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WAVE MOTION AND REPRESENTATION OF FUNDAMENTAL SOLUTION IN ELECTRO-MICROSTRETCH VISCOELASTIC SOLIDS

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Abstract. The homogeneous isotropic electro–microstretch viscoelastic solids have been taken into consideration for investigating the propagation of plane waves and fundamental solution. For two dimensional model, it is found that there exists two coupled longitudinal waves namely longitudinal displacement (LD) wave and longitudinal microstretch (LM) wave and two coupled transverse displacement and transverse microrotational waves (CD-I and CD-II). The phase velocities, attenuation coefficients, specific loss, penetration depth are computed numerically. The resulting quantities are depicted graphically to show the viscous effect. In addition, we construct the fundamental solution of the system of differential equations in the theory of an electro-microstretch viscoelastic solids in case of steady oscillations in terms of elementary functions. Some basic properties of the fundamental solution are established. Some special cases are also discussed.

1. Introduction

The study of viscoelastic behavior is of interest in several contexts. First, materials used in engineering applications may exhibit viscoelastic behavior as an unintentional side effect. Second, the mathematics underlying viscoelasticity is the theory within the applied mathematics community. Third, viscoelasticity is of interest in some branches of material science, metallurgy and solid-state-physics. Fourth, the casual links between viscoelasticity and microstructure is exploited in the use of viscoelastic tests as an inspection tools. In reality all materials deviate from Hooke's law in various ways, for example, by exhibiting viscouslike as well as characteristics.

Viscoelastic materials are those for which the relationship between stress and strain depends on time. All materials exhibit some viscoelastic response. In common metals such as steel, aluminum, copper etc. At room temperature and small strain, the behavior does not deviate much from linear. The Kelvin-Voigt model is one of the macroscopic mechanical models often used to describe the viscoelastic behavior of the material. This model represents the delayed elastic response subjected to stress when the deformation is time dependent but recoverable. The dynamical interaction of thermal and mechanical fields in solids has great practical applications in modern aeronautics, astronautics, nuclear reactors and high energy particle accelerators.

Eringen (1967) [1] extended the theory of micropolar elasticity to obtain linear constitutive theory for micropolar material possessing internal friction. A problem on micropolar viscoelastic waves has been discussed by McCarthy and Eringen (1969) [2]. Biswas et al. (1996) [3] studied the axisymmetric problems of wave propagation under the influence of gravity in a micropolar viscoelastic semi-infinite medium when a time varying axisymmetric loading has been applied on the surface of the medium. De Cicco and Nappa (1998) [4] discussed the problem of Saint Venant's principle for micropolar viscoelastic bodies. Kumar and Singh (2000) [5], Kumar and Partap (2008) [6], Kumar and Partap (2010) [7] investigated problems of waves and vibrations in micropolar and microstrech viscoelastic media.

Eringen (1990 a, b) [8, 9] developed a theory of thermo-microstretch elastic solid and fluids, in which he included microstructural expansions and contractions. Microstretch continuum is a model for Bravais lattice with a basis on the atomic level, and a two-phase dipolar solid with a core on the macroscopic level. For example, composite materials reinforced with chopped elastic fibres, porous media whose pores filled with gas or inviscid liquid, asphalt or other elastic inclusions and “solid-liquid” crystals etc. should be characterizable by microstretch solids. A comprehensive review on the micropolar continuum theory has been given in his book by Eringen (1999) [10].

Eringen (2003) [11] presented a continuum theory for micropolar electromagnetic thermoelastic solids. Eringen (2004) [12] further extended his theory of thermomicrostretch elastic solids (1990a) [8] to the electromagnetic interactions and termed it as an electromagnetic theory of microstretch elasticity. He presented constitutive relations and motion equations for isotropic thermo-microstretch elastic solids subjected to electro-magnetic fields. In the absence of magnetic flux vectors, the microstretch thermoelastic continuum will be subjected only to electric fields. We shall call such continuum materials as electro-microstretch thermoelastic solids.

Iesan and Pompei (1995) [13] investigated the problem on the equilibrium theory of microstretch elastic solid. Bofill and Quintanilla (1995) [14] obtained some qualitative result for the linear theory of thermo-microstretch elastic solids. Iesan (2006) [15] derived the basic equations for the microstretch piezoelectricity. Some theorems in the theory of microstretch thermopiezoelectricity were proved by Iesan and Quintanilla (2007) [16]. El-Karamany (2007) [17] proved some theorems in linear micropolar thermopiezoelectric / piezomagnetic continuum with relaxation time.

To investigate the boundary value problems of the theory of elasticity and thermoelasticity by potential method, it is necessary to construct a fundamental solution of the systems of partial differential equations and to establish their basic properties, respectively. Hetnarski (1964) [18] was the first to study the fundamental solutions in the classical theory of coupled thermoelasticity. Svanadze (1988) [19] obtained the fundamental solution for the linearlized equations of the theory of elastic mixtures. Svanadze (1996) [20] constructed the fundamental solution of the system of equations in thermoelasticity theory of mixtures having two elastic solids. The fundamental solution in the theory of microstrecth elastic solids and in the theory of thermoelasticity with microtemperatures were obtained by Svanadze (2004a, 2004b) [21, 22]. Svanadze and Cicco (2005) [23] constructed the fundamental solution of the system of differential equations in the case of steady oscillations in the theory of thermomicrostretch elastic solids. Svanadze, Tibullo and Zampoli (2006) [24] obtained the fundamental solution in the theory of micropolar thermoelasticity without energy dissipation.The fundamental solution in the theory of micropolar thermoelasticity for materials with void was constructed by Ciarletta, Scalia and Svanadze (2007) [25]. Svanadze and Tracinà (2011) [26] presented the fundamental solutions in the theory of thermoelasticity with microtemperatures for microstretch solids. Kumar and Kansal (2012) [27] obtained the fundamental solution in the theory of micropolar thermoelastic diffusion with voids. Sherief, Faltas and Ashmawy (2012) [28] investigated the fundamental solutions for axi-symmetric translational motion of a microstretch fluid. The comprehensive information on fundamentals solutions of differential equations is also given in the books [29-31].

In this article, the propagation of plane waves and fundamental solution in electro-microstretch solids has been investigated. The phase velocities have been computed numerically and depicted graphically. The representation of the fundamental solution of system of equations in the case of steady oscillations is considered in terms of elementary functions and basic properties of the fundamental solution are established.

2. Basic equations

Let $x=(x_1,x_2,x_3)$ be the points of the Euclidean three-dimensional space $E^3:|x|=\left(x_1^2+x_2^2+x_3^2\right)^{1/2}$, $D_x=(\partial/\partial x_1,\partial/\partial x_2,\partial/\partial x_3)$ and let t denote the time variable.

Following Eringen [8] and Iesan and Quintanilla [16], the equations of motion and the constitutive relations in a homogeneous isotropic electro-microstretch solid in the absence of body forces, body couples, stretch force and charge densities are given by

$$\left(\lambda_0+\mu_0\right) grad\ div\ \vec{u}+\left(\mu_0+K_0\right)\Delta\vec{u}+K_0\ curl\ \vec{\varphi}+\lambda_{01}\ grad\ \varphi^*=\rho\ddot{\vec{u}}, \tag{2.1}$$

$$\left(\gamma_0\Delta-2K_0\right)\vec{\varphi}+\left(\alpha_0+\beta_0\right) grad\ div\ \vec{\varphi}+K_0\ curl\ \vec{u}=\rho j\ddot{\vec{\varphi}}, \tag{2.2}$$

$$\left(a_{01}\Delta-\lambda_{30}\right)\varphi^*-\lambda_{01}\ div.\vec{u}+\lambda_{20}\ div.\vec{E}=\frac{\rho j_0}{2}\ddot{\varphi}^*, \tag{2.3}$$

