Materials Physics and Mechanics 16 (2013) 175-186 Received: February 22, 2013

ANALYSIS OF THERMOELASTIC DISC WITH RADIATION CONDITIONS ON THE CURVED SURFACES

Rajneesh Kumar[1*], N.K. Lamba[2], Vinod Varghese[3***]**

[1]Department of Mathematics, Kurukshetra University, Kurukshetra, Haryana, India

[2]Department of Mathematics, MJP Educational Campus, RTM Nagpur University, Nagpur (MS), 440033, India

[3]Reliance Industries Limited, Nagpur Manufacturing Division, Mauda, Nagpur (MS) 440 104, India

*e-mail: rajneesh_kuk@rediffmail.com

**e-mail: navneetkumarlamba@gmail.com

***e-mail: vinod.varghese@ril.com

Abstract. The principal aim of this paper is to investigate the thermoelastic problems in a nonhomogeneous thick annular disc in which sources are generated according to the linear function of the temperature, with compounded effect due to partial heating and boundary conditions of the radiation type. The solutions are based on theory of integral transformations with boundary conditions of radiation type on the curved surfaces, with independent radiation constants. The results are obtained in series form in terms of Bessel's functions. Some numerical results for the temperature change, the displacement, and the stress distributions are shown in figures.

1. Introduction

As a result of the increased usage of industrial and construction materials the interest in the thermal stress problems has grown considerably, typified by the annular fins of heat exchangers and brake disc rotors, because of its elementary geometry. Therefore, a number of theoretical studies concerning them have been reported so far. However, to simplify the analyses, almost all the studies were conducted on the assumption that the upper and lower surfaces of the discs or circular are insulated or heat is dissipated with uniform heat transfer coefficients throughout the surfaces. For example, Noda et al. [1] has considered a circular plate and discussed the transient thermoelastic-plastic bending problem, making use of the strain increment theorem. Khobragade et al. [2] has studied the distributed heat supply of a thin circular plate by using finite Hankel and Fourier transform with Dirichlet's type of boundary conditions. Varghese et al. [10] studied themoelastic response due to partially distributed heat supply of a hollow cylinder structure by using transform technique.

Nasser [8, 9] investigated problems due to heat sources in generalized thermoelastic body. Kulkarni et al. [3] determined quasi-static thermal stresses in a thick annular disc subjected to arbitrary initial temperature on the upper face with lower face at zero temperature. Most of the studies considered by various authors [3, 8, 9] have not considered any thermoelastic problem for thick plates with boundary conditions of radiation type, in which sources are generated according to the linear function of the temperatures, which will also, satisfies the time-dependent heat conduction equation. The significance of aforementioned transform [5, 6, 10] over the previous used or published integral transform techniques [2, 3] can be seen while obtaining the temperature or displacement of any height, with third kind boundary conditions of radiation type on the outside and inside surfaces, with independent radiation constant. From the previous literatures regarding thick disc as considered, it was observed by the author that no analytical procedure has been established for thick annular disc, considering internal heat sources generation within the body and specifically impacted by partial heating.

2. Formulation of the problem

We consider a thick annular disc with thickness $2h$, internal radius a and external radius b, occupying the space $D = \{(x, y, z) \in R^3 : a \le (x^2 + y^2)^{1/2} \le b, -h \le z \le h\}$, where $r = (x^2 + y^2)^{1/2}$. Let the disc in which internal sources are generated according to linear function of the temperature are subjected to partial heating $(Q_0 / \lambda)\exp(-\omega t)\delta(r - r_0)$ over the upper surface $(z = h)$ and $\delta(r - r_0)$ is the Dirac delta function.

2.1. Temperature distribution. The transient heat conduction equation with internal heat generation is given as follows

$$\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} + \frac{\partial^2\theta}{\partial z^2} + \frac{\Theta(r,z,t,\theta)}{\kappa} = \frac{1}{\kappa}\frac{\partial\theta}{\partial t}, \tag{1}$$

where $\Theta(r,z,t,\theta)$ is the internal source function, and $\kappa = \lambda / \rho C$, λ being the thermal conductivity of the material, ρ is the density, and C is the calorific capacity, assumed to be constant.

Following [5], we consider the undergiven functions as the superposition of the simpler function:

$$\Theta(r, z, t, \theta) = \Phi(r, z, t) + \psi(t)\,\theta(r, z, t) \tag{2}$$

and

$$\left.\begin{aligned} &T(r,z,t) = \theta(r,z,t)\exp\left[-\int_0^t \psi(\zeta)d\zeta\right],\\ &\chi(r,z,t) = \Phi(r,z,t)\exp\left[-\int_0^t \psi(\zeta)d\zeta\right],\\ &\text{or for the sake of brevity, we consider}\\ &\chi(r,z,t) = \frac{\delta(r-r_0)\,\delta(z-z_0)}{2\pi r_0}\exp(-\omega t), \end{aligned}\right\} \tag{3}$$

$a \le r_0 \le b, \; -h \le z_0 \le h, \; \omega > 0$.

