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EFFECT OF VISCOSITY ON WAVE PROPAGATION IN ANISOTROPIC THERMOELASTIC WITH GREEN-NAGHDI THEORY TYPE-II AND TYPE-III

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Abstract. The aim of the present paper is to study the wave propagation in anisotropic thermoviscoelastic medium in the context Green-Naghdi theories of type-II and type-III. It is found that there exist two quasi-longitudinal waves (qP, qT) and two transverse waves (qS1, qS2). The governing equations for homogeneous transversely isotropic thermoviscoelastic are reduced as a special case from the considered model. Different characteristics of waves like phase velocity, attenuation coefficient are computed from the obtained results. Viscous effect is shown graphically on different resulting quantities for Green-Naghdi theories of type-II and type-III. From the present investigation, some particular cases of interest are also deduced.

1. Introduction

The generalized theory of thermoelasticity is one of the modified versions of classical uncoupled and coupled theory of thermoelasticity. It have been developed in order to remove the paradox of physical impossible phenomena of infinite velocity of thermal signals in the classical coupled thermoelasticity. Hetnarski and Ignaczak [1] examined five generalizations of the coupled theory of thermoelasticity.

The first generalization is due to Lord and Shulman [2] who formulated the generalized thermoelasticity theory involving one thermal relaxation time. This theory is referred to as L-S theory or extended thermoelasticity theory in the Maxwell-Cattaneo law replaces the Fourier Law of heat conduction by introducing a single parameter that acts as a relaxation time, who obtained a wave-type equation by postulating a new law of heat conduction instead of classical Fourier's law. Green and Lindsay [3] developed a temperature rate- dependent thermoelasticity that includes two thermal relaxation times and does not violate the classical Fourier's law of heat conduction, when the body under consideration has a center of symmetry. One can refer to Hetnarski and Ignaczak [4] for a review and presentation of generalized theories of thermoelasticity.

Chadwick [5] and Chadwick [6] discussed propagation of plane harmonic waves in transversely isotropic and homogeneous anisotropic heat conduction solids respectively. Banerjee and Pao [7] studied the thermoelastic waves in anisotropic solids. Four characteristic wave velocities are found, three being analogous to those of isothermal elastic waves. The fourth wave, which is predominately a temperature disturbance, corresponds to the heat pulses known as second sound. Sharma [8] and Sharma et al. [9-10] investigated the thermoelastic waves in transversely isotropic material and cubic crystal and orthorhombic material respectively. Sharma et al. [11-12] studied the wave propagation in anisotropic solids in generalized theory of thermoelasticity. Sharma [13] discussed the existence of longitudinal and transverse in anisotropic thermoelastic media.

The third generalization of the coupled theory of thermoelasticity is developed by Hetnarski and Ignaczak [14] and is known as low-temperature thermoelasticity. This model is characterized by a system of non linear field equations. Low-temperature non linear models of heat conduction that predict wave like thermal signals and which are supposed to hold at low temperatures have also been proposed and studied in some works by Kosinski [15], Kosinski and Cimmelli [16].

The fourth generalization to the coupled theory of thermoelasticity introduced by Green and Naghdi and this theory is concerned with the thermoelasticity theory without energy dissipation, referred to as G-N theory of type II in which the classical Fourier law is replaced by a heat flux rate-temperature gradient relation. The heat transport equation does not involve a temperature rate term and as such this model admits undamped thermoelastic waves in thermoelastic material. The fourth generalization of the thermoelasticity theory involves a heat conduction law, which includes the conventional law and one that involves the thermal displacement gradient among the constitutive variables. This model is referred to as the G-N model III [17, 18], which involves dissipation in general and admits thermoelastic waves.

The fifth generalization of the coupled theory of thermoelasticity is developed by Tzau [19] and Chandrasekhariah [20] and is referred to dual phase- lag thermoelasticity. Tzou [19] considered microstructural effects into the delayed response in time in the macroscopic formulation by taking into account that the increase of the lattice temperature is delayed due to phonon-electron interactions on the macroscopic level. A macroscopic lagging response between the temperature gradient and the heat flux seems to a possible outcome due to such progressive interactions. Tzou [19] introduced two phase lags to both the heat flux vector and the temperature gradient and considered constitutive equations to describe the lagging behavior in the heat conduction in solids. Raychoudhuri [21] has recently introduced the three-phase-lag heat conduction equation in which the Fourier law of heat conduction is replaced by an approximation to a modification of the Fourier law with the introduction of three different phase-lags for the heat flux vector, the temperature gradient and the thermal displacement gradient.

Simonetti [22] investigated Lamb wave propagation in elastic plates coated with viscoelastic materials. Sharma [23] discussed the problem of Rayleigh-Lamb wave propagation in visco-thermoelastic plate. Baksi et al. [24] discussed the two-dimensional visco-elastic problems in generalized thermoelastic medium with heat source. Sharma *et al.* [25] investigated the Lamb wave's propagation in viscothermoelastic plate under fluid loadings. Kumar and Partap [26] discussed the vibration analysis of wave micropolar thermoviscoelasic plate. Kumar and Devi [27] investigated the plane wave propagation in anisotropic thermoelastic medium in the context of Green-Naghdi theory type-II and type-III. Kumar and Chawla [28] discussed the plane wave propagation in anisotropic thermoelastic with three-phase-lag and two-phase-lag model.

Keeping in view of these applications, we studied the propagation of waves in the context of Green-Naghdi theory type-II, for anisotropic thermoviscoelastic medium. As a special case, the basic equations for homogeneous transversely isotropic thermoviscoelasticity with Green-Naghdi theory type-II and type–III are reduced. Viscous effect is shown graphically on different characteristics of waves like phase velocities and attenuation coefficients. From the present investigation, some special cases of interest are also deduced.

2. Fundamental equations

The basic equations for homogeneous anisotropic thermoelastic solid, without body forces and heat sources are given as

Constitutive relations

$$\sigma_{ij} = c_{ijkl} e_{kl} - \beta_{ij} T, \qquad \beta_{ij} = c_{ijkl} \alpha_{kl}, \tag{1}$$

$$\rho S T_0 = \rho C^* T + \beta_{ij} T_0 e_{ij}, \qquad e_{ij} = (u_{i,j} + u_{j,i})/2. \tag{2}$$

Equations of motion in the absence of body force

$$\sigma_{ij,j} = \rho \ddot{u}_i. \tag{3}$$

The energy equation (without extrinsic heat supply) is

$$\rho \dot{S} T_0 = -q_{i,i}. \tag{4}$$

The Fourier law (in the Green-Naghdi theory type-II and type-III) is given by Chandrasekharaiah (Chandrasekharaiah, 1998 [20]) as

$$q_i = -(K_{ij} T_{,j} + K^*_{ij} \upsilon_{,j}). \tag{5}$$

In the above equations symbol (“,”) followed by a suffix denotes differentiation with respect to spatial coordinate and a superposed dot (“.”) denotes the derivative with respect to time respectively.

3 Formulation of the problem

We consider a homogeneous, thermally conducting, anisotropic viscoelastic solid in the undeformed state at the uniform temperature T_0.