$$D_{i,i}=0, \tag{2.4}$$

where $D_i=\chi_0^E E_i-\lambda_{10}\varepsilon_{rsi}\phi_{s,r}-\lambda_{20}\phi^*_{,i}$, $\lambda_0=\lambda+\lambda^*\frac{\partial}{\partial t}$, $\mu_0=\mu+\mu^*\frac{\partial}{\partial t}$, $K_0=K+K^*\frac{\partial}{\partial t}$, $\alpha_0=\alpha+\alpha^*\frac{\partial}{\partial t}$, $\beta_0=\beta+\beta^*\frac{\partial}{\partial t}$, $\gamma_0=\gamma+\gamma^*\frac{\partial}{\partial t}$, $\lambda_{01}=\lambda_0+\lambda_0^*\frac{\partial}{\partial t}$, $\lambda_{30}=\lambda_3+\lambda_3^*\frac{\partial}{\partial t}$, $\lambda_{21}=\lambda_2+\lambda_2^*\frac{\partial}{\partial t}$, $a_{01}=a_0+a_0^*\frac{\partial}{\partial t}$, $\chi_0^E=\chi^E+\chi^{*E}\frac{\partial}{\partial t}$; $\alpha^*,\beta^*,\gamma^*,\mu^*,K^*,\lambda^*,\lambda_0^*,\lambda_3^*,\lambda_2^*,a_0^*$ are material constants, ρ is the mass density, $\vec{u}=\left(u_1,u_2,u_3\right)$ is the displacement vector and $\vec{\varphi}=\left(\varphi_1,\varphi_2,\varphi_3\right)$ is the microrotation vector, φ^* is the microstretch scalar function, j is the microintertia, j_o is the microinertia of the microelements, $E=(E_1,E_2,E_3)$ is the electric field vector, D_i are the components of dielectric displacement vector, ε_{rsi} is the alternate tensor, χ^E is the dielectric susceptibility, Δ is the Laplacian operator.

In the above equations, a comma followed by a suffix denotes spatial derivative and a superposed dot denotes the derivative with respect to time respectively.

For two- dimensional problem, we have

$$\boldsymbol{u}=(u_1,0,u_3),\ \boldsymbol{\varphi}=(0,\varphi_2,0). \tag{2.5}$$

We define the following dimensionless quantities

$$x_i^{'}=\frac{\omega^*}{c_1}x_i,\ u_i^{'}=\frac{\rho c_1\omega^*}{\lambda_0}u_i,\ \varphi_i^{'}=\frac{\rho c_1^2}{\lambda_0}\varphi_i,\ \varphi^{*'}=\frac{\rho c_1^2}{\lambda_0}\varphi^*,$$

$$E_i^{'}=\frac{\lambda_2\omega^*}{c_1\lambda_0}E_i,\ t^{'}=\omega^* t,\ i=(1,2,3), \tag{2.6}$$

where $\omega^{*2}=\dfrac{K_0}{\rho j}$, $c_1^2=\dfrac{\lambda_0+2\mu_0+K_0}{\rho}$, ω^* is the characteristic frequency of the medium.

Using the dimensionless quantities (2.6) in equations (2.1)-(2.4), with the aid of (2.5) and after suppressing the primes yield

$$\delta^2\frac{\partial e}{\partial x_1}+\left(1-\delta^2\right)\Delta u_1-a_1\frac{\partial\varphi_2}{\partial x_3}+a_2\frac{\partial\varphi^*}{\partial x_1}=\frac{\partial^2u_1}{\partial t^2}, \quad (2.7)$$

$$\delta^2\frac{\partial e}{\partial x_3}+\left(1-\delta^2\right)\Delta u_3+a_1\frac{\partial\varphi_2}{\partial x_1}+a_2\frac{\partial\varphi^*}{\partial x_3}=\frac{\partial^2u_3}{\partial t^2}, \quad (2.8)$$

$$a_4\nabla^2\varphi_2+a_5\left(\frac{\partial u_1}{\partial x_3}-\frac{\partial u_3}{\partial x_1}\right)-a_6\varphi_2=\frac{\partial^2\varphi_2}{\partial t^2}, \quad (2.9)$$

$$\left(\delta_1^2\Delta-a_7\right)\varphi^*-a_8e+a_9\nabla e_1=\frac{\partial^2\varphi^*}{\partial t^2}, \quad (2.10)$$

$$a_{10}e_1-\Delta\varphi^*=0, \quad (2.11)$$

where $\left(a_1,a_2\right)=\dfrac{1}{\rho c_1^2}\left(K_0,\lambda_{01}\right)$, $\left(a_4,a_5,a_6\right)=\dfrac{1}{j\rho}\left(\dfrac{\gamma_0}{c_1^2},\dfrac{K_0}{\omega^{*2}},\dfrac{2K_0}{\omega^{*2}}\right)$, $\left(a_7,a_8\right)=\dfrac{2}{j_0\omega^{*2}}\left(\dfrac{\lambda_{30}}{\rho},\dfrac{\lambda_{01}}{\rho}\right)$,

$a_9=\dfrac{2c_1^2}{j_0\omega^{*2}}$, $a_{10}=\dfrac{\rho c_1^4\chi_0^E}{\lambda_{21}^2\omega^{*2}}$, $\delta^2=\dfrac{\lambda_0+\mu_0}{\rho c_1^2}$, $\delta_1^2=\dfrac{c_2^2}{c_1^2}$, $c_2^2=\dfrac{2a_{01}}{\rho j_0}$,

$$e=\frac{\partial u_1}{\partial x_1}+\frac{\partial u_3}{\partial x_3},\ e_1=\frac{\partial E_1}{\partial x_1}+\frac{\partial E_3}{\partial x_3},\ \Delta=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_3^2}. \quad (2.12)$$

To simplify the problem, introducing the potential functions ϕ,ψ and ψ_1 through the relations

$$u_1=\frac{\partial\phi}{\partial x_1}-\frac{\partial\psi}{\partial x_3},u_3=\frac{\partial\phi}{\partial x_3}+\frac{\partial\psi}{\partial x_1},\ E_i=\psi_{1,i}, \quad (2.13)$$

in the equations (2.7)-(2.11), we obtain

$$\Delta\phi+a_2\varphi^*=\ddot{\phi}, \quad (2.14)$$

$$\left(1-\delta^2\right)\Delta\psi+a_1\varphi_2=\ddot{\psi}, \quad (2.15)$$

$$\left(a_4\Delta-a_6\right)\varphi_2-a_5\Delta\psi=\ddot{\varphi}_2, \quad (2.16)$$

$$-a_8\Delta\phi+\left(\delta_1^2\Delta-a_7\right)\varphi^*+a_9\Delta\psi_1=\ddot{\varphi}^*, \quad (2.17)$$

$$a_{10}\Delta\psi_1-\Delta\varphi^*=0. \quad (2.18)$$

3. Plane wave solution

For plane harmonic waves, we assume the solution of the form

$$(\phi,\psi,\varphi_2,\varphi^*,\psi_1)=\left\{\bar{\phi},\bar{\psi},\bar{\varphi}_2,\bar{\varphi}^*,\bar{\psi}_1\right\}\exp[i(\xi(x_1l_1+x_3l_3)-\omega t)], \quad (3.1)$$

where ω is the circular frequency and ξ is the wave number. $\bar{\phi},\bar{\psi},\bar{\varphi}_2,\bar{\varphi}^*,\bar{\psi}_1$ are the constants that are independent of time t and coordinates x_m $(m=1,3)$. l_1 and l_3 are the direction cosines of the wave normal to the x_1x_3-plane with the property $l_1^2+l_3^2=1$.

Using (3.1) in (2.14)-(2.18), after some simplification, we obtain

$$\left(\omega^2-\xi^2\right)\overline{\phi}+a_2\overline{\varphi}^*=0, \tag{3.2}$$

$$\left(\omega^2-\left(1-\delta^2\right)\xi^2\right)\overline{\psi}+a_1\overline{\varphi}_2=0, \tag{3.3}$$

$$a_5\xi^2\overline{\psi}+\left(\omega^2-a_4\xi^2-a_6\right)\overline{\varphi}_2=0, \tag{3.4}$$

$$a_8\xi^2\overline{\phi}+\left(\omega^2-\delta_1^2\xi^2-a_7\right)\overline{\varphi}^*-a_9\xi^2\overline{\psi}_1=0, \tag{3.5}$$

$$a_{10}\psi_1-\varphi^*=0. \tag{3.6}$$

The system of equations (3.2), (3.5) and (3.6) has a non-trivial solution if the determinant of the coefficients $\left[\overline{\phi},\overline{\varphi}^*,\overline{\psi}_1\right]^T$ vanishes, which yields to the following polynomial characteristic equation

$$F_1\xi^4+F_2\xi^2+F_3=0, \tag{3.7}$$

where $F_1=\left(a_{10}\delta_1^2+a_9\right)$, $F_2=-a_9\omega^2-a_{10}\left(\delta_1^2\omega^2+b_{11}+a_2a_8\right)$, $F_3=\omega^2b_{11}a_{10}$, $b_{11}=\omega^2-a_7$.