Substituting equations (2) and (3) in equation (1), yield

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{\chi(r,z,t)}{\kappa} = \frac{1}{\kappa}\frac{\partial T}{\partial t}, \tag{4}$$

where κ is the thermal diffusivity of the material of the disc (which is assumed to be constant), subject to the initial and boundary conditions

$$T = T_0 \text{ at } t = 0, \tag{5}$$

$$\left.\begin{aligned} T + k_1 \frac{\partial T}{\partial r} &= 0, \quad \text{at} \quad r = a, \quad -h \le z \le h, \quad t > 0 \\ T + k_2 \frac{\partial T}{\partial r} &= 0, \quad \text{at} \quad r = b, \quad -h \le z \le h, \quad t > 0 \end{aligned}\right\} \tag{6}$$

$$\left.\begin{aligned} T + k_3 \frac{\partial T}{\partial z} &= (Q_0 / \lambda) \exp(-\omega t) \delta(r - r_0), \quad \text{at} \quad z = h, \quad a \le r \le b, \quad t > 0 \\ T + k_4 \frac{\partial T}{\partial z} &= 0, \qquad \text{at} \quad z = -h, \quad a \le r \le b, \quad t > 0 \end{aligned}\right\} \tag{7}$$

where $\delta(r - r_0)$ is the Dirac Delta function having $a \le r_0 \le b$; $\omega > 0$ is a constant; Q_0 is the heat flux with constant strength; λ is the thermal conductivity coefficient of the material and T_0 is the reference temperature.

2.2. Thermal displacements and thermal stress. The Navier's equations in the absence of body forces for axisymmetric two-dimensional thermoelastic problem can be expressed as [7]:

$$\left.\begin{aligned} \nabla^2 u_r - \frac{u_r}{r} + \frac{1}{1-2\upsilon} \frac{\partial e}{\partial r} - \frac{2(1+\upsilon)}{1-2\upsilon} \alpha_t \frac{\partial \theta}{\partial r} &= 0, \\ \nabla^2 u_z - \frac{1}{1-2\upsilon} \frac{\partial e}{\partial z} - \frac{2(1+\upsilon)}{1-2\upsilon} \alpha_t \frac{\partial \theta}{\partial z} &= 0, \end{aligned}\right\} \tag{8}$$

where u_r and u_z are the displacement components in the radial and axial directions, respectively and the dilatation e as

$$e = \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}.$$

The displacement functions in the cylindrical coordinate system are represented by the Goodier's thermoelastic displacement potential ϕ and Love's function L as [4]

$$u_r = \frac{\partial \phi}{\partial r} - \frac{\partial^2 L}{\partial r \partial z}, \tag{9}$$

$$u_z = \frac{\partial \phi}{\partial z} + 2(1-\upsilon) \nabla^2 L - \frac{\partial^2 L}{\partial^2 z}, \tag{10}$$

in which Goodier's thermoelastic potential must satisfy the equation

$$\nabla^2 \phi = \left(\frac{1+\upsilon}{1-\upsilon} \right) \alpha_t \theta, \tag{11}$$

and the Love's function L must satisfy the equation

$$\nabla^2 (\nabla^2 L) = 0, \tag{12}$$

where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$.

The component of the stresses are represented by the use of the potential ϕ and Love's function L as

$$\sigma_{rr} = 2G\left\{\left(\frac{\partial^2\phi}{\partial r^2} - \nabla^2\phi\right) + \frac{\partial}{\partial z}\left(\upsilon\nabla^2 L - \frac{\partial^2 L}{\partial r^2}\right)\right\}, \tag{13}$$

$$\sigma_{\theta\theta} = 2G\left\{\left(\frac{1}{r}\frac{\partial\phi}{\partial r} - \nabla^2\phi\right) + \frac{\partial}{\partial z}\left(\upsilon\nabla^2 L - \frac{1}{r}\frac{\partial L}{\partial r}\right)\right\}, \tag{14}$$

$$\sigma_{zz} = 2G\left\{\left(\frac{\partial^2\phi}{\partial r^2} - \nabla^2\phi\right) + \frac{\partial}{\partial z}\left((2-\upsilon)\nabla^2 L - \frac{\partial^2 L}{\partial z^2}\right)\right\}, \tag{15}$$

and

$$\sigma_{rz} = 2G\left\{\frac{\partial^2\phi}{\partial r\partial z} + \frac{\partial}{\partial r}\left((1-\upsilon)\nabla^2 L - \frac{\partial^2 L}{\partial z^2}\right)\right\}, \tag{16}$$

where G and υ are the shear modulus and Poisson's ratio respectively.

The boundary conditions on the traction free surface functions are

$$\sigma_{rz}\big|_{r=a} = \sigma_{rz}\big|_{r=b} = 0, \tag{17}$$

Equations (1) to (17) constitute the mathematical formulation of the problem.

3. Solution of the problem

3.1. Solution of the heat conduction problem. In order to solve equation (4) under the boundary condition (6), we firstly introduce the integral transform [6] of order n over the variable r. Let n be the parameter of the transform, then the integral transform and its inversion theorem are written as

$$\bar{g}(n) = \int_a^b r\, g(r)\, S_p(k_1, k_2, \mu_n r)\, dr, \quad g(r) = \sum_{n=1}^{\infty} (\bar{g}_p(n)/C_n)\, S_p(k_1, k_2, \mu_n r), \tag{18}$$

where $\bar{g}_p(n)$ is the transform of $g(r)$ with respect to nucleus $S_p(k_1, k_2, \mu_n r)$.

