

WAVE MOTION AND REPRESENTATION OF FUNDAMENTAL SOLUTION IN ELECTRO-MICROSTRETCH VISCOELASTIC SOLIDS

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Abstract. The homogeneous isotropic electro–microstretch viscoelastic solids have been taken into consideration for investigating the propagation of plane waves and fundamental solution. For two dimensional model, it is found that there exists two coupled longitudinal waves namely longitudinal displacement (LD) wave and longitudinal microstretch (LM) wave and two coupled transverse displacement and transverse microrotational waves (CD-I and CD-II). The phase velocities, attenuation coefficients, specific loss, penetration depth are computed numerically. The resulting quantities are depicted graphically to show the viscous effect. In addition, we construct the fundamental solution of the system of differential equations in the theory of an electro-microstretch viscoelastic solids in case of steady oscillations in terms of elementary functions. Some basic properties of the fundamental solution are established. Some special cases are also discussed.

1. Introduction

The study of viscoelastic behavior is of interest in several contexts. First, materials used in engineering applications may exhibit viscoelastic behavior as an unintentional side effect. Second, the mathematics underlying viscoelasticity is the theory within the applied mathematics community. Third, viscoelasticity is of interest in some branches of material science, metallurgy and solid-state-physics. Fourth, the casual links between viscoelasticity and microstructure is exploited in the use of viscoelastic tests as an inspection tools. In reality all materials deviate from Hooke's law in various ways, for example, by exhibiting viscoulike as well as characteristics.

Viscoelastic materials are those for which the relationship between stress and strain depends on time. All materials exhibit some viscoelastic response. In common metals such as steel, aluminum, copper etc. At room temperature and small strain, the behavior does not deviate much from linear. The Kelvin-Voigt model is one of the macroscopic mechanical models often used to describe the viscoelastic behavior of the material. This model represents the delayed elastic response subjected to stress when the deformation is time dependent but recoverable. The dynamical interaction of thermal and mechanical fields in solids has great practical applications in modern aeronautics, astronautics, nuclear reactors and high energy particle accelerators.

Eringen (1967) [1] extended the theory of micropolar elasticity to obtain linear constitutive theory for micropolar material possessing internal friction. A problem on

translational motion of a microstretch fluid. The comprehensive information on fundamentals solutions of differential equations is also given in the books [29-31].

In this article, the propagation of plane waves and fundamental solution in electro-microstretch solids has been investigated. The phase velocities have been computed numerically and depicted graphically. The representation of the fundamental solution of system of equations in the case of steady oscillations is considered in terms of elementary functions and basic properties of the fundamental solution are established.

2. Basic equations

Let $x = (x_1, x_2, x_3)$ be the points of the Euclidean three-dimensional space $E^3 : |x| = (x_1^2 + x_2^2 + x_3^2)^{1/2}$, $D_x = (\partial/\partial x_1, \partial/\partial x_2, \partial/\partial x_3)$ and let t denote the time variable.

Following Eringen [8] and Iesan and Quintanilla [16], the equations of motion and the constitutive relations in a homogeneous isotropic electro-microstretch solid in the absence of body forces, body couples, stretch force and charge densities are given by

$$(\lambda_0 + \mu_0) \text{grad div } \bar{u} + (\mu_0 + K_0) \Delta \bar{u} + K_0 \text{curl } \bar{\varphi} + \lambda_{01} \text{grad } \varphi^* = \rho \ddot{\bar{u}}, \quad (2.1)$$

$$(\gamma_0 \Delta - 2K_0) \bar{\varphi} + (\alpha_0 + \beta_0) \text{grad div } \bar{\varphi} + K_0 \text{curl } \bar{u} = \rho j \ddot{\bar{\varphi}}, \quad (2.2)$$

$$(a_{01} \Delta - \lambda_{30}) \varphi^* - \lambda_{01} \text{div } \bar{u} + \lambda_{20} \text{div } \bar{E} = \frac{\rho j_0}{2} \ddot{\varphi}^*, \quad (2.3)$$