In order to account for the material damping behavior, the material coefficient c_{ijkl} are assumed to be function of the time operator $D = \dfrac{\partial}{\partial t}$, i.e.

$$c_{ijkl} = \bar{c}_{ijkl},$$

$$\bar{c}_{ijkl} = c_{ijkl}(D). \tag{6}$$

Assumed that the viscoelastic nature of the material is described by the Voigt model of linear viscoelasticity (Kaliski [25]), we write

$$\bar{c}_{ijkl} = c_{ijkl}\left(1 + \tau \frac{\partial}{\partial t}\right). \tag{7}$$

The general system of equations for anistropic thermoviscoelastic material are obtained by using equations (1), (2) and (5), in equations (3) and (4), and with the aid of equation (7), the equation of motion and heat conduction are:

equations of motion

$$\bar{c}_{ijkl}e_{kl,j}-\bar{\beta}_{ij}T_{,j}=\rho\ddot{u}_i\,. \tag{8}$$

where $\bar{\beta}_{ij}=\bar{c}_{ijkl}\bar{\alpha}_{kl}$;
equation of heat conduction

$$K_{ij}\dot{T}_{,ji}+K^*_{ij}T_{,ji}=\left[(\rho C^*\ddot{T}+\bar{\beta}_{ij}T_0\ddot{e}_{ij})\right], \tag{9}$$

for Green-Naghdi theory of type-II $K_{ij}=0$.

4. Solution of the problem
For plane harmonic waves, we assume the solution of equations (8)-(9) of the form

$$(u_1,u_2,u_3,T)=(U_1,U_2,U_3,T^*)\exp[i(\xi x_l n_l-\omega t)], \tag{10}$$

where ω is the circular frequency and ξ is the complex wave number. U_1,U_2,U_3 give the polarization of propagating wave and T^* is the amplitude of temperature distribution in the medium. The vector $\mathbf{n}=(\cos\phi\sin\theta,\sin\phi\sin\theta,\cos\theta)$ is unit wave normal vector, defines the phase direction of the propagating wave represents the propagation along the general direction of (θ,φ) in Cartesian coordinate system (x_1,x_2,x_3), where θ is the polar angle with x_3-axis and φ is azimuth with x_1-axis.
Substituting equation (10) in equations (8)-(9), we obtain

$$[\bar{c}_{ijkl}n_l n_j\xi^2-\rho\omega^2\delta_{ik}]U_k-i\xi\bar{\beta}_{ij}n_jT^*=0\,, \tag{11}$$

$$i\xi\beta_{ij}n_j\omega^2T_0U_m+[(i\xi n_j)(i\xi n_i)(K^*_{ij}-i\omega K_{ij})-\rho C^*\omega^2]T^*=0 \tag{12}$$

where δ_{ik} is the Kronecker delta.
To facilitate the solution, following dimensionless quantities are introduced:

$$x_i{}'=\frac{\omega_1^*x_i}{v_1},\qquad u_i{}'=\frac{\omega_1^*u_i}{v_1},\qquad T=\frac{T}{T_0},\ \ t'=\omega_1^*t,$$

where

$$v_1^2=\frac{\bar{c}_{1111}}{\rho},\qquad \omega_1^*=\frac{\rho C^*v_1^2}{K_{11}}. \tag{13}$$

The Christoffel's tensor notation may be expressed as follows

$$\bar{D}_{ik}=\bar{c}_{ijkl}n_ln_j,\ \ \beta_i=\bar{\beta}_{ij}n_j,\qquad K=K_{ij}n_in_j,\quad K^*=K^*_{ij}n_in_j. \tag{14}$$

Using equations (13) and (14) in equations (11)-(12), we obtain

$$[(\bar{D}_{ik}/\rho v_1^2)\xi^2-\omega^2\delta_{ik}]U_k+i\xi(T_0\beta_i/\rho v_1^2)T^*=0, \tag{15}$$

$$-i\xi\omega^2v_1^2\beta_kU_k+[\xi^2(K^*-i\omega K)-\omega^2\rho C^*v_1^2]T^*=0. \tag{16}$$

The non-trivial solution of the system of equations (15)-(16) is ensured by the determinant equation

$$\begin{vmatrix} (a_1\xi^2-\rho\omega^2) & a_2\xi^2 & a_3\xi^2 & i\xi a_4 \\ a_5\xi^2 & (a_6\xi^2-\rho\omega^2) & a_7\xi^2 & i\xi a_8 \\ a_9\xi^2 & a_{10}\xi^2 & a_{11}\xi^2-\rho\omega^2 & i\xi a_{12} \\ i\xi\omega^2 & i\xi\omega^2 a_{13} & i\xi\omega^2 a_{14} & -a_{15}\xi^2+a_{16}\omega^2 \end{vmatrix} \quad (17)$$

where a_i, $i=1,2,3,4...16$ are given in Appendix A. The equation (17) yields to the following polynomial characteristics equation in ξ as

$$A_1\xi^8 + A_2\xi^6 + A_3\xi^4 + A_4\xi^2 + A_5 = 0. \quad (19)$$

where the coefficient A_i $i=1,2,3,4,5$ are given in Appendix A.

On solving equation (18), we obtain eight roots of ξ that is $\pm\xi_1, \pm\xi_2, \pm\xi_3$ and $\pm\xi_4$ corresponding to these roots, there exists four waves corresponding to descending order of their velocities namely a quasi P-wave (qP) and two quasi-transverse waves (qS1, qS2) and a quasi-thermal wave (qT).

The expressions of phase velocities, attenuation coefficients of these types of waves are given in Appendix A.

Transversely isotropic media. Applying the transformation:

$$x_1' = x_1\cos\varphi + x_2\sin\varphi, \qquad x_2' = -x_1\sin\varphi + x_2\cos\varphi, \qquad x_3' = x_3, \quad (19)$$

where ϕ is the angle of rotation in the $x_1 - x_2$ plane, in the equations (8)-(9) and with the aid of equation (10) and (13), the basic equations for homogeneous transversely isotropic thermoviscoelastic for Green-Naghdi theories of type-II and III, we obtain

$$(\xi^2\Delta_1 - b_3\omega^2)U_1 + b_4\xi^2 n_1 n_2 U_2 + b_5\xi^2 n_1 n_3 U_3 + i\xi n_1 b_6 T^* = 0, \quad (20)$$

$$\xi^2 n_1 n_2 U_1 + (\xi^2\Delta_2 - b_{10}\omega^2)U_2 + b_{11}\xi^2 n_2 n_3 U_3 + i\xi n_2 b_{12} T^* = 0, \quad (21)$$

$$\xi^2 n_1 n_3 U_1 + \xi^2 n_2 n_3 U_2 + (\xi^2\Delta_3 - b_{15}\omega^2)U_3 + i\xi n_3 b_{16} T^* = 0, \quad (22)$$

$$ib_{21}\xi n_1\omega^2 U_1 + ib_{21}\xi n_2\omega^2 U_2 + ib_{21}\xi n_3\breve{\beta}\omega^2 U_3 + [(\xi^2\Delta_4 b_{22} + n_3^2 b_{23}) + b_{20}\omega^2]T^* = 0, \quad (23)$$

where $v_1^2 = c_{11}/\rho$, b_i; $1,2,3...21,22,23$ are given in Appendix B.