Applying the transform defined by equation (18) to the equations (3), (4), (5) and (7), and taking into account equation (6), we obtain

$$\kappa\left[-\mu_n^2 \bar{T}(n,z,t) + \frac{\partial^2 \bar{T}(n,z,t)}{\partial z^2}\right] + \bar{\chi}(n,z,t) = \frac{\partial \bar{T}(n,z,t)}{\partial t}, \tag{19}$$

$$\bar{T} = \bar{T}_0, \tag{20}$$

$$\left.\begin{aligned} \bar{T} + k_3\frac{\partial \bar{T}}{\partial z} &= (Q_0/\lambda)\exp(-\omega t)\, r_0\, S_0(k_1, k_2, \mu_n r_0), \\ \bar{T} + k_4\frac{\partial \bar{T}}{\partial z} &= 0, \end{aligned}\right\} \tag{21}$$

$$\bar{\chi}(n,z,t) = r_0\, S_0(k_1, k_2, \mu_n r_0)\, \delta(z - z_0)\exp(-\omega t), \tag{22}$$

where $\bar{T}$ is the transformed function of T. The eigenvalues μ_n are the positive roots of the characteristic equation

$$J_0(k_1, \mu a)\, Y_0(k_2, \mu b) - J_0(k_2, \mu b)\, Y_0(k_1, \mu a) = 0.$$

The kernel function $S_0(k_1,k_2,\mu_n r)$ can be defined as

$$S_0(k_1,k_2,\mu_n r)=J_0(\mu_n r)[Y_0(k_1,\mu_n a)+Y_0(k_2,\mu_n b)]-Y_0(\mu_n r)[J_0(k_1,\mu_n a)+J_0(k_2,\mu_n b)]$$

with

$$\left.\begin{aligned} J_0(k_i,\mu r)&=J_0(\mu r)+k_i\,\mu\, J_0'(\mu r)\\ Y_0(k_i,\mu r)&=Y_0(\mu r)+k_i\,\mu\, Y_0'(\mu r)\end{aligned}\right\}\quad \text{for}\quad i=1,2$$

and

$$C_n=\int_a^b r[S_0(k_1,k_2,\,\mu_n b)]^2\,dr\,,$$

in which $J_0(\mu r)$ and $Y_0(\mu r)$ are Bessel functions of first and second kind of order $p=0$ respectively.

We introduce the another integral transform stated in [5] that responds to the boundary conditions given in equation (7) as

$$\overline{f}(m,t)=\int_{-h}^{h} f(z,t)P_m(z)\,dz,\quad f(z,t)=\sum_{m=1}^{\infty}\frac{\overline{f}(m,t)}{\lambda_m}P_m(z)\,, \tag{23}$$

Applying the transform defined by equation (23) in equations (19), (20) and (22), and using equation (21), we obtain

$$\kappa\left[-\mu_n^2\overline{T}^*(n,m,t)+\frac{P_m(h)}{k_3}(Q_0/\lambda)\,r_0\,S_0(k_1,k_2,\mu_n r_0)\exp(-\omega t)-a_m^2\overline{T}^*(n,m,t)\right]+\overline{\chi}^*(n,m,t)=\frac{d\overline{T}^*(n,m,t)}{dt}\,, \tag{24}$$

$$\overline{T}^*=\overline{T}_0^*\,, \tag{25}$$

$$\overline{\chi}^*(n,m,t)=r_0\,S_0(k_1,k_2,\mu_n r_0)\,P_m(z_0)\exp(-\omega t)\,, \tag{26}$$

where $\overline{T}^*$ is the transformed function of $\overline{T}$. The symbol ($\overline{\ }$) means a function in the transformed domain, and the nucleus is given by the orthogonal functions in the interval $-h\le z\le h$ as

$$P_m(z)=Q_m\cos(a_m z)-W_m\sin(a_m z)$$

where

$$Q_m=a_m(k_3+k_4)\cos(a_m h),$$

$$W_m=2\cos(a_m h)+(k_3-k_4)a_m\sin(a_m h),$$

$$\lambda_m=\int_{-h}^{h}P_m^2(z)\,dz=h[Q_m^2+W_m^2]+\frac{\sin(2a_m h)}{2a_m}[Q_m^2-W_m^2]\,.$$

The eigenvalues a_m are the positive roots of the characteristic equation

$$[k_3\,a\cos(ah)+\sin(ah)][\cos(ah)+k_4\,a\sin(ah)]=[k_4\,a\cos(ah)-\sin(ah)][\cos(ah)-k_3\,a\sin(ah)]\,.$$

Using Eq. (26), equation (24) yield

$$\frac{d\overline{T}^*}{dt} + \kappa(\mu_n^2 + a_m^2)\overline{T}^* = H(\mu_n,\, a_m)\,, \tag{27}$$

where

$$H(\mu_n,\, a_m) = \left\{ P_m(z_0) + \frac{Q_0\, P_m(h)\kappa}{\lambda\, k_3} \right\} r_0\, S_0(k_1, k_2, \mu_n r_0)\exp(-\omega\, t)\,.$$

