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EXTENSIONAL WAVES IN A TRANSVERSELY ISOTROPIC SOLID BAR IMMERSED IN AN INVISCID FLUID CALCULATED USING CHEBYSHEV POLYNOMIALS

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Abstract. The extensional vibration in a homogeneous transversely isotropic solid bar immersed in an inviscid fluid is studied using the linearized, three-dimensional theory of elasticity. The equations of motion of solid bar and fluid are respectively formulated using the constitutive equations of a transversely isotropic cylinder and the constitutive equations of an inviscid fluid. The solution of the frequency equations are obtained by Chebyshev polynomial series using the geometric boundary conditions. The computed non-dimensional frequencies are presented in the form of dispersion curves for the material Zinc. To compare the model with exiting literature, the longitudinal vibration of cylindrical bar without fluid are obtained and they show good agreement.

1. Introduction

In many structural applications the extensional loadings has taking interest because of high tensile strength and high corrosion resistance properties. The extensional modes often used to evaluate the material properties of thin metal wires, reinforcement filament in ultrasonic transducers and resonators. Applying Chebyshev polynomial series as the admissible function for each displacement component has distinct advantages like rapid convergence and better numerical stability in computation than other algebraic polynomial series.

The most general form of harmonic waves in a hollow cylinder of circular cross section of infinite length has been analyzed by Gazis [1]. Mirsky [2] investigated the wave propagation in transversely isotropic circular cylinders of infinite length and presented the frequency equation in Part I and numerical results in Part II. A method, for solving wave propagation in arbitrary cross-sectional cylinders and plates and to find out the phase velocities in different modes of vibrations namely longitudinal, torsional and flexural, by constructing frequency equations was devised by Nagaya [3-5]. He formulated the Fourier expansion collocation method for this purpose. Following Nagaya, Paul and Venkatesan [6] studied the wave propagation in an infinite piezoelectric solid cylinder of arbitrary cross section using Fourier expansion collocation method. The longitudinal waves inhomogeneous anisotropic cylindrical bars immersed in a fluid is studied by Dayal [7]. Guided waves in a transversely isotropic cylinder immersed in a fluid are analyzed by Ahmad [8]. Following Ahmad, Nagay [9] have studied the longitudinal guided wave propagation in a transversely isotropic rod immersed in fluid, later, Nagy with Nayfeh [10] discussed the viscosity-induced attenuation of longitudinal guided waves in fluid-loaded rods. Easwaran and Munjal [11] reported a note on the effect of wall compliance on lowest-order mode propagation in fluid-

$$\sigma_{r\theta} = 2c_{66}e_{r\theta}\,,\ \sigma_{\theta z} = 2c_{44}e_{\theta z}\,,\ \sigma_{rz} = 2c_{44}e_{rz}\,, \tag{2d}$$

where $\sigma_{rr,}\sigma_{zz,}\sigma_{rz}$ are the stress components, $e_{rr,}$ $e_{\theta\theta,}$ $e_{zz,}$ $e_{r\theta,}$ $e_{\theta z,}$ e_{rz} are the strain components, $c_{11}, c_{12}, c_{13}, c_{33}, c_{44}$ and $c_{66} = (c_{11} - c_{12})/2$ are the five independent elastic constants, ρ is the mass density of the material.
The strain e_{ij} are related to the displacements are given by

$$e_{rr} = u_{,r}\,, \quad e_{\theta\theta} = r^{-1}\left(u + v_{,\theta}\right),\ e_{zz} = w_{,z}\,, \tag{3a}$$

$$2e_{r\theta} = v_{,r} - r^{-1}\left(v - u_{,\theta}\right), \qquad 2e_{zr} = \left(u_{,z} + w_{,r}\right), \qquad 2e_{\theta z} = \left(v_{,z} + r^{-1}w_{,\theta}\right), \tag{3b}$$

in which u, and w are the displacement components along radial, axial directions respectively. The comma in the subscripts denotes the partial differentiation with respect to the variables.