Similarly, the system of equations (3.3) and (3.4) has a non-trivial solution if the determinant of the coefficients $\left[\overline{\psi},\overline{\varphi}_2\right]^T$ vanishes, which yields to the following polynomial characteristic equation

$$F_5\xi^4+F_6\xi^2+F_7=0, \tag{3.8}$$

where $F_5=a_4\left(1-\delta^2\right)$, $F_6=-\left[a_1a_5+a_4\omega^2+\left(1-\delta^2\right)\left(\omega^2-a_6\right)\right]$, $F_7=\omega^2\left(\omega^2-a_6\right)$.

Solving (3.7) we obtain four roots of ξ, in which two roots ξ_1 and ξ_2 correspond to positive x_3-direction and other two roots $-\xi_1$ and $-\xi_2$ correspond to negative x_3-direction. Now and after, we will restrict our work to positive x_3-direction. Corresponding to roots ξ_1 and ξ_2 there exist two waves in descending order of their velocities, namely LD-wave and LM-wave. Likewise, on solving (3.8) we obtain four roots of ξ, in which two roots ξ_3 and ξ_4 correspond to positive x_3-direction represents the two waves in descending order of their velocities, namely CD-I and CD-II waves.

We now derive the expressions for phase velocity.

(i) Phase velocity. The phase velocities are given by

$$V_i=\frac{\omega}{\left|\mathrm{Re}(\xi_i)\right|};\quad i=1,2,3,4,$$

where V_1, V_2, V_3, V_4 are the phase velocities of LD, LM, CD-I, and CD-II waves respectively.

(ii) Attenuation coefficient. The attenuation coefficients are defined as

$$Q_i=\mathrm{Im}(\xi_i);\ i=1,2,3,4,$$

where Q_1, Q_2, Q_3, Q_4 are the attenuation coefficients of LD, T, LM, CD-I, and CD-II waves respectively.

(iii) Specific loss. The specific loss is the ratio of energy ($\overline{W}$) dissipated in taking a specimen through a stress cycle, to the elastic energy (W) stored in the specimen when the strain is maximum. The specific loss is the most direct method of defining internal friction for a material. For a sinusoidal plane wave of small amplitude, Kolsky [32], shows that the specific loss $(\overline{W}/W)$ equals 4π times the absolute value of the ratios of imaginary part of ξ to the real part of ξ, i.e.

$$R_i = \left(\frac{\overline{W}}{W}\right)_i = 4\pi\left|\frac{\mathrm{Im}(\xi_i)}{\mathrm{Re}(\xi_i)}\right|; \quad i = 1,2,3,4 .$$

(iv) Penetration depth. The penetration depths are defined by

$$S_i = \frac{1}{|\mathrm{Im}(\xi_i)|}; \; i = 1,2,3,4$$

4. Steady oscillations

For steady oscillations ,we assume the displacement vector, microrotation and microstretch of the form

$$(\vec{u}(x,t), \vec{\varphi}(x,t), \varphi^*(x,t)) = Re[(\vec{u}, \vec{\varphi}, \varphi^*)e^{-\iota\omega t}], \tag{4.1}$$

Equations (2.1) - (2.4) with aid of (2.6), after simplification and suppressing the primes and with the help of (4.1), we obtain

$$\left(\delta^2\Delta + \omega^2\right)\vec{u} + \left(1-\delta^2\right) grad\; div\, \vec{u} + a_1\; curl\; \vec{\varphi} + a_2\; grad\; \varphi^* = 0,$$

$$\left(a_4\Delta + \mu^*\right)\vec{\varphi} + a_{17}\; grad\; div\; \vec{\varphi} + a_5\; curl\; \vec{u} = 0,$$

$$\left(\left(\delta_1^2 + \delta_2^2\right)\Delta + \zeta^*\right)\varphi^* - a_8\; div\, \vec{u} = 0\,, \tag{4.2}$$

where $\mu^* = \left(\omega^2 - a_6\right),\; \zeta^* = \left(\omega^2 - a_7\right),\quad a_{17} = \dfrac{\alpha_0 + \beta_0}{j\rho c_1^2},\; \delta_2^2 = \dfrac{2\lambda_{21}^2}{j_0\rho c_1^2\chi^E}$.

We introduce the matrix differential operator

$$F(D_x) = \left\|F_{gh}(D_x)\right\|_{7\times 7}, \tag{4.3}$$

where

$$F_{mn}(D_x) = \left(\delta^2\Delta + \omega^2\right)\delta_{mn} + \left(1-\delta^2\right)\frac{\partial^2}{\partial x_m \partial x_n},\;\; F_{m,n+3}(D_x) = a_1\sum_{r=1}^{3}\varepsilon_{mrn}\frac{\partial}{\partial x_r},\;\; F_{m7}(D_x) = a_2\frac{\partial}{\partial x_m},$$

$$F_{m+3,n+3}(D_x) = \left(a_4\Delta + \mu^*\right)\delta_{mn} + a_{17}\frac{\partial^2}{\partial x_m \partial x_n},\;\; F_{m+3,n}(D_x) = a_5\sum_{r=1}^{3}\varepsilon_{mrn}\frac{\partial}{\partial x_r},\;\; F_{m+3,7}(D_x) = F_{7,n+3} = 0,$$

$$F_{77}(D_x) = \left(\delta_1^2 + \delta_2^2\right)\Delta + \zeta^*,\;\; F_{7n}(D_x) = -a_8\frac{\partial}{\partial x_n};\;\; m,n = 1,2,3 \tag{4.4}$$

Here ε_{mrn} is the alternating tensor and δ_{mn} is the Kronecker delta function.

The system of equations (4.2) can be written as

$$F(D_x)U(x) = 0, \tag{4.5}$$

where $U=\left(\vec{u},\vec{\varphi},\varphi^{*}\right)$ is a seven component vector function on E^{3}.

Definition: The fundamental solution of the system of equations (4.2) (the fundamental matrix of operator F) is the matrix $G(x)=\left\|G_{gh}(x)\right\|_{7\times 7}$, satisfying condition [32].

$$F(D_x)G(x)=\delta(x)I(x), \tag{4.6}$$

where δ is the Dirac delta, $I=\left\|\delta_{gh}\right\|_{7\times 7}$ is the unit matrix and $x\in E^{3}$.

Now we construct $G(x)$ in terms of elementary functions.

5. Fundamental solution of system of equation of steady oscillations

Consider the system of equations

$$\delta^{2}\Delta\vec{u}+\left(1-\delta^{2}\right)grad\ div\,\vec{u}+a_{5}\ curl\,\vec{\varphi}-a_{8}\ grad\ \varphi^{*}+\omega^{2}\vec{u}=\vec{H}\,' \tag{5.1}$$

$$a_{1}\ curl\,\vec{u}+\left(a_{4}\Delta+\mu^{*}\right)\vec{\varphi}+a_{17}\ grad\ div\,\vec{\varphi}=\vec{H}\,'' \tag{5.2}$$

$$a_{2}div\,\vec{u}+\left(\left(\delta_{1}^{2}+\delta_{2}^{2}\right)\Delta+\zeta^{*}\right)\varphi^{*}=Z \tag{5.3}$$

where $\vec{H}\,'$, $\vec{H}\,''$ are three component vector function on E^{3} and Z is the scalar functions on E^{3}.

The system of equations (5.1)-(5.3) may be written in the form

$$F^{tr}(D_x)U(x)=Q(x), \tag{5.4}$$

where F^{tr} is the transpose of matrix F, $Q=(\boldsymbol{H}^{'},\boldsymbol{H}^{''},Z)$ and $x\in E^{3}$.

Applying the operator *div* to (5.1) and (5.2), yield

$$\left(\Delta+\omega^{2}\right)div\,\vec{u}-a_{8}\ \Delta\ \varphi^{*}=div\,\vec{H}\,',$$

$$\left(v^{*}\Delta+\mu_{0}^{*}\right)div\,\vec{\varphi}=div\,\vec{H}\,'',$$

$$a_{2}div\,\vec{u}+\left(\left(\delta_{1}^{2}+\delta_{2}^{2}\right)\Delta+\zeta^{*}\right)\varphi^{*}=Z\,, \tag{5.5}$$

where $v^{*}=a_{4}\Delta+a_{17}$.