$$D_{i,i} = 0, \quad (2.4)$$

where $D_i = \chi_0^E E_i - \lambda_{10} \varepsilon_{rsi} \phi_{s,r} - \lambda_{20} \phi_{,i}^*$, $\lambda_0 = \lambda + \lambda^* \frac{\partial}{\partial t}$, $\mu_0 = \mu + \mu^* \frac{\partial}{\partial t}$, $K_0 = K + K^* \frac{\partial}{\partial t}$, $\alpha_0 = \alpha + \alpha^* \frac{\partial}{\partial t}$, $\beta_0 = \beta + \beta^* \frac{\partial}{\partial t}$, $\gamma_0 = \gamma + \gamma^* \frac{\partial}{\partial t}$, $\lambda_{01} = \lambda_0 + \lambda_0^* \frac{\partial}{\partial t}$, $\lambda_{30} = \lambda_3 + \lambda_3^* \frac{\partial}{\partial t}$, $\lambda_{21} = \lambda_2 + \lambda_2^* \frac{\partial}{\partial t}$, $a_{01} = a_0 + a_0^* \frac{\partial}{\partial t}$, $\chi_0^E = \chi^E + \chi^{*E} \frac{\partial}{\partial t}$; $\alpha^*, \beta^*, \gamma^*, \mu^*, K^*, \lambda^*, \lambda_0^*, \lambda_3^*, \lambda_2^*, a_0^*$ are material constants, ρ is the mass density, $\bar{u} = (u_1, u_2, u_3)$ is the displacement vector and $\bar{\varphi} = (\varphi_1, \varphi_2, \varphi_3)$ is the microrotation vector, φ^* is the microstretch scalar function, j is the microinertia, j_0 is the microinertia of the microelements, $E = (E_1, E_2, E_3)$ is the electric field vector, D_i are the components of dielectric displacement vector, ε_{rsi} is the alternate tensor, χ^E is the dielectric susceptibility, Δ is the Laplacian operator.

In the above equations, a comma followed by a suffix denotes spatial derivative and a superposed dot denotes the derivative with respect to time respectively.

For two-dimensional problem, we have

$$\mathbf{u} = (u_1, 0, u_3), \quad \boldsymbol{\varphi} = (0, \varphi_2, 0). \quad (2.5)$$

We define the following dimensionless quantities

$$x'_i = \frac{\omega^*}{c_1} x_i, \quad u'_i = \frac{\rho c_1 \omega^*}{\lambda_0} u_i, \quad \varphi'_i = \frac{\rho c_1^2}{\lambda_0} \varphi_i, \quad \varphi'^* = \frac{\rho c_1^2}{\lambda_0} \varphi^*,$$

$$E'_i = \frac{\lambda_2 \omega^*}{c_1 \lambda_0} E_i, \quad t' = \omega^* t, \quad i = (1, 2, 3), \quad (2.6)$$

constants that are independent of time t and coordinates x_m ($m=1,3$). l_1 and l_3 are the direction cosines of the wave normal to the x_1x_3 - plane with the property $l_1^2 + l_3^2 = 1$.

Using (3.1) in (2.14)-(2.18), after some simplification, we obtain

$$(\omega^2 - \xi^2)\bar{\phi} + a_2\bar{\phi}^* = 0, \quad (3.2)$$

$$(\omega^2 - (1 - \delta^2)\xi^2)\bar{\psi} + a_1\bar{\phi}_2 = 0, \quad (3.3)$$

$$a_5\xi^2\bar{\psi} + (\omega^2 - a_4\xi^2 - a_6)\bar{\phi}_2 = 0, \quad (3.4)$$

$$a_8\xi^2\bar{\phi} + (\omega^2 - \delta_1^2\xi^2 - a_7)\bar{\phi}^* - a_9\xi^2\bar{\psi}_1 = 0, \quad (3.5)$$

$$a_{10}\psi_1 - \phi^* = 0. \quad (3.6)$$

The system of equations (3.2), (3.5) and (3.6) has a non-trivial solution if the determinant of the coefficients $[\bar{\phi}, \bar{\phi}^*, \bar{\psi}_1]^T$ vanishes, which yields to the following polynomial characteristic equation

$$F_1\xi^4 + F_2\xi^2 + F_3 = 0, \quad (3.7)$$

where $F_1 = (a_{10}\delta_1^2 + a_9)$, $F_2 = -a_9\omega^2 - a_{10}(\delta_1^2\omega^2 + b_{11} + a_2a_8)$, $F_3 = \omega^2b_{11}a_{10}$, $b_{11} = \omega^2 - a_7$.