Substituting the Eqs. (3) and (2) in the Eq. (1), results in the following three-dimensional displacement equations of motion:

$$c_{11}\left(u_{,rr} + r^{-1}u_{,r} - r^{-2}u\right) - r^{-2}\left(c_{11} + c_{66}\right)v_{,\theta} + r^{-2}c_{66}u_{,\theta\theta} +$$

$$+ c_{44}u_{,zz} + \left(c_{44} + c_{13}\right)w_{,rz} + r^{-1}\left(c_{66} + c_{12}\right)v_{,r\theta} = \rho u_{,tt}\,, \tag{4a}$$

$$r^{-1}\left(c_{12} + c_{66}\right)u_{,r\theta} + r^{-2}\left(c_{66} + c_{11}\right)u_{,\theta} + c_{66}\left(v_{,rr} + r^{-1}v_{,r} - r^{-2}v\right) +$$

$$+ r^{-2}c_{11}v_{,\theta\theta} + c_{44}v_{,zz} + r^{-1}\left(c_{44} + c_{13}\right)w_{,\theta z} = \rho v_{,tt}\,, \tag{4b}$$

$$c_{44}\left(w_{,rr} + r^{-1}w_{,r} + r^{-2}w_{,\theta\theta}\right) + r^{-1}\left(c_{44} + c_{13}\right)\left(u_{,z} + v_{,\theta z}\right) +$$

$$+ \left(c_{44} + c_{13}\right)u_{,rz} + c_{33}w_{,zz} = \rho w_{,tt}\,. \tag{4c}$$

For extensional wave ,it is assumed that the displacement along the hoop direction, v is zero and Eqs. (4) reduce to

$$c_{11}\left(u_{,rr} + r^{-1}u_{,r} - r^{-2}u\right) + c_{44}u_{,zz} + \left(c_{44} + c_{13}\right)w_{,rz} = \rho u_{,tt}\,, \tag{5a}$$

$$c_{44}\left(w_{,rr} + r^{-1}w_{,r}\right) + r^{-1}\left(c_{44} + c_{13}\right)\left(u_{,z}\right) + \left(c_{44} + c_{13}\right)u_{,rz} + c_{33}w_{,zz} = \rho w_{,tt}\,. \tag{5b}$$

In the inviscid fluid-solid interface, the perfect-slip boundary condition allows discontinuity in planar displacement components. That is, the radial component of displacement of the fluid and solid must be equal and the extensional components are discontinuous at the interface. The above coupled partial differential equations are also subjected to the following non-dimensional boundary conditions at the surfaces $r = a$

$$\left(\sigma_{rr} + p^{f}\right) = \left(\sigma_{rz}\right) = \left(u - u^{f}\right) = 0\,. \tag{6}$$

$$W\left(\bar{r},\bar{z}\right)=F_w\left(\bar{z}\right)\sum_{i=1}^{2}\sum_{j=1}^{2}d_{ij}A_{ij}P_i\left(\bar{r}\right)P_j\left(\bar{z}\right). \tag{12b}$$

The constants d_{ij} defined in the Eq. (12b) can be calculated from the equation

$$d_{ij}=\left(1+\bar{c}_{13}\right)\left(\alpha_i a\right)^2\Big/\left(\left(\alpha_i a\right)^2+\Omega^2-\bar{c}_{33}^{-2}\varsigma^2\right),\quad i=1,2, \tag{13}$$

where the boundary functions for the stress free boundary condition is taken $F_u\left(\bar{z}\right)=F_u^{-1}\left(\bar{z}\right)F_u^{1}\left(\bar{z}\right)$=1 unity, and *i, j* are the order of Chebyshev polynomial series, A_{ij} and B_{kl} are the coefficients of the polynomial. $P_s\left(\psi\right)\left(s=1,2,3,4,5;\psi=\bar{r},\bar{z}\right)$ is the Chebyshev polynomial which can be written in terms of cosine function as follows:

$$P_s\left(\psi\right)=\cos\left[\left(s-1\right)arc\cos\left(\psi\right)\right],\ s=1,2,3...,\quad \psi=\bar{r},\bar{z}. \tag{14}$$

The advantage of using Chebyshev polynomials over using other polynomial functions as trial function has been shown due to its simple and unified expression with cosine function and excellent mathematical properties in numerical approximation containing a set of orthogonal and complete series.