The general solution of equation (27) is a function

$$\overline{T}^*(n,m,t)\exp(\kappa(\mu_n^2 + a_m^2)t) = \frac{H(\mu_n, \alpha_m)}{\kappa(\mu_n^2 + a_m^2) - \omega}\exp(-\kappa(\mu_n^2 + a_m^2)t) + C\,. \tag{28}$$

Using (28) in equation (25) yield the value of C and Substituting the resulting value of C in equation (28) yield

$$\overline{T}^*(n,m,t) = \frac{H(\mu_n, a_m)}{\kappa(\mu_n^2 + a_m^2) - \omega}\exp(-\omega t) + \left[\overline{T}_0^* - \frac{H(\mu_n, a_m)}{\kappa(\mu_n^2 + a_m^2) - \omega} \right]\exp(-\kappa(\mu_n^2 + a_m^2)t)\,. \tag{29}$$

Applying inversion theorems of the transform rules defined by equations (18) on the equation (29), yield

$$\overline{T}(n,z,t) = \sum_{m=1}^{\infty}\frac{1}{\lambda_m}[\wp_{n,m}\exp(-\omega t) + (\overline{T}_0^* - \wp_{n,m})\exp(-\kappa(\mu_n^2 + a_m^2)t)]P_m(z)\,, \tag{30}$$

and then accomplishing inversion theorems of the transform rules defined by equations (23) on equation (30), the temperature T is obtained as:

$$T(r,z,t) = \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{ \sum_{m=1}^{\infty}\frac{1}{\lambda_m}[\wp_{n,m}\exp(-\omega t) + (\overline{T}_0^* - \wp_{n,m})\exp(-\kappa(\mu_n^2 + a_m^2)t)]P_m(z) \right\}$$

$$\times S_0(k_1, k_2, \mu_n r)\,, \tag{31}$$

where

$$\wp_{n,m} = \frac{H(\mu_n, a_m)}{\kappa(\mu_n^2 + a_m^2) - \omega}\,.$$

Taking into account the first equation of equation (3), the temperature distribution is finally represented by

$$\theta(r,z,t) = \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{ \sum_{m=1}^{\infty}\frac{1}{\lambda_m}[\wp_{n,m}\exp(-\omega t) + (\overline{T}_0^* - \wp_{n,m})\exp(-\kappa(\mu_n^2 + a_m^2)t)]P_m(z) \right\}$$

$$\times\, S_0(k_1, k_2, \mu_n r)\exp\left[\int_0^t \psi(\zeta)d\zeta \right]\,, \tag{32}$$

The function given in equation (32) represents the temperature at every instant and at all points of thick annular disc of finite height when there are conditions of radiation type with partial heating on the upper surface.

3.2. Solution of the thermal stress problem. Referring to the fundamental equation (1) and its solution (32) for the heat conduction problem, the solution for the displacement function are represented by the Goodier's thermoelastic displacement potential ϕ governed by equation (11) are represented by

$$\phi(r,z,t)=\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-1}{\lambda_m(\mu_n^2+a_m^2)}[\wp_{n,m}\exp(-\omega t)+(\overline{T}_0^*-\wp_{n,m})\exp(-\kappa(\mu_n^2+a_m^2)t)]P_m(z)\right\}$$

$$\times S_0(k_1,k_2,\mu_n r)\exp\left[\int_0^t\psi(\zeta)d\zeta\right]. \tag{33}$$

Similarly, the solution for Love's function L are assumed so as to satisfy the governed condition of equation (12) as

$$L(r,z,t)=\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-1}{\lambda_m(\mu_n^2+a_m^2)}[\wp_{n,m}\exp(-\omega t)+(\overline{T}_0^*-\wp_{n,m})\exp(-\kappa(\mu_n^2+a_m^2)t)]\right\}$$

$$\times[\sinh(\mu_n z)+z\cosh(\mu_n z)]S_0(k_1,k_2,\mu_n r)\exp\left[\int_0^t\psi(\zeta)d\zeta\right]. \tag{34}$$

In this manner two displacement functions in the cylindrical coordinate system ϕ and L are fully formulated. Now, in order to obtain the displacement components, we substitute the values of thermoelastic displacement potential ϕ and Love's function L in equations (9) and (10), one obtains

$$u_r=\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-1}{\lambda_m(\mu_n^2+a_m^2)}[\wp_{n,m}\exp(-\omega t)+(\overline{T}_0^*-\wp_{n,m})\exp(-\kappa(\mu_n^2+a_m^2)t)]\right\}$$

$$\times\left[P_m(z)-(\mu_n+1)\cosh(\mu_n z)-\mu_n z\sinh(\mu_n z)\right]S_0'(k_1,k_2,\mu_n r)\exp\left[\int_0^t\psi(\zeta)d\zeta\right], \tag{35}$$

$$u_z=\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-1}{\lambda_m(\mu_n^2+a_m^2)}[\wp_{n,m}\exp(-\omega t)+(\overline{T}_0^*-\wp_{n,m})\exp(-\kappa(\mu_n^2+a_m^2)t)]\right\}$$

$$\times[-a_m(Q_m\sin(a_m z)+W_m\cos(a_m z))-4\upsilon\mu_n\sin(\mu_n z)-\mu_n^2(\sinh(\mu_n z)+z\cosh(\mu_n z))]$$

$$\times S_0(k_1,k_2,\mu_n r)\exp\left[\int_0^t\psi(\zeta)d\zeta\right]. \tag{36}$$

