid bar with $\Omega = 0.2$.

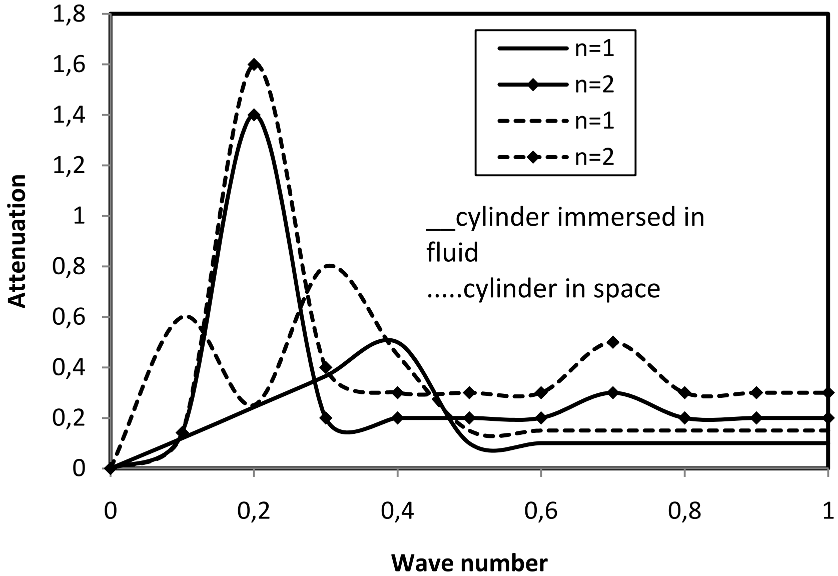

Fig. 4. Dispersion of attenuation with wave number of Zinc solid bar with $\Omega = 0.4$.

7. Conclusions

In this paper, the axisymmetric vibration in a finite, homogeneous transversely isotropic rotating solid bar immersed in a fluid is studied using the linearized, three-dimensional theory of elasticity**.** Two displacement potentional functions are introduced to uncouple the equations of motion. The computed non-dimensional phase velocity and attenuation are presented in the form of dispersion curves for the material Zinc. In addition, a comparative study is made between the non dimensional frequencies of bar in space and bar immersed in fluid among different axisymmetric modes with respect to wave number.

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