Equations $(5.5)_1$ and $(5.5)_3$ may be written in the form

$$N\left(\Delta\right)S=\bar{Q}\,, \tag{5.6}$$

where $S=\left(div\,\vec{u},\varphi^{*}\right)$, $\bar{Q}=\left(d_{1},d_{2}\right)=\left(div\,\vec{H}\,',Z\right)$, and

$$N\left(\Delta\right)=\left\|N_{mn}\left(\Delta\right)\right\|_{2\times 2}=\left\|\begin{matrix}\Delta+\omega^{2} & -a_{8}\ \Delta\\ a_{2} & \left(\delta_{1}^{2}+\delta_{2}^{2}\right)\Delta+\zeta^{*}\end{matrix}\right\|_{2\times 2}. \tag{5.7}$$

Equations $(5.5)_1$ and $(5.5)_3$ may be written as

$$\Gamma_{1}\left(\Delta\right)S=\Psi\,, \tag{5.8}$$

where

$$\Psi=\left(\Psi_1,\Psi_2\right),\quad \Psi_n=e^*\sum_{m=1}^{2}N_{mn}^*d_m$$

$$\Gamma_1(\Delta)=e^*\det N\left(\Delta\right),\;\; e^*=1\Big/\left(\delta_1^2+\delta_2^2\right),\;\; n=1,2, \tag{5.9}$$

and N_{mn}^* is the cofactor of the elements of the matrix N.
From (5.7) and (5.9), we see that

$$\Gamma_1(\Delta)=\prod_{m=1}^{2}(\Delta+\lambda_m^2), \tag{5.10}$$

where $\lambda_m^2,\ m=1,2$ are the roots of the equation $\Gamma_1(-k)=0$ (with respect to k).
From equation $(5.5)_2$, it follows that

$$(\Delta+\lambda_5^2)div\,\varphi=\frac{1}{v^*}div\vec{H}'', \tag{5.11}$$

where $\lambda_5^2=\mu_0^*/v^*$.
Applying the operators $a_4\Delta+\mu_0^*$ and $a_5\,curl$ to (5.1) and (5.2), respectively, yield

$$(a_4\Delta+\mu_0^*)\left(\delta^2\Delta\vec{u}+\left(1-\delta^2\right)grad\ div\ \vec{u}+\omega^2\vec{u}\right)+a_5(a_4\Delta+\mu_0^*)curl\vec{\varphi}$$
$$=(a_4\Delta+\mu_0^*)\left(\vec{H}'+a_8\ grad\varphi^*\right), \tag{5.12}$$

and

$$a_5(a_4\Delta+\mu_0^*)curl\ \varphi=-a_5a_1\ curl\ curl\ \vec{u}+a_5\ curl\ \vec{H}''. \tag{5.13}$$

Now

$$curl\ curl\ \vec{u}=grad\ div\ \vec{u}-\Delta\vec{u}\ . \tag{5.14}$$

Making use of (5.13) and (5.14) in (5.12), gives

$$(a_4\Delta+\mu_0^*)\left(\delta^2\Delta\vec{u}+\left(1-\delta^2\right)grad\ div\ \vec{u}+\omega^2\vec{u}\right)+a_5a_1\Delta\vec{u}-a_5a_1\ grad\ div\ \vec{u}$$
$$=(a_4\Delta+\mu_0^*)\left(\vec{H}'+a_8\ grad\varphi^*\right)-a_5\ curl\ \vec{H}''. \tag{5.15}$$

The above equation can also be written as

$$\left\{\left[(a_4\Delta+\mu_0^*)\delta^2+a_5a_1\right]\Delta+(a_4\Delta+\mu_0^*)\omega^2\right\}\vec{u}$$
$$=-\left[\left(1-\delta^2\right)(a_4\Delta+\mu_0^*)-a_5a_1\right]grad\ div\ \vec{u}+(a_4\Delta+\mu_0^*)\left[\vec{H}'+a_8\ grad\varphi^*\right]-a_5\ curl\ \vec{H}''. \tag{5.16}$$

Applying the operator $\Gamma_1(\Delta)$ to (5.16) and using (5.8), we get

$$\Gamma_1(\Delta)\left[a_4\delta^2\Delta^2+\left(\mu^*\delta^2+a_4\omega^2+a_5a_1\right)\Delta+\mu_0^*\omega^2\right]\vec{u}$$
$$=-\left[\left(1-\delta^2\right)(a_4\Delta+\mu_0^*)-a_5a_1\right]grad\ \Psi_1$$

$$+(a_4\Delta+\mu_0^*)\left[\Gamma_1(\Delta)\vec{H}'+a_8\ grad\ \Psi_2\right]-a_5\ \Gamma_1(\Delta)\ curl\ \vec{H}'' \quad (5.17)$$

The above equation can be written as

$$\Gamma_1(\Delta)\Gamma_2(\Delta)\vec{u}=\Psi', \quad (5.18)$$

where

$$\Gamma_2(\Delta)=f^*\det\begin{Vmatrix}\delta^2\Delta+\omega^2 & a_5\Delta\\ -a_1 & a_4\Delta+\mu_0^*\end{Vmatrix},\quad f^*=\frac{1}{\delta^2 a_4} \quad (5.19)$$

and

$$\Psi'=$$

$$f^*\left\{-\left[\left(1-\delta^2\right)(a_4\Delta+\mu_0^*)-a_5a_1\right]grad\ \Psi_1+(a_4\Delta+\mu_0^*)\left[\Gamma_1(\Delta)\vec{H}'+a_8\ grad\ \Psi_2\right]-a_5\ \Gamma_1(\Delta)\ curl\ \vec{H}''\right\}. \quad (5.20)$$

It can be seen that

$$\Gamma_2(\Delta)=(\Delta+\lambda_3^2)(\Delta+\lambda_4^2), \quad (5.21)$$

where $\lambda_3^2,\ \lambda_4^2$ are the roots of the equation $\Gamma_2(-k)=0$ (with respect to k).
Applying the operators $a_1\ curl$ and $(\delta^2\Delta+\omega^2)$ to (5.1) and (5.2), respectively, we obtain

$$a_1(\delta^2\Delta+\omega^2)curl\ \vec{u}=a_1curl\ H'-a_1a_5\ curl\ curl\ \vec{\varphi}, \quad (5.22)$$

$$(\delta^2\Delta+\omega^2)\left(a_4\Delta+\mu_0^*\right)\vec{\varphi}+a_{17}\ (\delta^2\Delta+\omega^2)grad\ div\ \vec{\varphi}$$

$$+a_1(\delta^2\Delta+\omega^2)\ curl\ \vec{u}=(\delta^2\Delta+\omega^2)\vec{H}''. \quad (5.23)$$

We know

$$curl\ curl\ \vec{\varphi}=grad\ div\ \vec{\varphi}-\Delta\vec{\varphi}. \quad (5.24)$$

Making use of (5.22) and (5.24) in (5.23), yield

$$(\delta^2\Delta+\omega^2)\left(a_4\Delta+\mu_0^*\right)\vec{\varphi}+a_{17}\ (\delta^2\Delta+\omega^2)grad\ div\ \vec{\varphi}$$

$$+a_1a_5\Delta\vec{\varphi}-a_1a_5grad\ div\ \vec{\varphi}=(\delta^2\Delta+\omega^2)\vec{H}''-a_1curl\ H'. \quad (5.25)$$

The above equation (5.25) may also be written as

$$\left\{\left[\left(a_4\Delta+\mu_0^*\right)\delta^2+a_1a_5\right]\Delta+\left(a_4\Delta+\mu_0^*\right)\omega^2\right\}\vec{\varphi}$$

$$=-\left[a_{17}\ (\delta^2\Delta+\omega^2)-a_1a_5\right]grad\ div\ \vec{\varphi}+(\delta^2\Delta+\omega^2)\vec{H}''-a_1curl\ H'. \quad (5.26)$$

Applying operator $(\Delta+\lambda_6^2)$ to (5.26) and using (5.11), we obtain

$$(\Delta+\lambda_6^2)\left[a_4\delta^2\Delta^2+\left(\mu_0^*\delta^2+a_4\omega^2+a_5a_1\right)\Delta+\mu_0^*\omega^2\right]\vec{\varphi}$$

$$= -a_1(\Delta+\lambda_6^2) curl\ H^{'} + (\Delta+\lambda_6^2)(\delta^2\Delta+\omega^2)\vec{H}"$$

$$-\left(1/v^*\right)\left[a_{17}\ (\delta^2\Delta+\omega^2) - a_1 a_5\right] grad\ div\vec{H}". \tag{5.27}$$