Thus, making use of the two displacement components, the dilation is established as

$$e=\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-1}{\lambda_m(\mu_n^2+a_m^2)}[\wp_{n,m}\exp(-\omega t)+(\overline{T}_0^*-\wp_{n,m})\exp(-\kappa(\mu_n^2+a_m^2)t)]\right\}$$

$$\times[-P_m(z)+(\mu_n+1)\cosh(\mu_n z)+\mu_n z\sinh(\mu_n z)-a_m^2P_m(z)-(4\upsilon+1)\mu_n^2\cosh(\mu_n z)$$

$$-\mu_n^3(\cosh(\mu_n z)+z\sinh(\mu_n z))]S_0(k_1,k_2,\mu_n r)\exp\left[\int_0^t\psi(\zeta)d\zeta\right]. \tag{37}$$

Then, the stress components can be evaluated by substituting the values of thermoelastic displacement potential ϕ from equation (33) and Love's function L from equation (34) in equations (13), (14), (15), and (16), one obtains

$$\sigma_{rr} = 2G\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{1}{\lambda_m}[\wp_{n,m}\exp(-\omega t)+(\overline{T}_0^{*}-\wp_{n,m})\exp(-\kappa(\mu_n^2+a_m^2)t)]\right\}$$

$$\times\{-P_m(z)[(\mu_n^2+a_m^2)^{-1}S_0''(k_1,k_2,\mu_n r)+S_0(k_1,k_2,\mu_n r)]-(\mu_n^2+a_m^2)^{-1}[2\upsilon\,\mu_n^2\cosh(\mu_n z)$$

$$\times S_0(k_1,k_2,\mu_n r)-[(\mu_n+1)\cosh(\mu_n z)+z\,\mu_n\sinh(\mu_n z)]S_0''(k_1,k_2,\mu_n r)]\}$$

$$\times\exp\left[\int_0^t\psi(\zeta)d\zeta\right], \tag{38}$$

$$\sigma_{\theta\theta} = 2G\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{1}{\lambda_m}[\wp_{n,m}\exp(-\omega t)+(\overline{T}_0^{*}-\wp_{n,m})\exp(-\kappa(\mu_n^2+a_m^2)t)]\right\}$$

$$\times\{-P_m(z)[r(\mu_n^2+a_m^2)^{-1}S_0'(k_1,k_2,\mu_n r)+S_0(k_1,k_2,\mu_n r)]-(\mu_n^2+a_m^2)^{-1}[2\upsilon\,\mu_n^2\cosh(\mu_n z)$$

$$\times S_0(k_1,k_2,\mu_n r)-[(\mu_n+1)\cosh(\mu_n z)+z\,\mu_n\sinh(\mu_n z)]r^{-1}S_0'(k_1,k_2,\mu_n r)]\}$$

$$\times\exp\left[\int_0^t\psi(\zeta)d\zeta\right], \tag{39}$$

$$\sigma_{zz} = 2G\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{1}{\lambda_m}[\wp_{n,m}\exp(-\omega t)+(\overline{T}_0^{*}-\wp_{n,m})\exp(-\kappa(\mu_n^2+a_m^2)t)]\right\}$$

$$\times\{-P_m(z)[(\mu_n^2+a_m^2)^{-1}S_0''(k_1,k_2,\mu_n r)+S_0(k_1,k_2,\mu_n r)]+[(2\upsilon+\mu_n)\mu_n^2\cosh(\mu_n z)$$

$$+\mu_n^3 z\sinh(\mu_n z)]S_0(k_1,k_2,\mu_n r)\}$$

$$\times\exp\left[\int_0^t\psi(\zeta)d\zeta\right], \tag{40}$$

$$\sigma_{rz} = 2G\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{1}{\lambda_m(\mu_n^2+a_m^2)}[\wp_{n,m}\exp(-\omega t)+(\overline{T}_0^{*}-\wp_{n,m})\exp(-\kappa(\mu_n^2+a_m^2)t)]\right\}$$

$$\times\{a_m[Q_m\sin(a_m z)+W_m\cos(a_m z)]S_0(k_1,k_2,\mu_n r)+[2\upsilon\mu_n\sinh(\mu_n z)$$

$$+\mu_n^2[\sinh(\mu_n z)+z\cosh(\mu_n z)]]S_0'(k_1,k_2,\mu_n r)\}$$

$$\times\exp\left[\int_0^t\psi(\zeta)d\zeta\right]. \tag{41}$$

4. Special case and numerical calculations

Setting

$$\psi(\zeta)=-\zeta,\; T_0=0 \tag{42}$$

$$\Rightarrow\; \int_0^t\psi(\zeta)\,d\zeta=-t^2/2,\; \overline{T}_0^{*}=0\,. \tag{43}$$

Substituting the value of equation (43) in equations (32)-(41), we obtained the expressions for the temperature and stresses respectively for our numerical discussion.

