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EFFECT OF ROTATION IN AN AXISYMMETRIC VIBRATION OF A TRANSVERSELY ISOTROPIC SOLID BAR IMMERSED IN AN INVISCID FLUID

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Abstract. Effect of rotation in a axis symmetric vibration of a finite, homogeneous transversely isotropic solid bar immersed in a fluid is studied using the linearized, three-dimensional theory of elasticity. The equations of motion of solid and fluid are respectively formulated using the constitutive equations of a transversely isotropic solid bar and the constitutive equations of an inviscid fluid. Two displacement potentional functions are introduced to uncouple the equations of motion. The computed non-dimensional frequency, phase velocity and attenuation are presented in the form of dispersion curves for the material Zinc.

1. Introduction

The axisymmetric waves of a rotating bar have taking interest in many structural applications because of high tensile strength and high corrosion resistance properties. The axisymmetric modes often used to evaluate the material properties of thin metal wires, reinforcement filament in ultrasonic transducers and resonators. A thorough knowledge of various wave propagation characteristics, as a function of material and geometrical parameters is necessary for a wide range of applications, from geophysical prospecting in cased holes, non-destructive evaluation of oil and gas pipelines, to the insulated fiber optic cables for data transmission. The most general form of harmonic waves in a hollow cylinder of circular cross section of infinite length has been analyzed by Gazis [1]. Mirsky [2] investigated analyzed the wave propagation in transversely isotropic circular cylinders of infinite length and presented the frequency equation in Part I and numerical results in Part II. A method, for solving wave propagation in arbitrary cross-sectional cylinders and plates and to find out the phase velocities in different modes of vibrations namely longitudinal, torsional and flexural, by constructing frequency equations was devised by Nagaya [3-5]. He formulated the Fourier expansion collocation method for this purpose. Following Nagaya, Paul and Venkatesan [6] studied the wave propagation in an infinite piezoelectric solid cylinder of arbitrary cross section using Fourier expansion collocation method. The longitudinal waves inhomogeneous anisotropic cylindrical bars immersed in a fluid are studied by Dayal [7]. Guided waves in a transversely isotropic cylinder immersed in a fluid are analyzed by Ahmad [8]. Following Ahmad, Nagay [9] have studied the longitudinal guided wave propagation in a transversely isotropic rod immersed in fluid, later, Nagy with Nayfeh [10] discussed the viscosity-induced attenuation of longitudinal guided waves in fluid-loaded rods. Easwaran and Munjal [11] reported a note on the effect of wall compliance on lowest-order mode propagation in fluid-

$$\sigma_{zz} = c_{13}e_{rr} + c_{13}e_{\theta\theta} + c_{33}e_{zz}\,, \tag{2b}$$

$$\sigma_{rz} = 2c_{44}e_{rz}\,. \tag{2c}$$

Here $\sigma_{rr,}\ \sigma_{zz,}\ \sigma_{rz}$ are the stress components, $e_{rr,}\ e_{\theta\theta,}\ e_{zz,}\ e_{r\theta,}\ e_{\theta z,}\ e_{rz}$ are the strain components, $c_{11}, c_{12}, c_{13}, c_{33}$, c_{44} and $c_{66} = \left(c_{11} - c_{12}\right)/2$ are the five independent elastic constants, ρ is the mass density of the material. The displacement equation of motion has the additional terms with a time dependent centripetal acceleration $\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{u})$ and $2\,\mathbf{\Omega}\times\mathbf{u}_{,t}$, where, $\mathbf{u} = (u, 0, w)$ is the displacement vector and $\mathbf{\Omega} = (0, \Omega, 0)$ is a constant, the comma notation used in the subscript denotes the partial differentiation with respect to the variables.

The strain e_{ij} are related to the displacements by

$$e_{rr} = u_{,r}\,, \quad e_{\theta\theta} = r^{-1}\left(u\right), \qquad e_{zz} = w_{,z}\,, \tag{3a}$$

$$2e_{zr} = \left(u_{,z} + w_{,r}\right), \tag{3b}$$

in which u, and w are the displacement components along radial and axial directions respectively. The comma in the subscripts denotes the partial differentiation with respect to the variables.