4. Solution of fluid medium

In cylindrical polar coordinates r, θ and z, the acoustic pressure and radial displacement equations of motion for an in viscid fluid are of the form Berliner [16]

$$p^f=-B^f\left(u_{,r}^f+r^{-1}(u^f)+w_{,z}^f\right), \tag{15}$$

and

$$c_f^{-2}u_{,tt}^f=\Delta,_r, \tag{16}$$

respectively, where B^f, is the adiabatic bulk modulus, ρ^f is the density, $c^f=\sqrt{B^f/\rho^f}$ is the acoustic phase velocity in the fluid, and $\left(u^f,w^f\right)$ is the displacement vector.

$$\Delta=\left(u_{,r}^f+r^{-1}(u^f)+w_{,z}^f\right). \tag{17}$$

Substituting

$$u^f=\phi_{,r}^f \text{ and } w^f=\phi_{,z}^f, \tag{18}$$

and seeking the solution of (15) in the form

$$\phi^f(r,\theta,z,t)=\left[\phi^f\left(r\right)\right]e^{i\omega t}. \tag{19}$$

The fluid represents the oscillatory waves propagating away is given by

For the solid the elastic constants are $c_{11} = 1.628\times10^{11}\,Nm^{-2}$, $c_{12} = 0.362\times10^{11}\,Nm^{-2}$, $c_{13} = 0.508\times10^{11}\,Nm^{-2}$, $c_{33} = 0.627\times10^{11}\,Nm^{-2}$, $c_{44} = 0.385\times10^{11}\,Nm^{-2}$ and density $\rho = 7.14\times10^{3}\,kg\,m^{-3}$ and for the fluid the density $\rho^{f} = 1000\,kg\,m^{-3}$ and phase velocity $c^{f} = 1500\,ms^{-1}$.

A comparison is made for the non-dimensional frequencies in case of free and clamped edges with respect to the velocity ratio for the symmetric and anti symmetric modes of the solid bar immersed in fluid in Tables 1 and 2. From these tables, it is clear that as the sequential number of the velocity ratio increases, the non dimensional frequencies also increases for both the free and clamped solid bar. The present solutions for frequency are compared with those of Leissa and So [21] and Zhou [22] for the solid bar without fluid interaction in Table 3 with *L/a* = 2.The first five frequency parameters for the extensional vibration are considered. A good agreement has been achieved. When a solid medium such as the solid bar is surrounded by fluid medium, guided waves are transmitted across the interface. Thus bulk waves are excited in the embedding medium, radiating away from the solid medium.

Table 1. The non-dimensional frequencies for first three symmetric extensional modes of the free and clamped edges of solid bar with velocity ratio.

Velocity ratio (c_0)	Free edge						Clamped edge				
	S_1		S_2		S_3		S_1		S_2		S_3
0.1	0.0635		0.1236		0.1681		0.0259		0.1039		0.1284
0.2	0.1801		0.2636		0.2747		0.1702		0.1213		0.2530
0.4	0.2063		0.3928		0.4492		0.1950		0.3563		0.4220
0.6	0.3967		0.4727		0.5391		0.2837		0.4702		0.5295
0.8	0.5010		0.6036		0.7853		0.5029		0.5031		0.6752

Table 2. The non-dimensional frequencies for first three anti symmetric extensional modes of the free and clamped edges of solid bar with velocity ratio.