The numerical computations have been carried out for Aluminum metal with parameter a=2.65 cm, b=3.22 cm, h=2.00 cm; modulus of elasticity $E = 6.9 \times 10^6$ N/cm^2; shear modulus $G = 2.7 \times 10^6$ N/cm^2; Poisson ratio $\upsilon = 0.281$; thermal expansion coefficient α_t = 25.5×10^6 cm/(cm- ºC); thermal diffusivity κ = 0.86 cm^2/sec; thermal conductivity λ = 0.48 cal sec^{-1}/(cm ºC) with $\mu_n = 1.07171$, 2.12479, 3.16511, 4.20764, 5.25165, 6.29656, 7.34205, 8.38792, 9.43405, 10.48039, 12.57346, 13.62014, 14.66689, 15.71369, 16.76055, 17.80744, 18.85436 which are the positive roots of the transcendental equation $J_0(k_1,\mu a)\, Y_0(k_2,\mu b) - J_0(k_2,\mu b)\, Y_0(k_1,\mu a) = 0$ and $a_m = 1.33975$, 2.63999, 3.94298, 5.24845, 6.55578, 7.86414, 9.43329, 11.00298, 12.57286, 14.14295, 15.7131, 17.2835, 18.85398, 20.45789, 22.89799 which are the positive roots of the transcendental equation

$$[k_3\, a\cos(ah) + \sin(ah)]\ [\cos(ah) + k_4\, a\sin(ah)] = [k_4\, a\cos(ah) - \sin(ah)]\ [\cos(ah) - k_3\, a\sin(ah)].$$

In the foregoing analysis are performed by setting the radiation coefficients constants, $k_i = 0.86\,(i = 1,3)$ and $k_i = 1\,(i = 2,4)$, so as to obtain considerable mathematical simplicities.

In order to examine the influence of partial heating on the upper surface of thick disc, we performed the numerical calculation for time t = 0.1, 0.3, 0.5, 0.7, 0.9 and numerical variations in radial direction on the upper surface (z = 1) where partial heat supply is applied are depicted in the following figures with the help of computer program.

The derived numerical results for the equations (32)-(41) has been illustrated graphically (refer Figs. 1-5) for the thick disc with internal heat source, and partial heating on its flat surface at z = 1.

Figure 1 shows the variation of temperature change along the radial of the thick annular disc at the heated surface z = 1. It is evident that temperature function rise gradually increases with time.

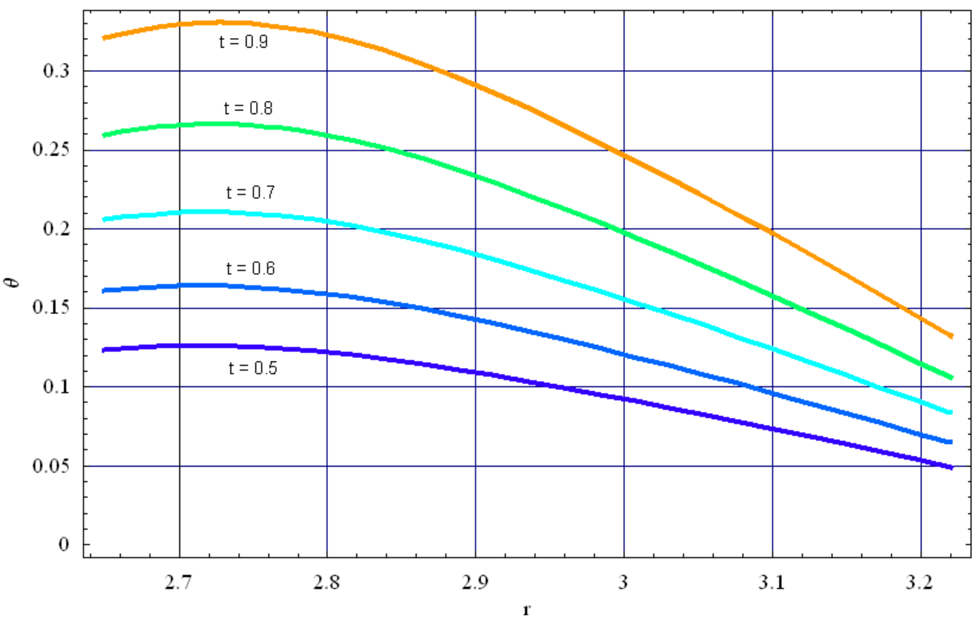

Fig. 1. Temperature distribution.

As shown in Fig. 2 the variation of thermal stress in the radial direction at the midpoint of the disc. From the figure, the location of points of minimum stress occurs at the end points through-the-radial direction, while the thermal stress response are maximum at the interior and so that outer edges tends to expand more than the inner surface leading inner part being under tensile stress.