Substituting the Eqs. (3) and (2) in the Eq. (1), results in the following three-dimensional displacement equations of motion:

$$c_{11}\left(u_{,rr} + r^{-1}u_{,r} - r^{-2}u\right) + c_{44}u_{,zz} + \left(c_{44} + c_{13}\right)w_{,rz} + \rho\left(\Omega^2 u + 2\Omega w_{,t}\right) = \rho u_{,tt}\,, \tag{4a}$$

$$c_{44}\left(w_{,rr} + r^{-1}w_{,r}\right) + r^{-1}\left(c_{44} + c_{13}\right)\left(u_{,z}\right) + \left(c_{44} + c_{13}\right)u_{,rz} + c_{33}w_{,zz} + \rho\left(\Omega^2 w + 2\Omega u_{,t}\right) = \rho w_{,tt}\,, \tag{4b}$$

In an inviscid fluid-solid interface, the perfect-slip boundary condition allows discontinuity in planar displacement components. That is, the radial component of displacement of the fluid and solid must be equal and the longitudinal components are discontinuous at the interface. The above coupled partial differential equations are also subjected to the following non-dimensional boundary conditions at the surfaces $r = a$:

$$\left(\sigma_{rr} + p^f\right) = \left(\sigma_{rz}\right) = \left(u - u^f\right) = 0\,. \tag{5}$$

3. Solution to solid medium

The Equation (4) is coupled partial differential equations of the three displacement components. This system of equations can be uncoupled by eliminating two of the three displacement components through two of the three equations, but this results in a partial differential equations of fourth order. To uncouple the Eq. (4), we follow Mirsky [2] and assuming the solution of Eqs. (4) as follows:

$$u(r, z, t) = \left[\left(\phi_{,r}\right)\right]e^{i(kz+\omega t)}, \tag{6a}$$

$$w(r, z, t) = \left(i/a\right)\left[W_n\right]e^{i(kz+\omega t)}\,, \tag{6b}$$

The Bessel functions J_n is used when the roots $(\alpha_i a)^2, (i = 1, 2)$ are real or complex and the modified Bessel function I_n is used when the roots are imaginary.
The constants d_i defined in the Eq. (11b) can be calculated from the equation

$$d_i = (1+\bar{c}_{13})(\alpha_i a)^2 \Big/ \left((\alpha_i a)^2 + \varpi^2 - \bar{c}_{33}^2 \varsigma^2 + \Gamma\right), \quad i = 1, 2, \tag{13}$$

where $(\alpha_3 a)^2 = (\varpi^2 - \varsigma^2 + \Gamma)/\bar{c}_{66}$. If $(\alpha_3 a)^2 < 0$, the Bessel function J_n is replaced by the modified Bessel function I_n.

4. Solution of fluid medium

In cylindrical polar coordinates r, θ, and z, the acoustic pressure and radial displacement equations of motion for an invicid fluid are of the form [16]:

$$p^f = -B^f \left(u^f_{r,r} + r^{-1}(u^f_r) + u^f_{z,z}\right), \tag{14}$$

and

$$c_f^{-2} u^f_{r,tt} = \Delta,_r \tag{15}$$

respectively,
where B^f is the adiabatic bulk modulus, ρ^f is the density, $c^f = \sqrt{B^f/\rho^f}$ is the acoustic phase velocity in the fluid, (u^f_r, u^f_z) is the displacement vector, and

$$\Delta = \left(u^f_{r,r} + r^{-1}(u^f_r) + u^f_{z,z}\right). \tag{16}$$

Substituting

$$u^f_r = \phi^f_{,r} \text{ and } u^f_z = \phi^f_{,z}, \tag{17}$$

and seeking the solution of Eq. (15) in the form:

$$\phi^f(r,\theta,z,t) = \left[\phi^f(r)\right] e^{i(kz+\omega t)}, \tag{18}$$

the fluid represents the oscillatory waves propagating away is given by

$$\phi^f = A_3 H^{(1)}(\delta a x), \tag{19}$$

where $(\delta a)^2 = \varpi^2/(\bar{\rho}^f \bar{B}^f) - \varsigma^2$, in which $\bar{\rho}^f = \rho/\rho^f$, $\bar{B}^f = B^f/c_{44}$, $H_n^{(1)}$ is the Hankel function of the first kind. If $(\delta_2 a)^2 < 0$, then the Hankel function of first kind is to be replaced by K_n, where K_n is the modified Bessel function of the second kind. By substituting Eq. (19) in (14) along with (19) and (20), the acoustic pressure for the fluid can be expressed as:

Table 1. The non-dimensional frequencies for different axisymmetric modes (S1, S2, S3) for the solid bar in space and immersed in fluid with $\Omega = 0.2$.

Wave number	Bar in space						Bar immersed in fluid				
	S1		S2		S3		S1		S2		S3
0.1	0.0435		0.1236		0.1681		0.0359		0.1139		0.1284
0.2	0.1801		0.2036		0.2747		0.1702		0.1213		0.2530
0.4	0.2263		0.3928		0.4492		0.1950		0.3563		0.4220
0.6	0.3967		0.4727		0.5391		0.2737		0.4702		0.5295
0.8	0.5310		0.6036		0.7853		0.5029		0.5031		0.6752
1.0	0.6400		0.7308		1.5288		0.7727		0.6092		0.8349
1.2	0.9025		1.7015		1.5824		0.8081		0.9231		0.9142

Table 2. The non-dimensional frequencies for different axisymmetric modes (S1, S2, and S3) for the solid bar in space and immersed in fluid with $\Omega = 0.4$.

Wave number	Bar in space						Bar immersed in fluid				
	S1		S2		S3		S1		S2		S3
0.1	0.0239		0.0765		0.1572		0.0215		0.0442		0.1408
0.2	0.0751		0.2419		0.2345		0.0554		0.1969		0.2205
0.4	0.1674		0.4977		0.5337		0.1444		0.3888		0.4473
0.6	0.1894		0.5385		0.7292		0.2387		0.5023		0.5941
0.8	0.3964		0.6952		0.9408		0.5504		0.6050		0.7303
1.0	0.4051		0.9714		1.4579		0.5551		0.8512		0.8070
1.2	0.7478		1.1350		1.6707		0.6038		1.0230		1.2107

A comparison is made for the non-dimensional frequencies in case of $\Omega = 0.2$, $\Omega = 0.4$ with respect to the wave number for the free and fluid loaded cylinder in Tables 1 and 2, for first three axisymmetric modes of vibration respectively. From these tables, it is clear that as the sequential number of the wave number increases, the non dimensional frequencies also increases for both the free and fluid loaded cylinder. Also, it is clear that the non dimensional frequency exhibits higher amplitudes for $\Omega = 0.4$ compared with the $\Omega = 0.2$.

The dispersion of phase velocities with the wave number is discussed in Fig. 1 and Fig. 2 for both the rotational speed ($\Omega = 0.2$ and $\Omega = 0.4$) of the solid cylinder in different modes of vibration. In Fig. 1 the phase velocity is decreasing at small wave number between 0 and 4 and become steady for higher values of the wave number for remaining modes of vibration with $\Omega = 0.2$. For the case of $\Omega = 0.4$ there is a small deviation on the phase velocity in Fig. 2 due to the damping effect of fluid medium and rotational speed. The energy transmission occurs only on the surface of the bar because the cylinder acts as the semi infinite medium. The phase velocities of higher modes attain large values at vanishing wave number. From the Figs.1 and 2 it is observed that the non-dimensional phase velocity of the fundamental mode is non dispersive and decreases rapidly in the presence of liquid with increasing wave number.