Similarly, the system of equations (3.3) and (3.4) has a non-trivial solution if the determinant of the coefficients $[\bar{\psi}, \bar{\phi}_2]^T$ vanishes, which yields to the following polynomial characteristic equation

$$F_5\xi^4 + F_6\xi^2 + F_7 = 0, \quad (3.8)$$

where $F_5 = a_4(1 - \delta^2)$, $F_6 = -[a_1a_5 + a_4\omega^2 + (1 - \delta^2)(\omega^2 - a_6)]$, $F_7 = \omega^2(\omega^2 - a_6)$.

Solving (3.7) we obtain four roots of ξ , in which two roots ξ_1 and ξ_2 correspond to positive x_3 - direction and other two roots $-\xi_1$ and $-\xi_2$ correspond to negative x_3 - direction. Now and after, we will restrict our work to positive x_3 - direction. Corresponding to roots ξ_1 and ξ_2 there exist two waves in descending order of their velocities, namely LD-wave and LM-wave. Likewise, on solving (3.8) we obtain four roots of ξ , in which two roots ξ_3 and ξ_4 correspond to positive x_3 - direction represents the two waves in descending order of their velocities, namely CD-I and CD-II waves.

We now derive the expressions for phase velocity.

(i) Phase velocity. The phase velocities are given by

$$V_i = \frac{\omega}{|\text{Re}(\xi_i)|}; \quad i = 1, 2, 3, 4,$$

where V_1, V_2, V_3, V_4 are the phase velocities of LD, LM, CD-I, and CD-II waves respectively.

(ii) Attenuation coefficient. The attenuation coefficients are defined as

$$Q_i = \text{Im}(\xi_i); \quad i = 1, 2, 3, 4,$$

where Q_1, Q_2, Q_3, Q_4 are the attenuation coefficients of LD, T, LM, CD-I, and CD-II

where $U = (\bar{u}, \bar{\varphi}, \varphi^*)$ is a seven component vector function on E^3 .

Definition: The fundamental solution of the system of equations (4.2) (the fundamental matrix of operator F) is the matrix $G(x) = \|G_{gh}(x)\|_{7 \times 7}$, satisfying condition [32].

$$F(D_x)G(x) = \delta(x)I(x), \quad (4.6)$$

where δ is the Dirac delta, $I = \|\delta_{gh}\|_{7 \times 7}$ is the unit matrix and $x \in E^3$.

Now we construct $G(x)$ in terms of elementary functions.

5. Fundamental solution of system of equation of steady oscillations

Consider the system of equations

$$\delta^2 \Delta \bar{u} + (1 - \delta^2) \text{grad div } \bar{u} + a_5 \text{curl } \bar{\varphi} - a_8 \text{grad } \varphi^* + \omega^2 \bar{u} = \vec{H}' \quad (5.1)$$

$$a_1 \text{curl } \bar{u} + (a_4 \Delta + \mu^*) \bar{\varphi} + a_{17} \text{grad div } \bar{\varphi} = \vec{H}'' \quad (5.2)$$

$$a_2 \text{div } \bar{u} + ((\delta_1^2 + \delta_2^2) \Delta + \zeta^*) \varphi^* = Z \quad (5.3)$$

where \vec{H}' , \vec{H}'' are three component vector function on E^3 and Z is the scalar functions on E^3 .

The system of equations (5.1)-(5.3) may be written in the form

$$F^{tr}(D_x)U(x) = Q(x), \quad (5.4)$$

where F^{tr} is the transpose of matrix F , $Q = (\vec{H}', \vec{H}'', Z)$ and $x \in E^3$.

Applying the operator div to (5.1) and (5.2), yield

$$(\Delta + \omega^2) \text{div } \bar{u} - a_8 \Delta \varphi^* = \text{div } \vec{H}',$$

$$(v^* \Delta + \mu_0^*) \text{div } \bar{\varphi} = \text{div } \vec{H}'',$$

$$a_2 \text{div } \bar{u} + ((\delta_1^2 + \delta_2^2) \Delta + \zeta^*) \varphi^* = Z, \quad (5.5)$$

where $v^* = a_4 \Delta + a_{17}$.