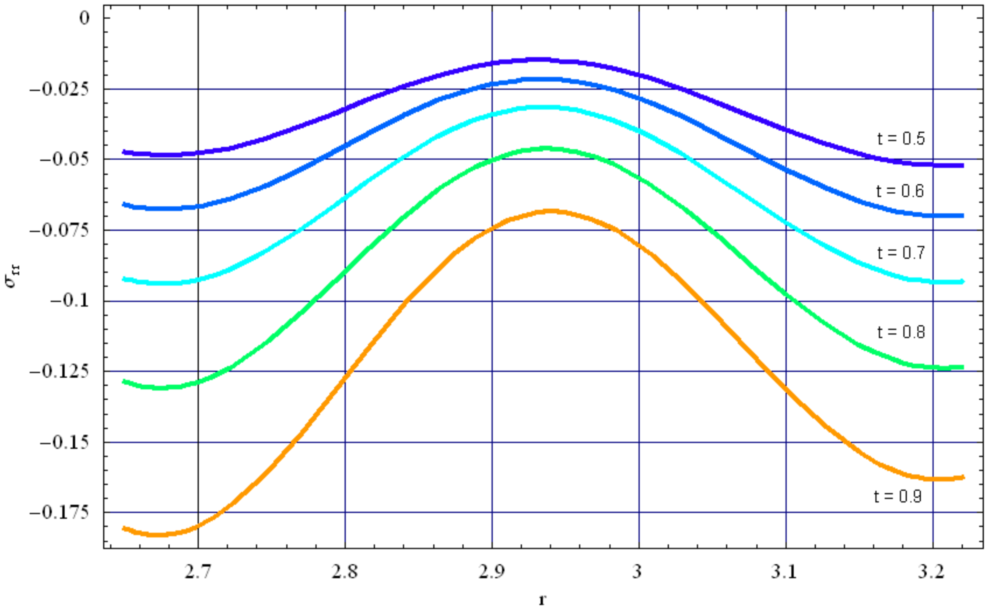

Fig. 2. Radial stress distribution.

Figure 3 shows the tangential stress distribution $\sigma_{\theta\theta}$ following a decreasing trend along the radial direction due to compressive stresses within concentric region.

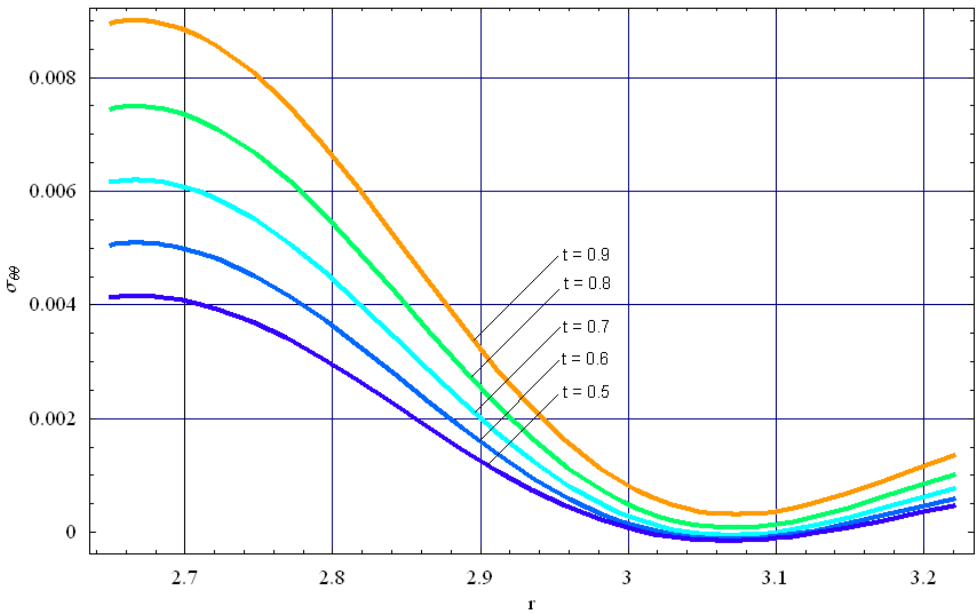

Fig. 3. Tangential stress distribution.

Figure 4 shows the variation of the axial stress distribution σ_{zz}, which is similar in nature, but comparatively large in magnitude to radial stress component.

Figure 5 shows the variation of the shearing stress σ_{rz} along the radial direction of the thick annular disc at the heated surface z = 1. This shear stress follows a sinusoidal nature with high crest and troughs along the radial direction.