In Figure 3, the dispersion of attenuation coefficients with respect to the wave number of the solid bar is presented for the rotational speed $\Omega = 0.2$. The magnitude of the attenuation coefficient increases monotonically, attaining the maximum in $0.1 \le k \le 0.5$ for first two modes of the solid bar both in space and immersed in fluid , and slashes down to become asymptotically linear in the remaining range of wave number. The dispersion of attenuation coefficients with respect to the wave number for $\Omega = 0.4$ of the solid bar is presented in Fig. 4,

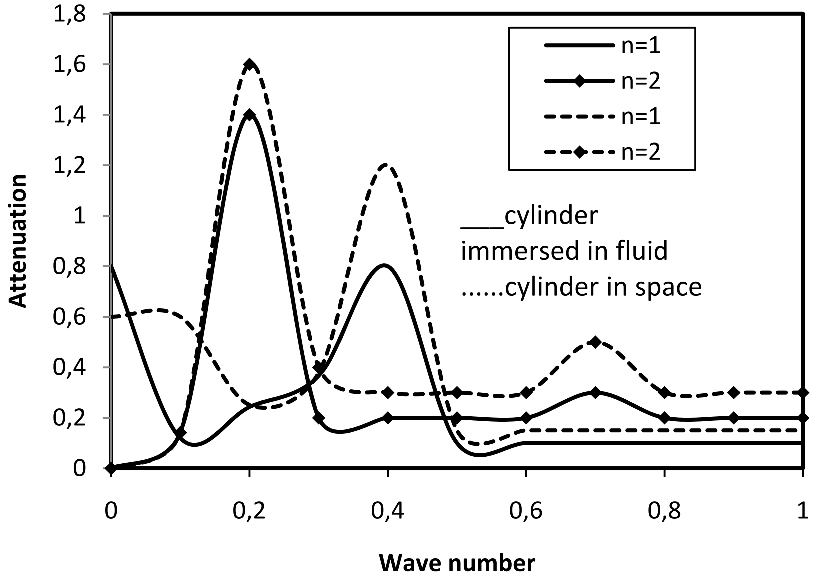

Fig. 3. Dispersion of attenuation with wave number of Zinc solid bar with $\Omega = 0.2$.

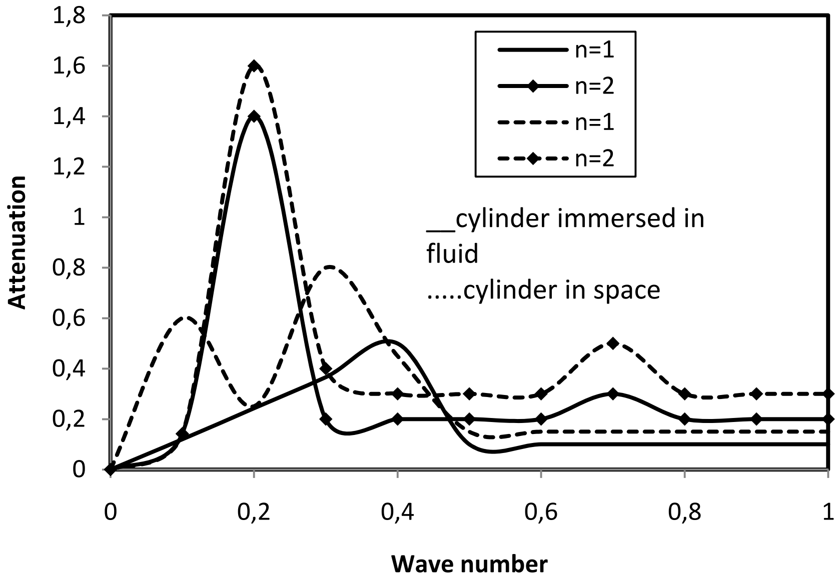

Fig. 4. Dispersion of attenuation with wave number of Zinc solid bar with $\Omega = 0.4$.

7. Conclusions

In this paper, the axisymmetric vibration in a finite, homogeneous transversely isotropic rotating solid bar immersed in a fluid is studied using the linearized, three-dimensional theory of elasticity. Two displacement potentional functions are introduced to uncouple the equations of motion. The computed non-dimensional phase velocity and attenuation are presented in the form of dispersion curves for the material Zinc. In addition, a comparative study is made between the non dimensional frequencies of bar in space and bar immersed in fluid among different axisymmetric modes with respect to wave number.

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