Solving equations (20)-(23) for non trivial solution of the system, we obtain the characteristic equation as

$$B_1\xi^8 + B_2\xi^6 + B_3\xi^4 + B_4\xi^2 + B_5 = 0, \quad (24)$$

where the coefficients $B_i (i=1,2,3,4,5)$ are given in Appendix B.

The equation (24) has complete information about phase velocities and attenuation coefficients in transversely isotropic thermoviscoelastic medium.

5. Special cases

Now we will study the propagation of plane harmonic waves in different principal planes as follows:

(i) For propagation in the x_1x_3-plane i.e. $\boldsymbol{n}=(n_1,0,n_3)$; $n_1^2+n_3^2=1$ the characteristic equation (24) reduces to

$$\xi^2\Delta_2^*-b_{10}\omega^2=0, \tag{25}$$

$$E_1\xi^6+E_2\xi^4+E_3\xi^2+E_4=0 \tag{26}$$

where $\Delta_2^*=b_7n_1^2+b_9n_3^2$ and the coefficients E_i; $i=1,2,3,4$ are given in Appendix C.

(ii) For propagation in the x_2x_3-plane i.e. $\boldsymbol{n}=(0,n_2,n_3)$; $n_2^2+n_3^2=1$ the characteristic equation (24) reduces to

$$\xi^2\Delta_{11}^*-b_3\omega^2=0, \tag{27}$$

$$E_{11}\xi^6+E_{22}\xi^4+E_{33}\xi^2+E_{44}=0, \tag{28}$$

where $\Delta_{11}^*=b_1n_2^2+b_2n_3^2$ and the coefficients E_{ii}; $i=1,2,3,4$ are given in Appendix D.

(iii) For propagation in the x_1x_2-plane i.e. $\boldsymbol{n}=(n_1,n_2,0)$; $n_1^2+n_2^2=1$ the characteristic equation (24) reduces to

$$\xi^2\Delta_{111}^*-b_{15}\omega^2=0, \tag{29}$$

$$E_{111}\xi^6+E_{222}\xi^4+E_{333}\xi^2+E_{444}=0, \tag{30}$$

where $\Delta_{111}^*=b_{13}(n_1^2+n_2^2)$ and the coefficients E_{iii}, $i=1,2,3,4$ are given in Appendix E.

(iv) For $\theta=90^\circ$ and propagation in the x_1x_3-plane i.e. $\boldsymbol{n}=(n_1,0,0)$ the characteristic equation (24) reduces to

$$\xi^2\Delta_{1111}^*-b_{10}\omega^2=0, \tag{31}$$

$$\xi^2\Delta_{2222}^*-\mathrm{b}_{10}\omega^2=0, \tag{32}$$

$$\mathrm{E}_{1111}\xi^4+\mathrm{E}_{2222}\xi^2+\mathrm{E}_{3333}=0, \tag{33}$$

where the coefficients E_{iiii}, $i=1,2,3$ are given in Appendix F.

Equations (25), (27), (29), (31), and (32) correspond to purely transverse wave mode that decouple from the rest of the motion and are not affected by the thermal parameters.

6. Particular cases

1. If we take $\tau\to 0$, in equation (24) we obtain the corresponding results for thermoviscoelastic medium in the context of Green-Naghdi theory type-II and type-III and the obtained results are similar as obtained by Kumar and Devi [27].

2. If we take $\overline{c}_{11} = \overline{c}_{22} = \overline{c}_{33},\ \overline{c}_{12} = \overline{c}_{13},\ \overline{c}_{44} = \overline{c}_{66},\ \beta_1 = \beta_2 = \beta_3,\ K_1 = K_3 = K,$ $K_1^* = K_3^* = K^*$ in equation (24), we obtain the corresponding results for cubic crystal thermoviscoelastic materials.

3. If we take $\overline{c}_{11} = \overline{c}_{33} = \overline{\lambda} + 2\overline{\mu},\ \overline{c}_{12} = \overline{c}_{13} = \overline{\lambda},\ c_{44} = \overline{\mu},\ \beta_1 = \beta_3,\ K_1 = K_3 = K,$ $K_1^* = K_3^* = K^*$ in equation (24), then the corresponding results are reduced for isotropic thermoviscoelastic materials.

7. Numerical results and discussion

In order to illustrate theoretical results derived in the proceeding sections, we now present some numerical results. Following (Sharma MD [13]), we take the physical data of Dolomite rock is considered as anisotropic thermoelastic medium (in two-suffixed notations) are given

$c_{11} = 106.8\times10^{12}\ \mathrm{N\cdot m^{-2}}$, $c_{12} = 27.1\times10^{12}\ \mathrm{N\cdot m^{-2}}$, $c_{13} = 9.68\times10^{12}\ \mathrm{N\cdot m^{-2}}$,
$c_{14} = -0.03\times10^{12}\ \mathrm{N\cdot m^{-2}}$, $c_{15} = 0.12\times10^{12}\ \mathrm{N\cdot m^{-2}}$, $c_{16} = 99.0\times10^{12}\ \mathrm{N\cdot m^{-2}}$,
$c_{23} = 18.22\times10^{12}\ \mathrm{N\cdot m^{-2}}$, $c_{24} = 1.49\times10^{12}\ \mathrm{N\cdot m^{-2}}$, $c_{25} = 0.13\times10^{12}\ \mathrm{N\cdot m^{-2}}$,
$c_{26} = -0.58\times10^{12}\ \mathrm{N\cdot m^{-2}}$, $c_{33} = -54.57\times10^{12}\ \mathrm{N\cdot m^{-2}}$, $c_{34} = 2.44\times10^{12}\ \mathrm{N\cdot m^{-2}}$,
$c_{35} = -1.69\times10^{12}\ \mathrm{N\cdot m^{-2}}$, $c_{36} = -0.75\times10^{12}\ \mathrm{N\cdot m^{-2}}$, $c_{44} = 25.97\times10^{12}\ \mathrm{N\cdot m^{-2}}$,
$c_{56} = 1.44\times10^{12}\ \mathrm{N\cdot m^{-2}}$, $c_{46} = 0.43\times10^{12}\ \mathrm{N\cdot m^{-2}}$, $c_{55} = 26.05\times10^{12}\ \mathrm{N\cdot m^{-2}}$,
$c_{66} = 37.82\times10^{12}\ \mathrm{N\cdot m^{-2}}$, with the assumption of thermoelastic parameters as $C^* = 1\times10^{12}\ \mathrm{N\ m^{-2}/K}$, $T_0 = 300\mathrm{K}$, $\beta_0 = 30\times10^3\ \mathrm{N\ m^{-2}/K}$, $K_0 = 10\times10^3\ \mathrm{W\ m^{-1}\ deg^{-1}}$.