Velocity ratio (c_0)	Free edge						Clamped edge				
	S_1		S_2		S_3		S_1		S_2		S_3
0.1	0.0249		0.0865		0.1572		0.0215		0.0342		0.1408
0.2	0.0651		0.2219		0.2345		0.0554		0.1969		0.2205
0.4	0.1774		0.4977		0.5337		0.1444		0.3888		0.4473
0.6	0.1994		0.5385		0.7292		0.2487		0.5023		0.5941
0.8	0.3964		0.6952		0.9408		0.5504		0.6050		0.7303

Dispersion curves. The results of extensional (symmetric and anti symmetric) modes of vibrations are plotted in the following figures with respect to the parameters aspect ratio a/b and L/a. The notations ES1, ES2 and EAS1, EAS2 denote the extensional symmetric and anti-symmetric mode respectively, “1” and “2” refer to the first and the second modes.

down in $0 \le L/a \le 3$ with a small oscillation in the starting L/a ratios, and become linear due to the fluid medium in the rest. From Figs. 3 and 4, it is clear that the effects of length to radius ratio of the solid bar are quite pertinent due to the combined effect of mechanical property and damping effect of the fluid medium. When the ratio of the densities of the fluid and elastic material is small (0.14), then the mode spectrum of fluid loaded bar is slightly different from that of free bar.

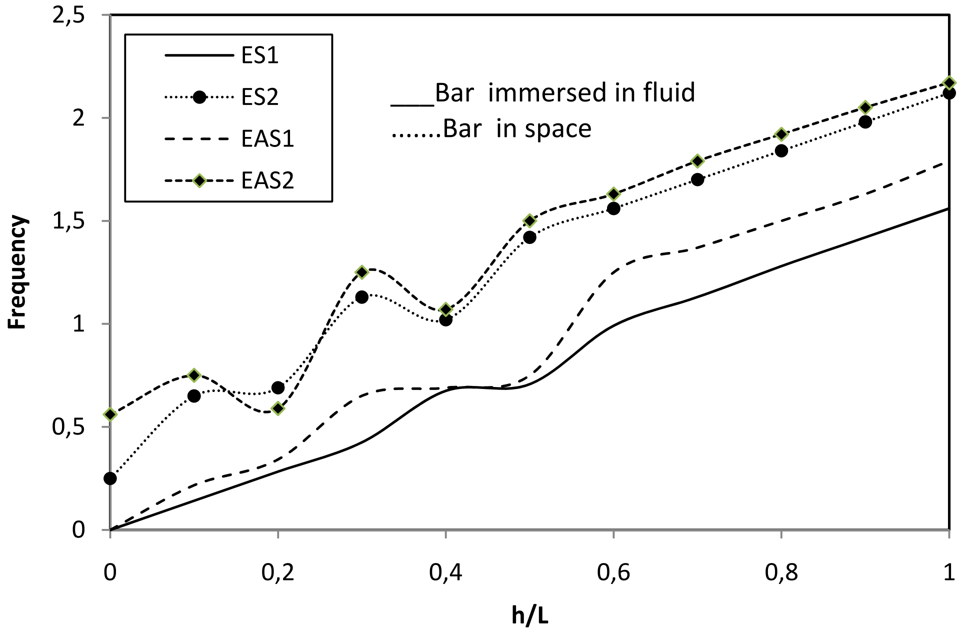

Fig. 2. Variation of frequency with *h/L* for anti symmetric mode of Zinc solid bar.