Equations (5.5)₁ and (5.5)₃ may be written in the form

$$N(\Delta)S = \bar{Q}, \quad (5.6)$$

where $S = (\text{div } \bar{u}, \varphi^*)$, $\bar{Q} = (d_1, d_2) = (\text{div } \vec{H}', Z)$, and

$$N(\Delta) = \|N_{mn}(\Delta)\|_{2 \times 2} = \begin{vmatrix} \Delta + \omega^2 & -a_8 \Delta \\ a_2 & (\delta_1^2 + \delta_2^2) \Delta + \zeta^* \end{vmatrix}_{2 \times 2}. \quad (5.7)$$

Equations (5.5)₁ and (5.5)₃ may be written as

$$\Gamma_1(\Delta)S = \Psi, \quad (5.8)$$

where

$$+(a_4\Delta + \mu_0^*)\left[\Gamma_1(\Delta)\vec{H}' + a_8 \text{grad } \Psi_2\right] - a_5 \Gamma_1(\Delta) \text{curl } \vec{H}'' \quad (5.17)$$

The above equation can be written as

$$\Gamma_1(\Delta)\Gamma_2(\Delta)\vec{u} = \Psi', \quad (5.18)$$

where

$$\Gamma_2(\Delta) = f^* \det \begin{vmatrix} \delta^2\Delta + \omega^2 & a_5\Delta \\ -a_1 & a_4\Delta + \mu_0^* \end{vmatrix}, \quad f^* = \frac{1}{\delta^2 a_4} \quad (5.19)$$

and

$$\Psi' = f^* \left\{ - \left[(1 - \delta^2)(a_4\Delta + \mu_0^*) - a_5 a_1 \right] \text{grad } \Psi_1 + (a_4\Delta + \mu_0^*) \left[\Gamma_1(\Delta)\vec{H}' + a_8 \text{grad } \Psi_2 \right] - a_5 \Gamma_1(\Delta) \text{curl } \vec{H}'' \right\}. \quad (5.20)$$

It can be seen that

$$\Gamma_2(\Delta) = (\Delta + \lambda_3^2)(\Delta + \lambda_4^2), \quad (5.21)$$

where λ_3^2, λ_4^2 are the roots of the equation $\Gamma_2(-k) = 0$ (with respect to k).

Applying the operators $a_1 \text{curl}$ and $(\delta^2\Delta + \omega^2)$ to (5.1) and (5.2), respectively, we obtain

$$a_1(\delta^2\Delta + \omega^2)\text{curl } \vec{u} = a_1\text{curl } H' - a_1 a_5 \text{curl } \text{curl } \vec{\varphi}, \quad (5.22)$$

$$\begin{aligned} &(\delta^2\Delta + \omega^2)(a_4\Delta + \mu_0^*)\vec{\varphi} + a_{17}(\delta^2\Delta + \omega^2)\text{grad } \text{div } \vec{\varphi} \\ &+ a_1(\delta^2\Delta + \omega^2)\text{curl } \vec{u} = (\delta^2\Delta + \omega^2)\vec{H}'' . \end{aligned} \quad (5.23)$$

We know

$$\text{curl } \text{curl } \vec{\varphi} = \text{grad } \text{div } \vec{\varphi} - \Delta \vec{\varphi}. \quad (5.24)$$

Making use of (5.22) and (5.24) in (5.23), yield

$$\begin{aligned} &(\delta^2\Delta + \omega^2)(a_4\Delta + \mu_0^*)\vec{\varphi} + a_{17}(\delta^2\Delta + \omega^2)\text{grad } \text{div } \vec{\varphi} \\ &+ a_1 a_5 \Delta \vec{\varphi} - a_1 a_5 \text{grad } \text{div } \vec{\varphi} = (\delta^2\Delta + \omega^2)\vec{H}'' - a_1 \text{curl } H' . \end{aligned} \quad (5.25)$$

The above equation (5.25) may also be written as

$$\begin{aligned} &\left\{ \left[(a_4\Delta + \mu_0^*)\delta^2 + a_1 a_5 \right] \Delta + (a_4\Delta + \mu_0^*)\omega^2 \right\} \vec{\varphi} \\ &= - \left[a_{17}(\delta^2\Delta + \omega^2) - a_1 a_5 \right] \text{grad } \text{div } \vec{\varphi} + (\delta^2\Delta + \omega^2)\vec{H}'' - a_1 \text{curl } H' . \end{aligned} \quad (5.26)$$