The symmetric matrices {1, 0.1, 0.2; 0.1, 1.1, 0.15; 0.2, 0.15, 0.9} and {01, 0.02, 0.03; 0.02, 0.04, 0.05; 0.03, 0.05, 0.06} are multiplied by β_0 and K_0 to define the general anisotropy tensors $\{\beta_{ij}\}$ and $\{K_{ij}\}$, respectively, in which $\theta = 75^0$ has been fixed. Following Dhaliwal and Singh [29], we take the physical data of cobalt for transversely isotropic thermoelastic material as

$c_{11} = c_{22} = 3.071\times10^{11}\ \mathrm{N\cdot m^{-2}}$, $c_{12} = c_{21} = 1.650\times10^{11}\ \mathrm{N\cdot m^{-2}}$, $c_{13} = c_{23} = 1.027\times10^{11}\ \mathrm{N\cdot m^{-2}}$,
$c_{44} = c_{55} = 1.510\times10^{11}\ \mathrm{N\cdot m^{-2}}$, $c_{33} = 3.581\times10^{11}\ \mathrm{N\cdot m^{-2}}$, $\rho = 8.836\times10^3\ \mathrm{Kg\ m^{-3}}$,
$C^* = 4.27\times10^2\ \mathrm{J\ Kg^{-1}K^{-1}}$, $T_0 = 298\ \mathrm{K}$, $K_1 = .690\times10^2\ \mathrm{W\ m^{-1}\ deg^{-1}}$,
$K_3 = .690\times10^2\ \mathrm{W\ m^{-1}\ deg^{-1}}$, $\beta_1 = 7.04\times10^6\ \mathrm{N\ m^{-2}\ deg^{-1}}$, $\beta_3 = 6.90\times10^6\ \mathrm{N\ m^{-2}\ deg^{-1}}$,
$K_1^* = \overline{c}_{11}C^*/4$, $K_3^* = \overline{c}_{33}C^*/4$ with non dimensional parameters $\omega = 0.2$.

Figures 1(a)-1(d) and 2(a)-2(d) exhibit the variations of phase velocities $(V_i, 1, 2, 3, 4)$ and attenuation coefficient $(Q_i, 1, 2, 3, 4)$ w.r.t. θ for the anisotropic case and Figs. 3, 4 depict the variations of phase velocities $(V_i, 1, 2, 3, 4)$ and attenuation coefficient $(Q_i, 1, 2, 3, 4)$ w.r.t. θ for the transversely isotropic case. In all the figures GN-II corresponds to Green Naghdi type-II and GN-III corresponds to Green-Naghdi type-III. VGN-II and VGN-III correspond to viscous effect on GN-II and GN-III.

Figure 1(a) shows that the values of phase velocity for GN-II and GN-III increases for smaller values of θ and for higher values of θ the values of phase velocity decreases, whereas for the case of with viscous effect, the values of phase velocity increase for all values of θ. It is noticed that due to viscosity effect the values of V_1 remain more. Figure 1(b) exhibits the variation of phase velocity V_2 w.r.t. θ and it indicates that the values of phase velocity increase for initial values of θ whereas for higher values of θ the values of V_2

decrease. It is evident that the values of V_2 in case of with viscous effect remain more in comparison to without viscous effect. Figure 1(c) depicts the variation of phase velocity V_3 w.r.t. θ and it indicates that the behavior and variation of V_3 is similar as V_2 whereas the magnitude values of V_3 are different. Figure 1(d) shows variation of phase velocity V_4 w.r.t. θ and it indicates that the values of V_4 increase for smaller values of θ whereas for higher values of θ the values of V_4 decreases.

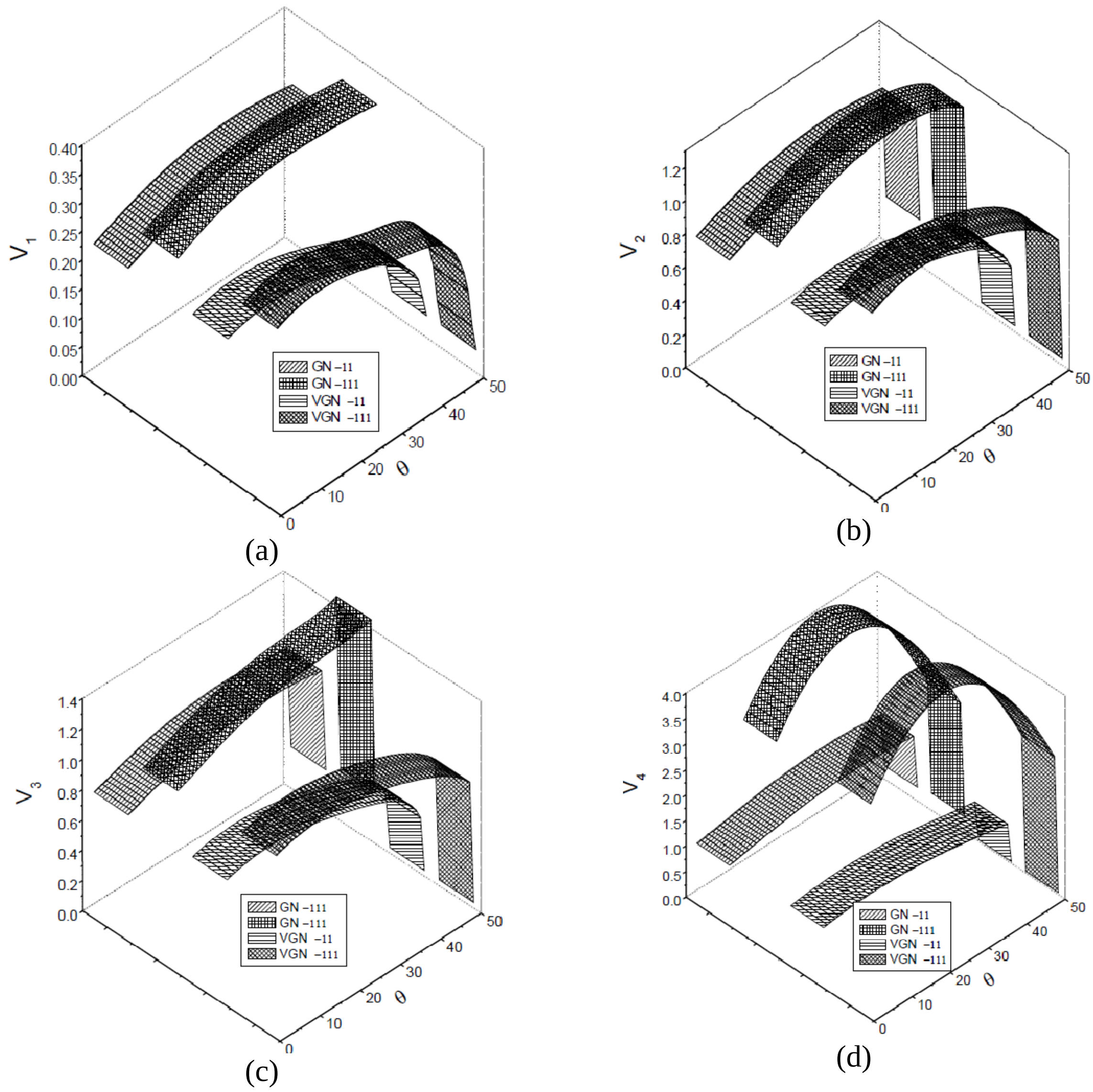


Fig. 1. Variation of phase velocity (a) V_1, (b) V_2, (c) V_3, and (d) V_4 w.r.t. angle θ_0.