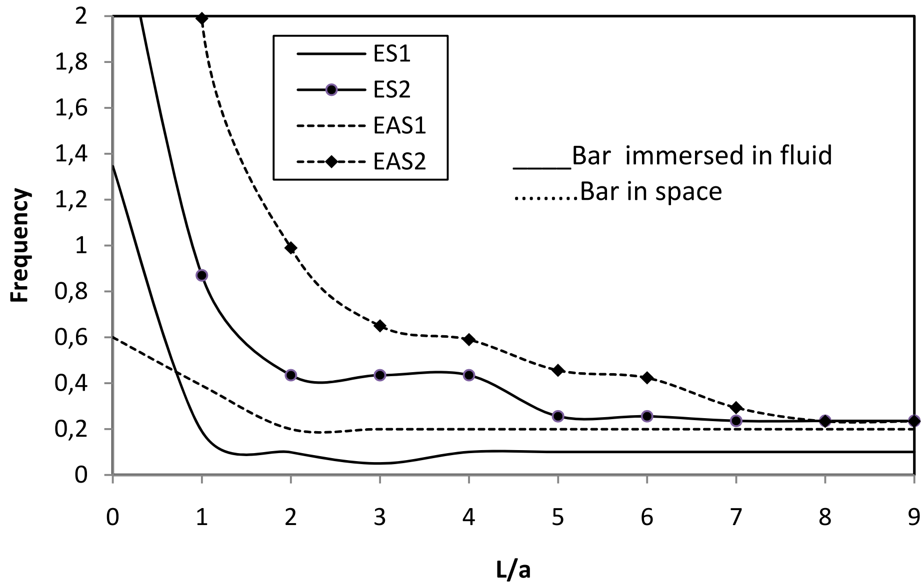

Fig. 3. Variation of frequency with *L/a* for symmetric mode of Zinc solid bar.

7. Conclusions

In this paper, the extensional vibration in a finite, homogeneous transversely isotropic solid bar immersed in a in viscid fluid is studied using the linearized, three-dimensional theory of elasticity**.** Two displacement potential functions are introduced to uncouple the equations of motion. In the present analysis, a set of Chebyshev polynomials multiplied by a boundary function which satisfies the geometric boundary conditions of the cylinder are taken as the trial functions. The computed non-dimensional frequencies are presented in the form of dispersion curves for the material Zinc. In addition, a comparative study is made to prove the feasibility of the model with exiting literature and they show good agreement.

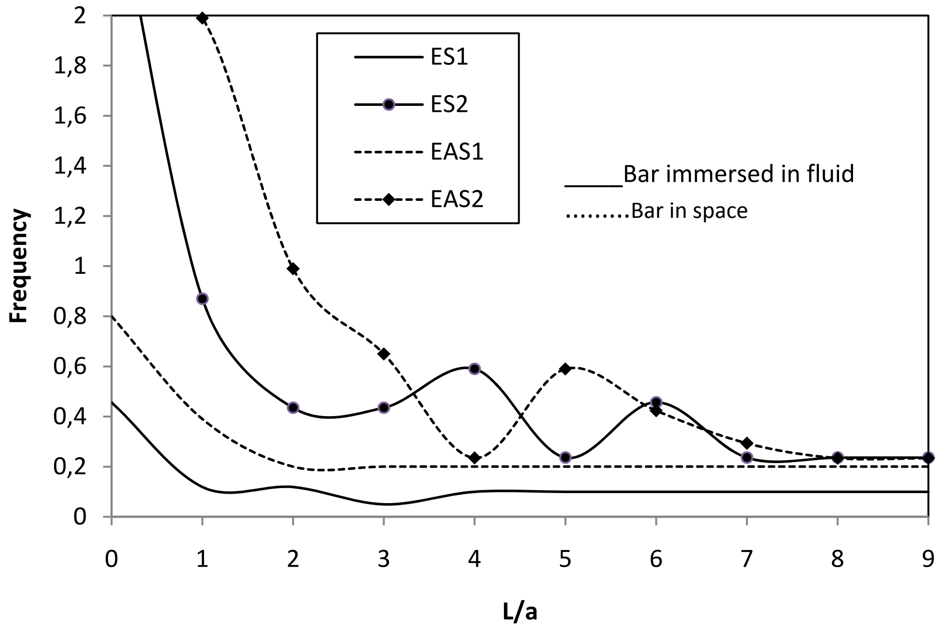

Fig. 4. Variation of frequency with *L/a* for anti symmetric mode of Zinc solid bar.

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