The above equation may also be rewritten in the form

$$\Gamma_2(\Delta)(\Delta+\lambda_5^2)\vec{\varphi} = \Psi^{"}, \tag{5.28}$$

where

$$\Psi^{"} = f^*\left\{-a_1(\Delta+\lambda_6^2) curl\ \vec{H}^{'} + (\Delta+\lambda_6^2)(\delta^2\Delta+\omega^2)\vec{H}" - \left(1/v^*\right)\left[a_{17}\ (\delta^2\Delta+\omega^2) - a_1 a_5\right] grad\ div\vec{H}"\right\} \tag{5.29}$$

From (5.8), (5.18), and (5.28), we obtain,

$$\Theta(\Delta)U(x) = \hat{\Psi}(x), \tag{5.30}$$

where

$$\hat{\Psi} = \left(\Psi^{'}, \Psi^{"}, \Psi_2\right),$$

$$\Theta(\Delta) = \left\|\Theta_{gh}(\Delta)\right\|_{7\times 7},$$

$$\Theta_{mm}(\Delta) = \Gamma_1(\Delta)\Gamma_2(\Delta) = \prod_{q=1}^{4}(\Delta+\lambda_q^2),\ \Theta_{m+3,n+3}(\Delta) = \Gamma_2(\Delta)(\Delta+\lambda_5^2) = \prod_{q=3}^{5}(\Delta+\lambda_q^2),$$

$$\Theta_{77}(\Delta) = \Gamma_1(\Delta),\ \Theta_{gh}(\Delta) = 0,\quad m = 1,2,3,\quad g,h = 1,2,3,\ldots\ldots\ldots\ldots,7,\ g \neq h. \tag{5.31}$$

Equations (5.9), (5.20) and (5.29) can be rewritten in the form

$$\Psi^{'} = \left[f^*(a_4\Delta+\mu^*)\Gamma_1(\Delta)J + q_{11}(\Delta) grad\ div\right]\vec{H}^{'} + q_{21}(\Delta) curl\ \vec{H}^{"} + q_{31}(\Delta) grad\ Z + q_{41}(\Delta),$$

$$\Psi^{"} = q_{12}(\Delta) curl\ \vec{H}^{'} + f^*\left\{(\Delta+\lambda_6^2)(\delta^2\Delta+\omega^2)J + q_{22}(\Delta) grad\ div\right\}\vec{H}",$$

$$\Psi_2 = q_{13}(\Delta) div\ \vec{H}^{'} + q_{33}(\Delta)Z,$$

$$\Psi_3 = q_{14}(\Delta) div\ \vec{H}^{'} + q_{34}(\Delta)Z + q_{44}(\Delta)L, \tag{5.32}$$

where $J = \left\|\delta_{gh}\right\|_{2\times 2}$ is the unit matrix.
In (5.32), we have used the following notations:

$$q_{m1}(\Delta) = f^* e^*\left\{(a_4\Delta+\mu_0^*)\left[a_8 N_{m2}^*\right] - \left[\left(1-\delta^2\right)(a_4\Delta+\mu_0^*) - a_5 a_1\right] N_{m1}^*\right\},$$

$$q_{21}(\Delta) = -f^* a_1\Gamma_1(\Delta),\ q_{12}(\Delta) = -f^* a_1(\Delta+\lambda_6^2),\quad q_{22}(\Delta) = -\left(f^*/v^*\right)\left[a_{17}\ (\delta^2\Delta+\omega^2) - a_1 a_5\right],$$

$$q_{1p}(\Delta) = e^* N_{1,p-1}^*,\ (p=3),\ q_{rs}(\Delta) = e^* N_{r-1,s-1}^*,\ (r,s=3), m = 1,3. \tag{5.33}$$

Now from equations (5.32), we have

$$\hat{\Psi}(x) = R^{tr}(D_x)Q(x) \tag{5.34}$$

where R^{tr} is the transpose of matrix R and $R = \|R_{mn}\|_{7\times 7}$,

$$R_{mn}(D_x) = f^*(a_4\Delta + \mu_0^*)\Gamma_1(\Delta)\delta_{mn} + q_{11}(\Delta)\frac{\partial^2}{\partial x_m \partial x_n}, \quad R_{m,n+3}(D_x) = q_{12}(\Delta)\sum_{r=1}^{3}\varepsilon_{mrn}\frac{\partial}{\partial x_r},$$

$$R_{mp}(D_x) = q_{1,p-4}(\Delta)\frac{\partial}{\partial x_m}, \quad R_{m+3,n}(D_x) = q_{21}(\Delta)\sum_{r=1}^{3}\varepsilon_{mrn}\frac{\partial}{\partial x_r}, \quad R_{pn}(D_x) = q_{p-4,1}(\Delta)\frac{\partial}{\partial x_n},$$

$$R_{m+3,n+3}(D_x) = f^*(\Delta + \lambda_6{}^2)(\delta^2\Delta + \omega^2)\delta_{mn} + q_{22}(\Delta)\frac{\partial^2}{\partial x_m \partial x_n}, \quad R_{p,s}(D_x) = q_{p-4,s-4}(\Delta),$$

$$R_{m+3,7}(D_x) = R_{m+3,p}(D_x) = R_{p,m+3}(D_x) = 0, \quad m,n = 1,2,3;\ p,s = 7. \tag{5.35}$$

From (5.4), (5.30) and (5.34), we obtain

$$\Theta U = R^{tr} F^{tr} U \quad . \tag{5.36}$$

It implies that

$$R^{tr} F^{tr} = \Theta,$$

and hence

$$F(D_x) R(D_x) = \Theta(\Delta), \tag{5.37}$$

We assume that

$$\lambda_m^2 \neq \lambda_n^2 \neq 0, \quad m,n = 1,2,3,4,5, \quad m \neq n\,. \tag{5.38}$$

Let

$$Y(x) = \|Y_{rs}(x)\|_{8\times 8}, \quad Y_{mm}(x) = \sum_{n=1}^{5} r_{1n}\varsigma_n(x), \quad Y_{m+3,m+3}(x) = \sum_{n=4}^{6} r_{2n}\varsigma_n(x),$$

$$Y_{77}(x) = Y_{88}(x) = \sum_{n=1}^{3} r_{3n}\varsigma_n(x), \quad Y_{vw} = 0, \quad v,w = 1,2,.............,8;\ v \neq w;\ \ m = 1,2,3, \tag{5.39}$$

where

$$\varsigma_n(x) = -\frac{1}{4\pi|x|}\exp(i\lambda_n|x|), \quad n = 1,2,..........,6$$

$$r_{1\ell} = \prod_{m=1, m\neq \ell}^{5}(\lambda_m^2 - \lambda_\ell^2)^{-1}, \quad \ell = 1,2,3,4,5$$

$$r_{2v} = \prod_{m=4, m\neq v}^{6}(\lambda_m^2 - \lambda_v^2)^{-1}, \quad v = 4,5,6$$

$$r_{3w} = \prod_{m=1, m\neq w}^{3}(\lambda_m^2 - \lambda_w^2)^{-1}, \quad w = 1,2,3 \tag{5.40}$$

We will prove the following Lemma:

Lemma: The matrix Y defined above is the fundamental matrix of operator $\Theta(\Delta)$, that is

$$\Theta(\Delta)Y(x) = \delta(x)I(x)\,. \tag{5.41}$$

Proof: To prove the lemma, it is sufficient to prove that

$$\Gamma_1(\Delta)\Gamma_2(\Delta)Y_{11}(x) = \delta(x), \quad \Gamma_2(\Delta)(\Delta+\lambda_6^2)Y_{33}(x) = \delta(x), \quad \Gamma_1(\Delta)Y_{66}(x) = \delta(x)\,, \tag{5.42}$$

we find that

$$r_{11}+r_{12}+r_{13}+r_{14}+r_{15} = 0, \quad \sum_{j=2}^{5} r_{1j}(\lambda_1^2-\lambda_j^2) = 0, \quad \sum_{j=3}^{5} r_{1j}\prod_{m=1}^{2}(\lambda_m^2-\lambda_j^2) = 0, \quad \sum_{j=4}^{5} r_{1j}\prod_{m=1}^{3}(\lambda_m^2-\lambda_j^2) = 0,$$

$$r_{15}\prod_{m=1}^{4}(\lambda_m^2-\lambda_5^2) = 1, \quad (\Delta+\lambda_m^2)\varsigma_n(x) = \delta(x)+(\lambda_m^2-\lambda_n^2)\varsigma_n(x), \quad m,n = 1,2,3,4,5 \tag{5.43}$$