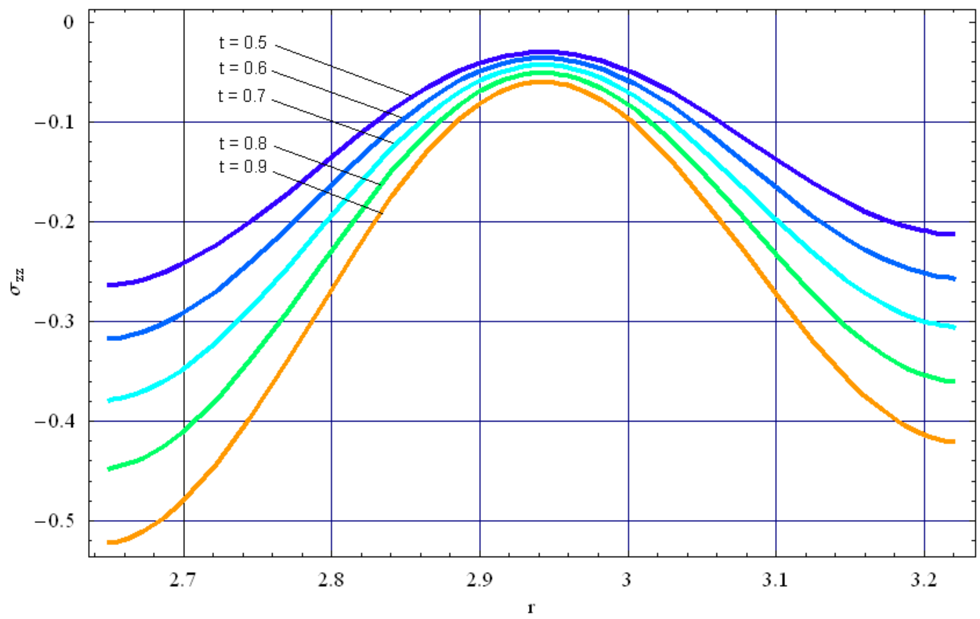

Fig. 4. Axial stress distribution.

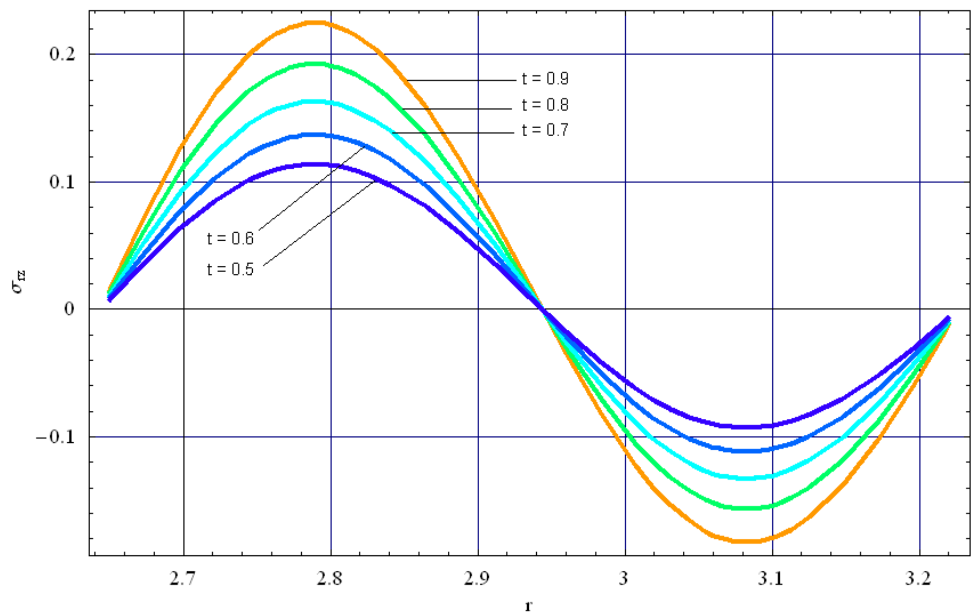

Fig. 5. Shear stress distribution.

5. Conclusions

In this study, we have treated thermoelastic problem of a thick annular disc in which sources are generated according to the linear function of the temperature. We successfully established and obtained the temperature distribution, displacements and stress functions with additional partial heating $(Q_0/\lambda)\exp(-\omega t)\delta(r-r_0)$ available at the edge $z=h$ of the disc. Then, in order to examine the validity of boundary value problem, we analyze, as a particular case with mathematical model for $\psi(\zeta)=-\zeta$ and numerical calculations were carried out. We may conclude that the system of equations proposed in this study can be adapted to design of useful structures or machines in engineering applications in the determination of thermoelastic behaviour with radiation.

References

[1] M. Ishihara, Y. Tanigawa, R. Kawamura, N. Noda // *Journal of Thermal Stresses* **20 (2)** (1997) 203.

[2] N.L. Khobragade, K.C. Deshmukh // *Journal of Thermal Stresses* **28** (2005) 171.

[3] V.S. Kulkarni, K.C. Deshmukh // Sadhana **32 (5)** (2007) 561.

[4] A.E.H. Love, *A Treatise on the Mathematical Theory of Elasticity* (Dover publications, Inc, New York, 1944).

[5] E. Marchi, A. Fasulo // *Atti della Reale Accademia delle scienze di Torino* **1** (1967) 373.

[6] E. Marchi, G. Zgrablich // *Proc. Edinburgh Math. Soc*. **14 (2)** (1964) 159.

[7] N. Noda, R. Hetnarski, Y. Tanigawa, *Thermal Stresses* (Taylor and Francis, New York, 2003).

[8] M.E. Nasser // *J. Therm. Stresses* **27** (2004) 227.

[9] M.E. Nasser // *J. Therm. Stresses* **28** (2005) 1227.

[10] V. Varghese, N.W. Khobragade // *International Journal of Applied Mathematics & Statistics* **14 (M09)** (2009) 37.