Figure 2(a) exhibits the variation of attenuation coefficient (Q_1) w.r.t. θ and it indicates that the values of Q_1 increase for smaller values of θ whereas for higher values of θ, the values of Q_1 increase monotonically. It is noticed that due to viscosity effect the values of Q_1 remain smaller. Figure 2(b) shows that the values of Q_2 increase for smaller values of θ whereas for higher values of θ, the values of Q_2 decrease. It is noticed that due to viscosity effect, the values of Q_2 remain smaller for initial values of θ. Figure 2(c) depicts the variation of Q_3 w.r.t. θ and it indicates that the behavior and variation of Q_3 is similar as Q_1 whereas the magnitude values of Q_3 are different. Figure 2(d) shows the variation of attenuation coefficient Q_4 w.r.t. θ, and it indicates that the values of Q_4 increases for smaller values of θ although for higher values of θ the values of Q_4 decrease.

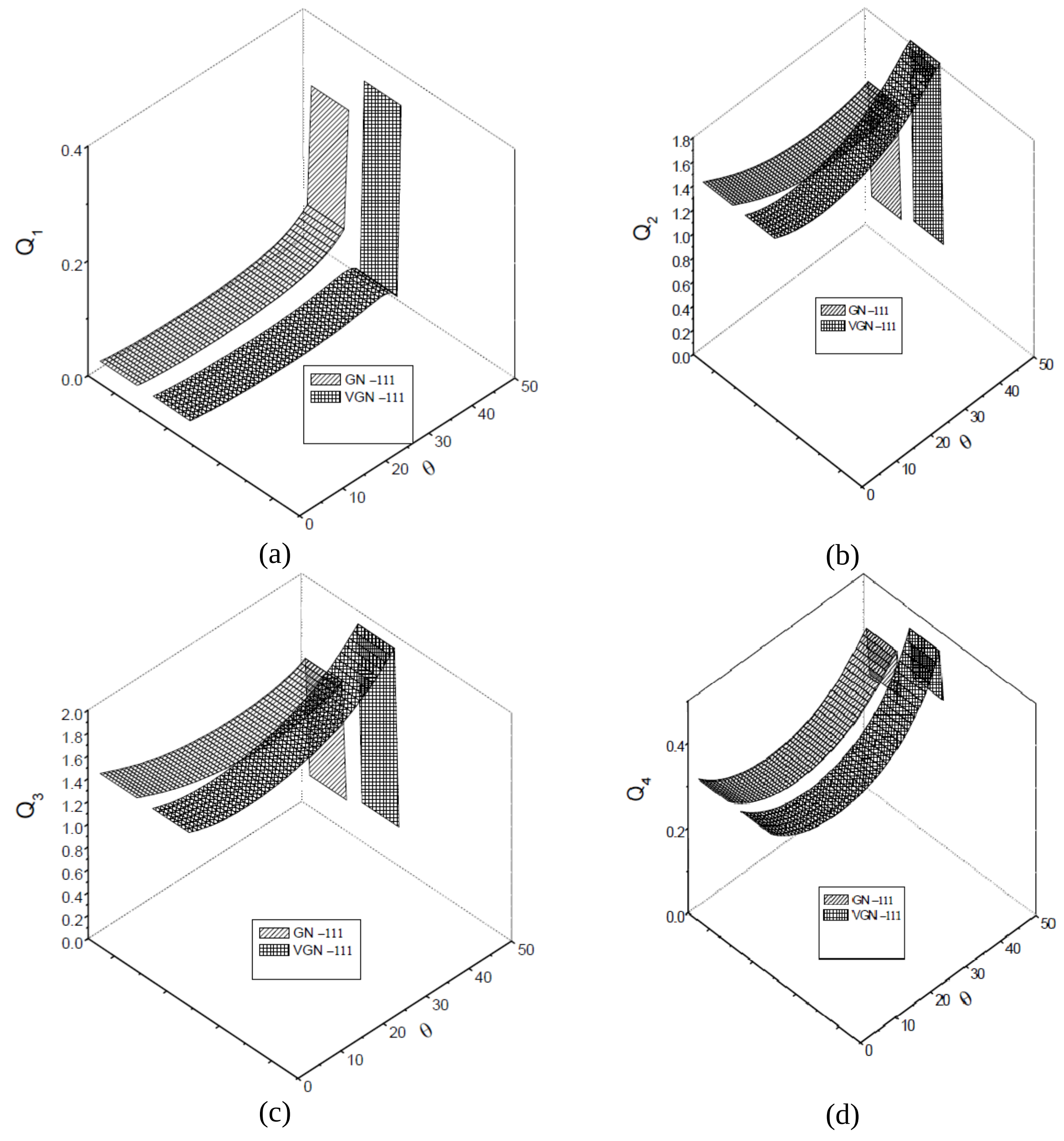
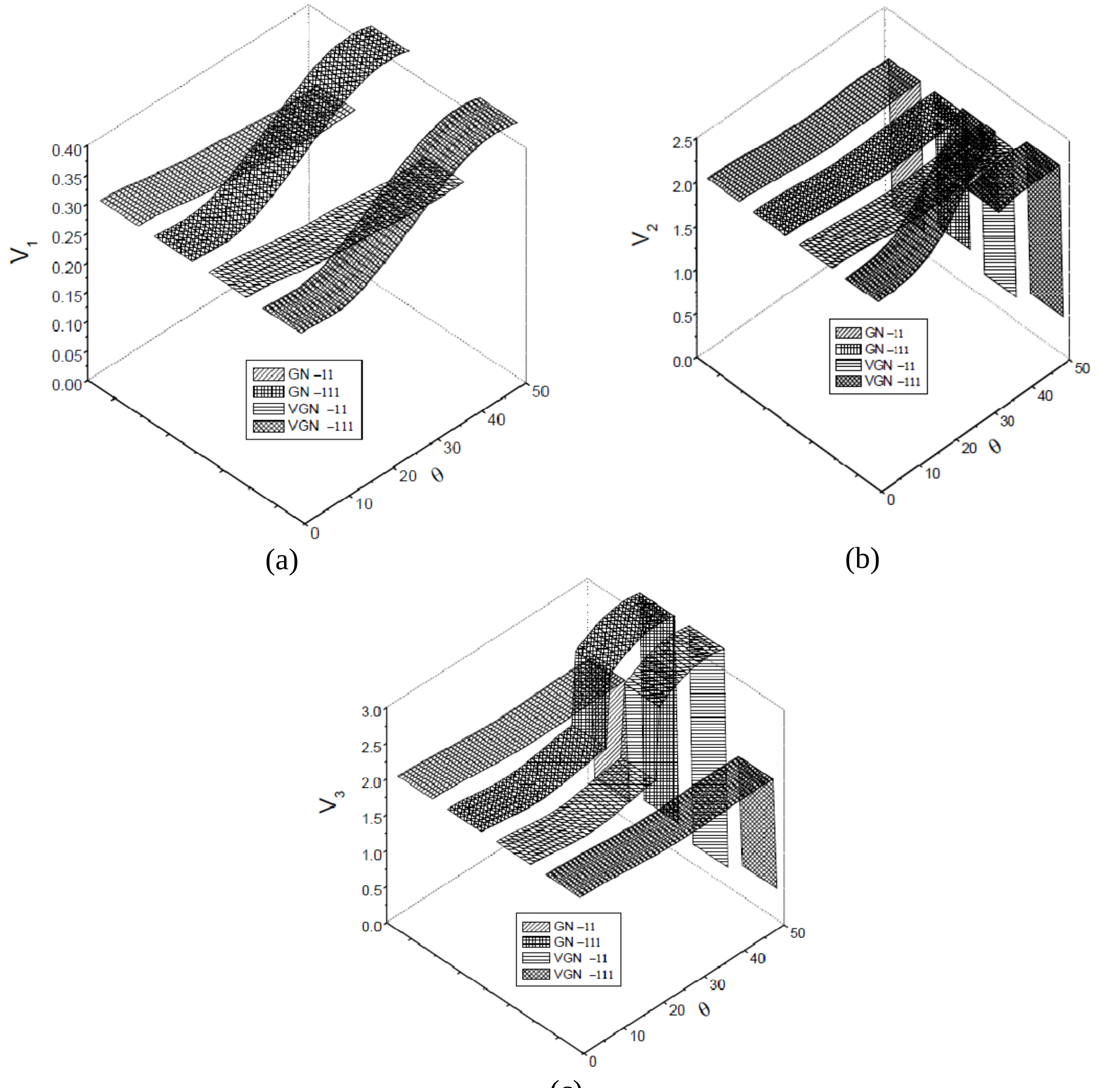