Now consider

$$\Gamma_1(\Delta)\Gamma_2(\Delta)Y_{11}(x) = (\Delta+\lambda_2^2)(\Delta+\lambda_3^2)(\Delta+\lambda_4^2)(\Delta+\lambda_5^2)\sum_{n=1}^{5} r_{1n}[\delta(x)+(\lambda_1^2-\lambda_n^2)\varsigma_n(x)],$$

$$= (\Delta+\lambda_2^2)(\Delta+\lambda_3^2)(\Delta+\lambda_4^2)(\Delta+\lambda_5^2)\sum_{n=2}^{5} r_{1n}(\lambda_1^2-\lambda_n^2)\varsigma_n(x),$$

$$= (\Delta+\lambda_3^2)(\Delta+\lambda_4^2)(\Delta+\lambda_5^2)\sum_{n=2}^{5} r_{1n}(\lambda_1^2-\lambda_n^2)[\delta(x)+(\lambda_2^2-\lambda_n^2)\varsigma_n(x)],$$

$$= (\Delta+\lambda_3^2)(\Delta+\lambda_4^2)(\Delta+\lambda_5^2)\sum_{n=3}^{5} r_{1n}(\lambda_1^2-\lambda_n^2)(\lambda_2^2-\lambda_n^2)\varsigma_n(x),$$

$$= (\Delta+\lambda_4^2)(\Delta+\lambda_5^2)\sum_{n=3}^{5} r_{1n}(\lambda_1^2-\lambda_n^2)(\lambda_2^2-\lambda_n^2)[\delta(x)+(\lambda_3^2-\lambda_n^2)\varsigma_n(x)],$$

$$= (\Delta+\lambda_5^2)\sum_{n=4}^{5} r_{1n}(\lambda_1^2-\lambda_n^2)(\lambda_2^2-\lambda_n^2)(\lambda_3^2-\lambda_n^2)(\lambda_4^2-\lambda_n^2)[\delta(x)+(\lambda_4^2-\lambda_n^2)\varsigma_n(x)],$$

$$= (\Delta+\lambda_5^2)\varsigma_5(x) = \delta(x) \quad . \tag{5.44}$$

Similarly, $(5.42)_2$ and $(5.42)_3$ can be proved.
We introduce the matrix

$$G(x) = R(D_x)Y(x)\,. \tag{5.45}$$

From (5.37), (5.41) and (5.45), we obtain

$$F(D_x)G(x) = F(D_x)R(D_x)Y(x) = \delta(x)I(x)\,. \tag{5.46}$$

Hence, $G(x)$ is a solution to (5.5).
Therefore we have proved the following theorem.
***Theorem**:* The matrix $G(x)$ defined by (5.45) is the fundamental solution of system of equations (4.2).

6. Basic properties of the matrix $G(x)$

Property 1. Each column of the matrix $G(x)$ is the solution of the system of equations (4.2) at every point $x \in E^3$ except at the origin.

Property 2. The matrix $G(x)$ can be written in the form

$$G = \left\| G_{gh} \right\|_{7\times 7},$$

$$G_{mn}(x) = R_{mn}(D_x)Y_{11}(x), \;\; G_{m,n+3}(x) = R_{m,n+3}(D_x)Y_{33}(x),$$

$$G_{mp}(x) = R_{mp}(D_x)Y_{66}(x), \;\; m = 1, 2, \ldots\ldots\ldots\ldots, 7 \;\;\; n = 1, 2, 3, \; p = 7 . \tag{6.1}$$

7. Particular cases

If we neglect the electro effect, we obtain the same result for fundamental solution as discussed by Svanadze [21] by changing the dimensionless quantities in to physical quantities in case of microstretch elastic solid.

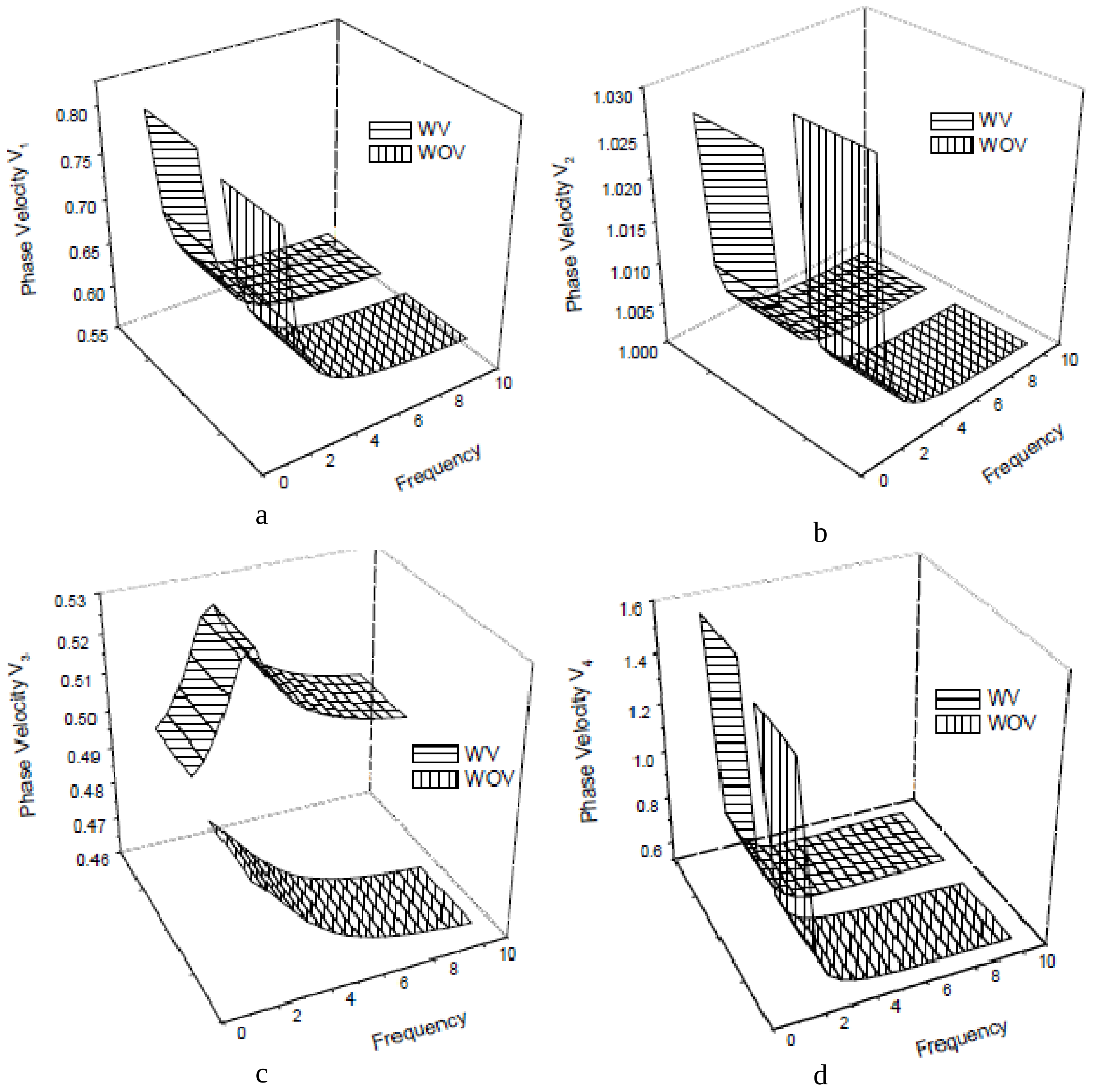

Fig. 1. Variations of phase velocities V_1, V_2, V_3, and V_4 w.r.t. frequency ω.