Fig. 2. Variation of attenuation coefficient (a) Q_1, (b) Q_2, (c) Q_3, and (d) Q_4 w.r.t. angle θ.

Figure 3(a) depicts the variation of phase velocity V_1 w.r.t. angle θ and it indicates that the values of V_1 increase for smaller values of θ although for higher values of θ, the values of V_1 become dispersionless. It is noticed that the values of V_1 due to viscosity effect remain more in comparison with without viscous effect. Figure 3(b) depicts the variation of phase velocity V_2 w.r.t. angle θ. The values of V_2 increase for smaller values of θ whereas for higher values of θ, the values of V_2 decrease. Figure 3(c) depicts the variation of phase velocity V_3 w.r.t. angle θ. It is evident that the behavior and variation of V_3 is similar as V_2 although the magnitude values of V_3 are different.



(a) (b) (c)

Fig. 3. Variation of phase velocity (a) V_1, (b) V_2, and (c) V_3 w.r.t. angle θ_0.

Figure 4(a) exhibits the variation attenuation coefficient Q_1 w.r.t. angle θ and it indicates that the values of Q_1 increase for smaller values of θ although for higher values of θ, the values of Q_1 increases monotonically. It is noticed that the values of Q_1 in case of without viscous effect remain more in comparison with viscous effect. Figure 4(b) exhibits the variation attenuation coefficient Q_3 w.r.t. angle θ. It is noticed that the values of Q_3 increases for all values of θ and for comparison, it is evident that the values of Q_3 due to viscosity effect remain smaller in comparison with without viscous effect. Figure 4(c) depicts the variation attenuation coefficient Q_3 w.r.t. angle θ. It is evident that the behavior and variation of Q_3 is similar as Q_1 although the magnitude values of Q_3 are different.

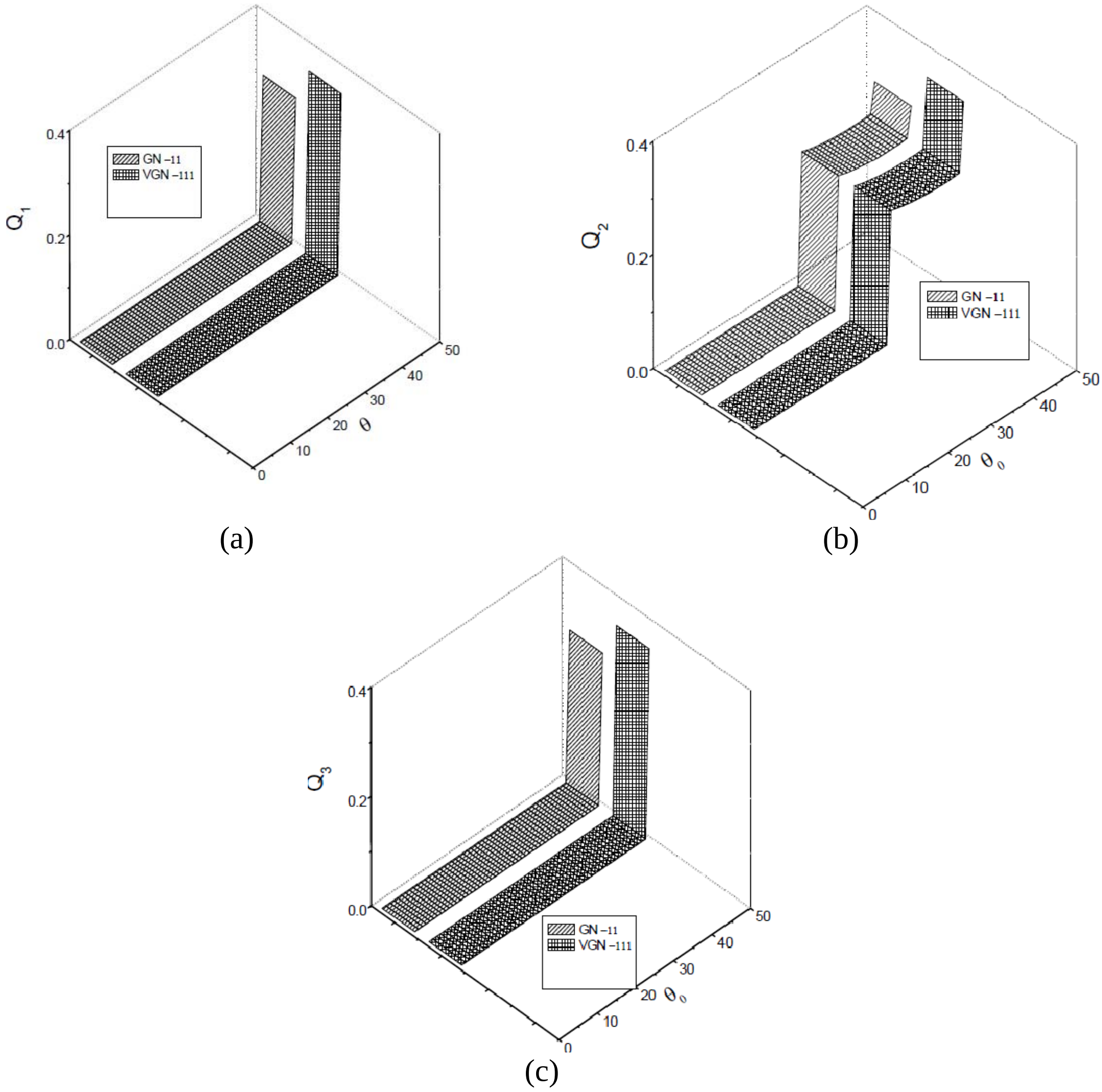


Fig. 4. Variation of attenuation coefficient (a) Q_1, (b) Q_2, and (c) Q_3 w.r.t. angle θ.

8. Conclusions

The propagation of plane wave in anisotropic thermoviscoelastic medium in the context of the theory Green-Naghdi theory type-II and (GN-II) and Green-Naghdi theory type-III (GN-III) have been investigated. The governing equations for homogeneous transversely isotropic GN-II and GN-III are reduced as a special case and obtained that three coupled quasi waves and one quasi-transverse wave which is decoupled from rest of the motion. From the obtained results the different characteristics of waves like phase velocity, attenuation coefficient are computed numerically and presented graphically.