8. Numerical results and discussion

With the view to illustrate theoretical results obtained in the preceding sections and to compare these with microstretch viscoelastic solid,following Eringen (1984) [33], the values of physical constants are taken as

$$\lambda = 9.4\times10^{10} N\,m^{-2}, \quad \mu = 4.0\times10^{10} N\,m^{-2}, \quad K = 1.0\times10^{10} N\,m^{-2}, \quad \rho = 1.74\times10^{3} kg\,m^{-3},$$
$$j = 0.2\times10^{-19} m^2,\ \gamma = 0.779\times10^{-9} N,\ \lambda_2 = 1.7\times10^{-4} C\,m^{-1},\ \chi^E = 3.18\times10^{2} C^2 N^{-1} m^{-2},$$

and, the microstretch parameters are taken as

$$j_0 = 0.19\times10^{-19} m^2,\ a_0 = 0.779\times10^{-9} N,\ \lambda_{01} = 0.5\times10^{10} Nm^{-2},\ \lambda_{11} = 0.5\times10^{10} Nm^{-2}.$$

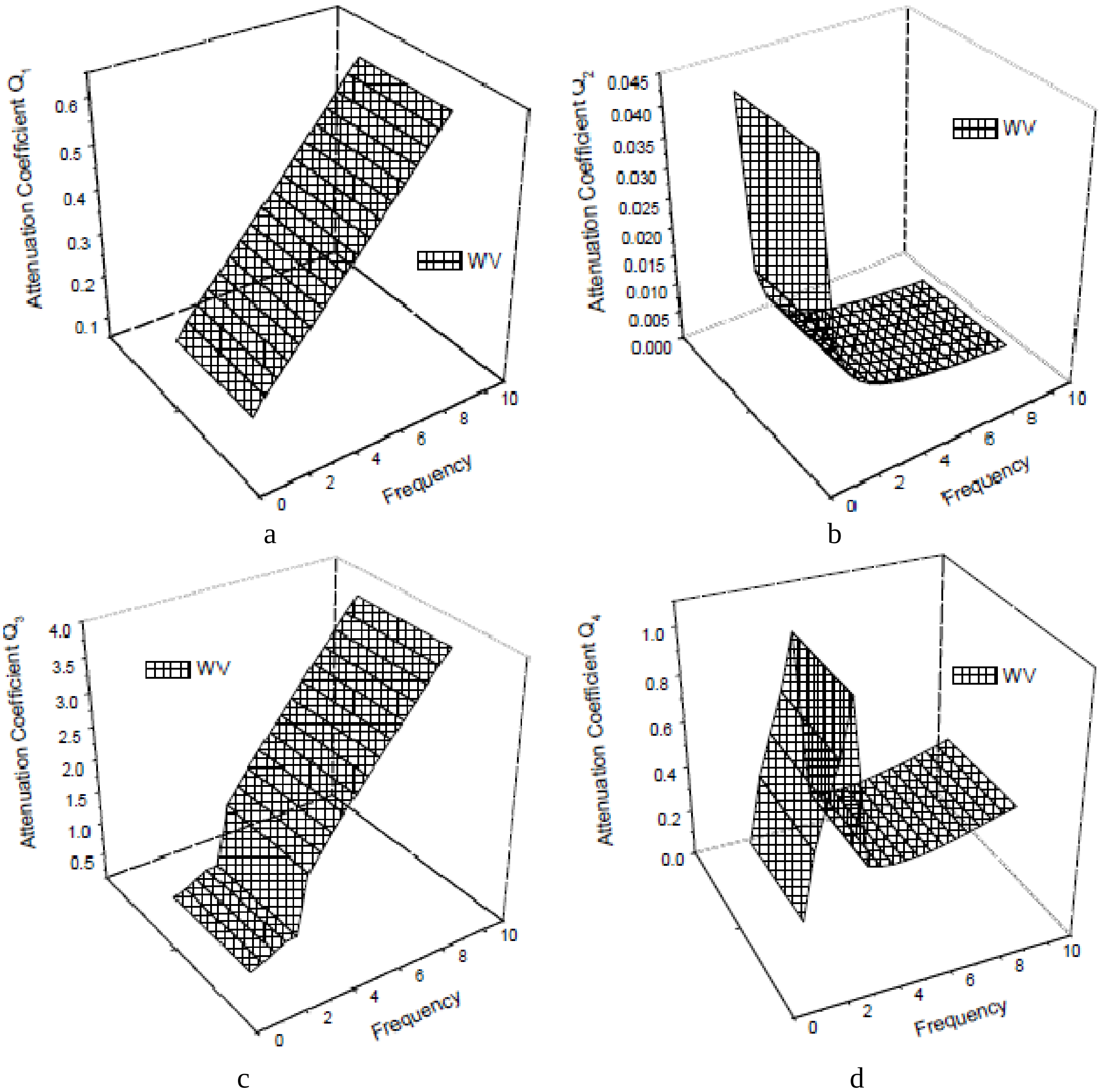

Fig. 2. Variations of attenuation coefficients Q_1, Q_2, Q_3, and Q_4 w.r.t. frequency ω.

For a particular model of electro-microstretch viscoelastic medium, the relevant parameters are expressed as

$\psi_j = \psi(1 - i\omega R_j)$, j=1, 2……, 9; $\psi = \lambda, \mu, K, \lambda_0, \lambda_2, a_0, \chi^E, \gamma, \lambda_3$,

and $R_1 = 0.05$, $R_2 = 0.1$, $R_3 = 0.2$, $R_4 = 0.5$, $R_5 = 0.3$, $R_6 = 0.15$, $R_7 = 0.45$, $R_8 = 0.5$, $R_9 = 1$.

The software Matlab 7.0.4 has been used to determine the values of phase velocities, attenuation coefficients, specific losses and penetration depth of plane waves i.e. LD, LM, CD-I, and CD-II. The variations of resulting quantities with respect to frequency have been shown in Figs. 1-4, respectively. In all the Figures, WV and WOV correspond to electro-microstretch viscoelastic solids and microstretch elastic solids respectively.

Figures 1(a)-(d) show the variation of phase velocities, the curves with and the curves with show the electro-microstretch viscoelastic solid & electro-microstretch elastic solid.

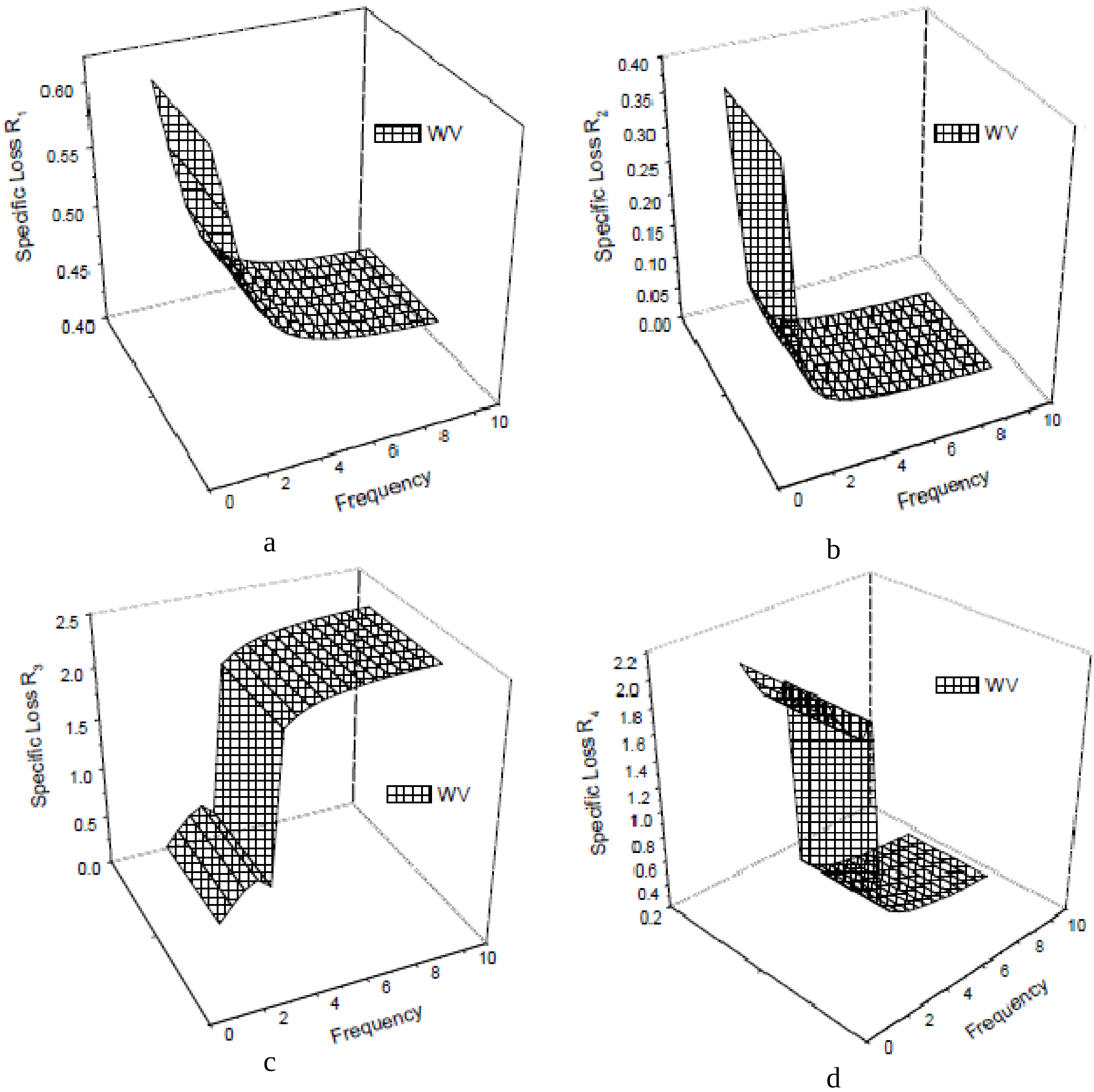

Fig. 3. Variation of specific losses R_1, R_2, R_3, and R_4 w.r.t. frequency ω.