From the numerical results, we conclude that in anisotropic case, the values of phase velocities V_1, V_2, V_3, and V_4 in case of VGN (type-II) and VGN (type-III) remain more in comparison with GN-II and GN-III whereas the values of attenuation coefficients Q_1, Q_2, Q_3, and Q_4 remain more in case of GN-III in comparison with VGN (type-III). It is noticed that in case of transversely isotropic the values of phase velocities and attenuation coefficients are similar with the case of anisotropic although the magnitude values are different.

Nomenclature

$c_{ijkm}(=c_{kmij}=c_{ijkm}=c_{ijmk})$ - elastic parameter,
$\vec{u}$ - displacement vector,
β_{ij} - thermoelastic coupling tensor,
ρ - density at constant strain,
C^* - specific heat at constant strain,
T_0 - reference temperature assumed to be such that $\left|\frac{T}{T_0}\right| << 1$,
q_i - the heat flux vector,
S - entropy per unit mass,
$T(x_1,x_2,x_3,t)$ - temperature distribution from the reference temperature T_0,
$\sigma_{ij}(=\sigma_{ji})$ - component of stress tensor,
e_{ij} - component of strain tensor,
$K_{ij}(=K_{ji})$ - coefficient of thermal conductivity,
$K^*_{ij}(=K^*_{ji})$ - thermal coefficients which are characteristics of GREEN-NAGHADI-II theory,
$\omega -$ circular frequency,
$\xi -$ complex wave number,
U_1, U_2, U_3 - the polarization of propagating wave,
T^* - amplitude of temperature distribution in the medium,
$\mathbf{n}=(\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta)$ - unit wave normal vector,
θ - polar angle with x_3-axis,
ϕ - azimuth with x_1-axis,
φ - angle of rotation in the x_1-x_2 plane.

Appendix A

(i) Phase velocity
The phase velocity are given by

$$V_i=\frac{\omega}{\mathrm{Re}(\xi_i)}, \quad i=1,2,3,4, \tag{A.1}$$

where V_1,V_2,V_3,V_4 are the velocities of qP1, qS1, qS2 and qP2 waves respectively.
(ii) Attenuation coefficient
The attenuation coefficient is defined as

$$Q_i=\mathrm{Im}\, g(\xi_i), \text{ i=1, 2, 3, 4}, \tag{A.2}$$

where Q_i, $i=1,2,3,4$ are the attenuation coefficient of qP1, qS1, qS2 and qP2 waves respectively.

$$(a_1,a_2,a_{13},a_4,a_5,a_6,a_7,a_8,a_9,a_{10},a_{11},a_{12})=\frac{1}{\rho v_1^2}(\bar{D}_{11},\bar{D}_{12},\bar{D}_{13},\beta_1 T_0,\bar{D}_{21},\bar{D}_{22},\bar{D}_{23},\beta_2 T_0,\bar{D}_{31},\bar{D}_{32},\bar{D}_{33},\beta_3 T_0),$$

$$(a_{13}, a_{14}, a_{15}, a_{16}) = \frac{1}{\beta_1}(\beta_2, \beta_3, \frac{K^* - i\omega\omega_1^* K}{v_1^2}, \rho C^*), \ K_{ij}^* = \frac{\bar{c}_{ij} C^*}{4},$$

$$A_1 = -a_{16}, \ A_2 = (a_1 + a_6 + a_{11})a_{16} + (a_{15} + a_8 a_{13}) + a_4(1 + a_{12}),$$

$$A_3 = -a_1(a_6 a_{16} + (a_{15} + a_{11}a_{16} + a_4 a_{12}) + a_8 a_{13}) - a_6(a_{15} + a_{11}a_{16} + a_4 a_{12}) - a_{11}a_{15} + a_7(a_{10}a_{16} + a_{13}a_{12}) +$$

$$a_8(a_{10}a_{14} + a_{13}a_{11}) + a_2 a_5 a_{16} - a_2 a_8 + a_3(a_9 a_{16} + a_{12}) + a_4 a_5 a_{12} - a_4 a_6 + a_4(a_9 a_{14} - a_{11}),$$

$$A_4 = a_1 a_6(a_{15} + a_{11}a_{16} + a_4 a_{12}) + a_{11}a_{15} - a_7(a_{10}a_{16} + a_{13}a_{12}) - a_8(a_{10}a_{14} - a_{13}a_{11}) + a_{15}(a_{11}a_6 - a_7 a_{10}) -$$

$$a_2 a_5(a_{15} + a_{11}a_{16} - a_{12}a_{14}) + a_2 a_7(a_9 a_{16} + a_{12}) + a_2 a_8(a_9 a_{14} + 1) + a_3 a_5(a_{10}a_{16} - a_{12}a_{13}) -$$

$$a_3 a_9(a_6 a_{16} + a_{12} + a_{15}) + a_3 a_8(-a_9 a_{13} + a_{10}) + a_4 a_5(a_{10}a_{14} - a_{11}a_{13}) - a_4 a_6(a_{10}a_{14} - a_{11}) +$$

$$a_4 a_7(a_9 a_{13} - a_{10}), \ A_5 = a_1 a_{15}(a_7 a_{10} - a_1 a_6) + a_2 a_{15}(a_{11}a_5 - a_7 a_9) + a_3 a_5(-a_{10}a_{15} + a_6 a_9).$$

Appendix B

$$(b_1, b_2, b_3, b_4, b_5, b_6) = \frac{1}{\bar{c}_{11}}(\bar{c}_{66}, \bar{c}_{44}, \rho v_1^2, \bar{c}_{12} + \bar{c}_{16}, \bar{c}_{13} + \bar{c}_{44}, \beta_1 T_0),$$

$$(b_7, b_8, b_9, b_{10}, b_{11}, b_{12}) = \frac{1}{\bar{c}_{66} + \bar{c}_{12}}(\bar{c}_{66}, \bar{c}_{11}, \bar{c}_{44}, \rho v_1^2, \beta_1 T_0, \beta_3 T_0),$$

$$(b_{13}, b_{14}, b_{15}, b_{16}) = \frac{1}{\bar{c}_{44} + \bar{c}_{13}}(\bar{c}_{44}, \bar{c}_{33}, \rho v_1^2, \beta_3 T_0),$$

$$(b_{17}, b_{18}, b_{19}, b_{20}, b_{21}) = (K_3^*, \omega_1^* K_1, \omega_1^* K_3, \rho C^* c_1^2, \beta_1 v_1^2), \ b_{22} = (-1 + i\omega b_{18}), b_{23} = (-b_{17} + i\omega b_{19}),$$

$$B_1 = b_3 b_{10} b_{15} b_{20}, \ B_2 = (-b_{10}\Delta_1 + b_3\Delta_2)b_{15}b_{20} + b_3 b_{10}(-b_{20}\Delta_3 + b_{15}b_{24} - b_{21}b_{16}n_3^2\beta) -$$

$$b_{21}b_{15}(b_3 b_{12} n_2^2 + b_6 b_{10} n_1^2), \ B_3 = -\Delta_1(b_{15}b_{20}\Delta_2 + b_{10}(-b_{20}\Delta_3 + b_{15}b_{24} - b_{21}b_{16}n_3^2\beta) + b_{12}b_{21}b_{15}n_2^2) +$$