Figures 2, 3 and 4 show the variation of attenuation coefficients, specific losses and penetration depth with respect to frequency, respectively.

Figures 1(a)-(d) show the variation of phase velocities V_1, V_2, V_3, and V_4 with respect to ω with and without viscous effect. Figures 1(a) and 1(b) show that phase velocities V_1 and V_2 decrease smoothly with frequency ω and attain minimum value for higher values of ω. V_2 attains higher values in comparison to V_1. Due to the effect of viscosity V_1 remains more whereas the values of V_2 are small in comparison to that of without viscous effect. Figure 1(c) shows that due to the effect of viscosity, phase velocity V_3 first increases sharply but decreases with small variation for higher values of ω and in absence of viscous effect V_3 decreases smoothly till it attains the minimum value. Figure 1(d) depicts that phase velocity V_4 has the same behavior and variation as V_1 with difference in their magnitude values.

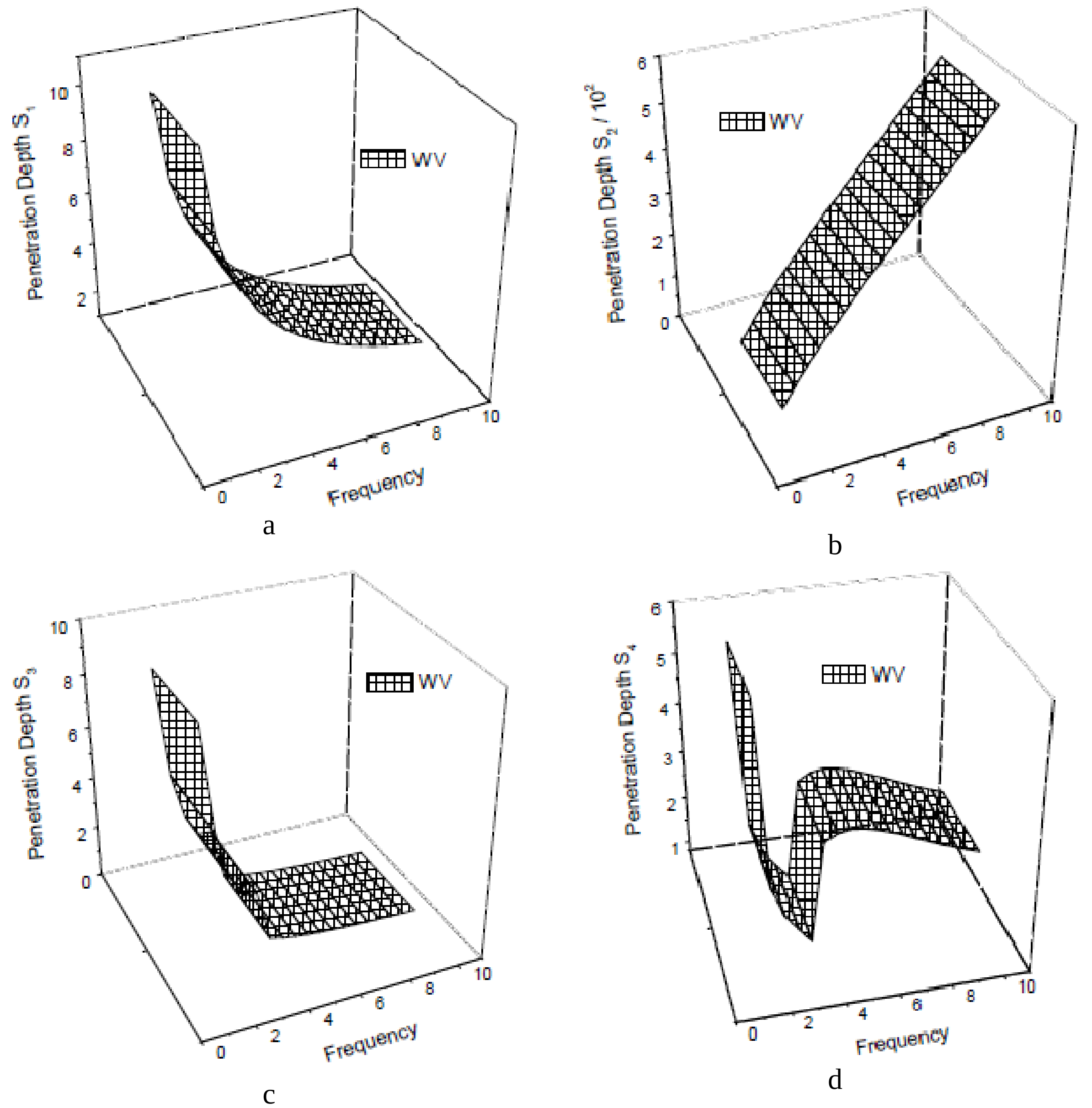

Fig. 4. Variation of penetration depths S_1, S_2, S_3, and S_4 w.r.t. frequency ω.

Figures 2(a)-(d) show the variation of attenuation coefficients Q_1, Q_2, Q_3, and Q_4 with respect to ω with viscous effect. Figure 2(a) shows that attenuation coefficient Q_1 increases sharply with ω. Figure 2(b) depicts that Q_2 decreases sharply for $\omega \leq 4$ and decreases with small variation for other values of ω. It is clear from Fig. 2(c) that initially there is small increase in values of attenuation coefficient Q_3 but it increases sharply for higher values of ω. Figure 2(d) shows that Q_4 fluctuates for $\omega < 5$ and becomes constant for other values of ω. The value of attenuation coefficient Q_3 remains more in comparison to the other roots.
Figures 3(a)-(d) show the variation of specific loss R_1, R_2, R_3, and R_4 with respect to ω with viscous effect. Figures 3(a) and 3(b) show that specific losses R_1 and R_2 decrease sharply for

$\omega \le 4$ and show small variation and appear to be constant for other values of ω. Figure 3(c) depicts that R_3 first increases for $\omega \le 4$ and then shows small variation attenuation coefficient Q_3 but increases sharply for higher values of ω. Figure 3(d) shows that behavior and variation of R_4 are opposite to that of R_3 but with different magnitude values.

Figures 4(a)-(d) show the variation of penetration depth S_1, S_2, S_3, and S_4 with respect to ω with viscous effect. Figure 4(a) shows that penetration depth S_1 decreases smoothly for all values of ω. It is clear from Fig. 4(b) that S_2 increases sharply for all ω. The values of S_2 are demagnified by dividing the original values by 10^2. Figure 4(c) depicts that S_3 decreases sharply for $\omega \le 4$, but becomes constant with small magnitude values for $\omega > 4$. Figure 4(d) shows that S_4 fluctuates for $\omega < 4$ and then decreases smoothly for $\omega \ge 4$.

9. Conclusion

The propagation of plane waves and representation of fundamental solution in a homogeneous, isotropic electro-microstretch viscoelastic solid medium has been studied. For two dimensional model, there exist two coupled longitudinal waves, namely longitudinal displacement wave (LD-wave), longitudinal microstretch wave (LM-wave) and two coupled transverse waves viz. (CD-I and CD-II waves) affected by viscosity. Appreciable viscosity effect has been observed on phase velocity and attenuation coefficients for LD, LM, CD-I and CD-II waves. The fundamental solution $G(x)$ of the system of equations (4.2) makes it possible to investigate three- dimensional boundary value problems in the theory of electro-microstretch viscoelastic solids by potential method [31].

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