$$b_3\Delta_2(-b_{20}\Delta_3 + b_{15}b_{24} - b_{21}b_{16}n_3^2\beta) + b_3(\Delta_3 b_{10}b_{24} + b_{11}b_{20}n_2 n_3 + (b_{11}b_{16} + b_{12}\beta)b_{21}n_2^2 n_3^2 - b_{12}b_{21}n_2^2\Delta_3) -$$

$$n_1^2 n_2^2 b_{20}(b_4 b_{15} - b_6 b_{21}) - b_5 b_{10} n_1^2 n_3^2(b_{20} + b_{16}b_{21}) + b_6 b_{21} n_1^2(b_{15}(n_2^2 - \Delta_2) + b_{10}(n_2 n_3\beta - \Delta_3)),$$

$$B_4 = -\Delta_1\Delta_2(-b_{20}\Delta_3 + b_{15}b_{24} - b_{21}b_{16}n_3^2\beta) + b_{10}b_{24}\Delta_1\Delta_3 - b_{11}b_{20}n_2^2 n_3^2\Delta_1 + b_{21}n_2^2 n_3^2\Delta_1(b_{11}b_{16} + b_{12}\beta) -$$

$$b_{12}b_{21}n_2^2\Delta_1\Delta_3 - b_3(\Delta_2\Delta_3 b_{24} + n_2^2 n_3^2 b_{11}b_{24}) + n_1^2 n_2^2 n_3^2(-b_4 b_{21}b_{16}\beta + b_4 b_{11}b_{20} + b_4 b_{11}b_{21}b_{16} +$$

$$b_4 b_{12}b_{21}\beta + b_5 b_{20}) - n_1^2 n_2^2(b_4(b_{20}\Delta_3 + b_{15}b_{24}) + b_{12}b_{15}b_{21}) - b_6 b_{21}\Delta_3) - n_1^2 n_3^2(-b_5 b_{20}\Delta_2 + b_5 b_{10}b_{24} -$$

$$b_5 b_{16}b_{21}\Delta_2 - b_5 b_{21}\beta n_1^2 n_2^2 n_3 + b_5 b_6 b_{21}\beta n_2^2 n_3^2 n_1 + b_6 b_{21} n_1^2\Delta_2(\beta n_3^2 n_2 - \Delta_3),$$

$$B_5 = -\Delta_1(\Delta_2\Delta_3 b_{24} + n_2^2 n_3^2 b_{11}b_{24}) - n_1^2 n_2^2 n_3^2 b_{24}(b_4 b_{11} + b_5) + b_4 n_1^2 n_2^2\Delta_3(b_{24} - b_{12}b_{21}) + n_1^2 n_3^2 b_5 b_{24}\Delta_2,$$

$$\Delta_1 = n_1^2 + b_1 n_2^2 + b_2 n_3^2, \ \Delta_2 = b_7 n_1^2 + b_8 n_2^2 + b_9 n_3^2, \ \Delta_3 = b_{13}(n_1^2 + n_2^2) + b_{14}n_3^2, \ \Delta_4 = n_1^2 + n_2^2 + n_3^2 = 1.$$

Appendix C

$$E_1 = b^*_{24}(\Delta^*_1\Delta^*_3 - b_5 n_1^2 n_3^2),\; E_2 = \omega^2\{(-b_{15}b^*_{24} + \Delta^*_3 b_{20} + b_{16}b_{21}n_3^2\breve{\beta})\Delta^*_1 - b_3 b^*_{24}\Delta^*_3 -$$

$$b_5 n_1^2 n_3^2(b_{20} + b_{16}b_{21}n_1 n_3) + b_6 b_{21} n_1^2(-n_3^2\beta + \Delta^*_3)\},$$

$$E_3 = \omega^4\{-b_{15}b_{20}\Delta^*_1 + b_3(b_{15}b^*_{24} - \Delta^*_3 b_{20} - b_6 b_{21} n_3^2\breve{\beta}) - b_6 b_{21} b_{15} n_1^2,$$

$$E_4 = b_3 b_{15} b_{20}\omega^6,\; \Delta^*_1 = n_1^2 + b_2 n_3^2,\; \Delta^*_3 = b_{13}n_1^2 + b_{14}n_3^2,\; b^*_{24} = \Delta^*_4 b_{22} + n_3^2 b_{23}.$$

Appendix D

$$E_{11} = b^{**}_{24}(\Delta^{**}_2\Delta^{**}_3 - b_{11}n_2^2 n_3^2),\; E_{22} = \omega^2\{(-b_{15}b^{**}_{24} + \Delta^{**}_3 b_{20} + b_{16}b_{21}n_3^2\breve{\beta})\Delta^{**}_2 - b_{10}b^{**}_{24}\Delta^{**}_3 -$$

$$b_{11}n_2^2 n_3^2(b_{20} + b_{16}b_{21}) + b_{12}b_{21}n_2(-n_3\breve{\beta} + \Delta^{**}_3)\},$$

$$E_{33} = \omega^4\{-b_{15}b_{20}\Delta^{**}_2 - b_{10}(b_{15}b^{**}_{24} - \Delta^{**}_3 b_{20} - b_{16}b_{21}n_3^2\breve{\beta}) - b_{12}b_{21}b_{15}n_2^2,$$
$$E_{44} = b_{10}b_{15}b_{20}\omega^6,\; \Delta^{**}_2 = b_8 n_2^2 + b_2 n_3^2,\; \Delta^*_3 = b_{13}n_1^2 + b_{14}n_3^2,\; b^*_{24} = \Delta^{**}_4 b_{22} + n_3^2 b_{23}.$$

Appendix E

$$E_{111} = b^{***}_{24}(\Delta^{***}_1\Delta^{***}_2 - b_4 n_1^2 n_2^2),\; E_{222} = \omega^2\{(-b_{15}b^{***}_{24} + \Delta^{***}_3 b_{20} + b_{16}b_{21}n_3^2\breve{\beta})\Delta^{***}_1 - b_3 b^{***}_{24}\Delta^{***}_3 -$$

$$b_5 n_1^2 n_3^2(b_{20} + b_{16}b_{21}n_1 n_3) + b_6 b_{21} n_1^2(-n_3^2\beta + \Delta^{***}_3)\},$$

$$E_{333} = \omega^4\{b_3 b_{10}\Delta^{***}_2 - b_{20}(\Delta^{***}_2 b_3 + \Delta^{***}_1 b_{10} + b_6 b_{21} b_{10} n_1^2),$$

$$E_{444} = b_{10}b_3 b_{20}\omega^6,\; \Delta^{***}_1 = n_1^2 + b_2 n_3^2,\; \Delta^{***}_3 = n_1^2 + b_2 n_3^2,\; b^{***}_{24} = \Delta^{***}_4 b_{22} + b_{23}.$$

Appendix F

$$\Delta^*_{1111} = b_7 n_1^2,\;\; b^{****}_{24} = \Delta^{****}_3 b_{22} + b_{23},\;\; E_{1111} = n_1^2, b^{****}_{24},$$

$$\Delta^*_{4444} = n_1^2,\;\; E_{2222} = \omega^2(n_1^2(b_{20} + b_6 b_{21}) - b_3 b^{****}_{24}),\;\; E_{3333} = -b_3 b_{20}\omega^4.$$

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