Applying operator $(\Delta + \lambda_6^2)$ to (5.26) and using (5.11), we obtain

$$(\Delta + \lambda_6^2) \left[a_4 \delta^2 \Delta^2 + (\mu_0^* \delta^2 + a_4 \omega^2 + a_5 a_1) \Delta + \mu_0^* \omega^2 \right] \vec{\varphi}$$

where R^{tr} is the transpose of matrix R and $R = \|R_{mn}\|_{7 \times 7}$,

$$\begin{aligned} R_{mn}(D_x) &= f^*(a_4\Delta + \mu_0^*)\Gamma_1(\Delta)\delta_{mn} + q_{11}(\Delta)\frac{\partial^2}{\partial x_m \partial x_n}, \quad R_{m,n+3}(D_x) = q_{12}(\Delta)\sum_{r=1}^3 \varepsilon_{mrn} \frac{\partial}{\partial x_r}, \\ R_{mp}(D_x) &= q_{1,p-4}(\Delta)\frac{\partial}{\partial x_m}, \quad R_{m+3,n}(D_x) = q_{21}(\Delta)\sum_{r=1}^3 \varepsilon_{mrn} \frac{\partial}{\partial x_r}, \quad R_{pn}(D_x) = q_{p-4,1}(\Delta)\frac{\partial}{\partial x_n}, \\ R_{m+3,n+3}(D_x) &= f^*(\Delta + \lambda_6^2)(\delta^2\Delta + \omega^2)\delta_{mn} + q_{22}(\Delta)\frac{\partial^2}{\partial x_m \partial x_n}, \quad R_{p,s}(D_x) = q_{p-4,s-4}(\Delta), \\ R_{m+3,7}(D_x) &= R_{m+3,p}(D_x) = R_{p,m+3}(D_x) = 0, \quad m, n = 1, 2, 3; p, s = 7. \end{aligned} \quad (5.35)$$

From (5.4), (5.30) and (5.34), we obtain

$$\Theta U = R^{tr} F^{tr} U \quad (5.36)$$

It implies that

$$R^{tr} F^{tr} = \Theta,$$

and hence

$$F(D_x)R(D_x) = \Theta(\Delta), \quad (5.37)$$

We assume that

$$\lambda_m^2 \neq \lambda_n^2 \neq 0, \quad m, n = 1, 2, 3, 4, 5, \quad m \neq n. \quad (5.38)$$

Let

$$\begin{aligned} Y(x) &= \|Y_{rs}(x)\|_{8 \times 8}, \quad Y_{mm}(x) = \sum_{n=1}^5 r_{1n} \varsigma_n(x), \quad Y_{m+3,m+3}(x) = \sum_{n=4}^6 r_{2n} \varsigma_n(x), \\ Y_{77}(x) &= Y_{88}(x) = \sum_{n=1}^3 r_{3n} \varsigma_n(x), \quad Y_{vw} = 0, \quad v, w = 1, 2, \dots, 8; v \neq w; m = 1, 2, 3, \end{aligned} \quad (5.39)$$

where

$$\begin{aligned} \varsigma_n(x) &= -\frac{1}{4\pi|x|} \exp(i\lambda_n|x|), \quad n = 1, 2, \dots, 6 \\ r_{1\ell} &= \prod_{m=1, m \neq \ell}^5 (\lambda_m^2 - \lambda_\ell^2)^{-1}, \quad \ell = 1, 2, 3, 4, 5 \\ r_{2v} &= \prod_{m=4, m \neq v}^6 (\lambda_m^2 - \lambda_v^2)^{-1}, \quad v = 4, 5, 6 \\ r_{3w} &= \prod_{m=1, m \neq w}^3 (\lambda_m^2 - \lambda_w^2)^{-1}, \quad w = 1, 2, 3 \end{aligned} \quad (5.40)$$

We will prove the following Lemma:

Lemma: The matrix Y defined above is the fundamental matrix of operator $\Theta(\Delta)$, that is

Property 2. The matrix $G(x)$ can be written in the form

$$G = \left\| G_{gh} \right\|_{7 \times 7},$$

$$G_{mn}(x) = R_{mn}(D_x)Y_{11}(x), \quad G_{m,n+3}(x) = R_{m,n+3}(D_x)Y_{33}(x),$$

$$G_{mp}(x) = R_{mp}(D_x)Y_{66}(x), \quad m = 1, 2, \dots, 7 \quad n = 1, 2, 3, \quad p = 7. \quad (6.1)$$

7. Particular cases

If we neglect the electro effect, we obtain the same result for fundamental solution as discussed by Svanadze [21] by changing the dimensionless quantities in to physical quantities in case of microstretch elastic solid.

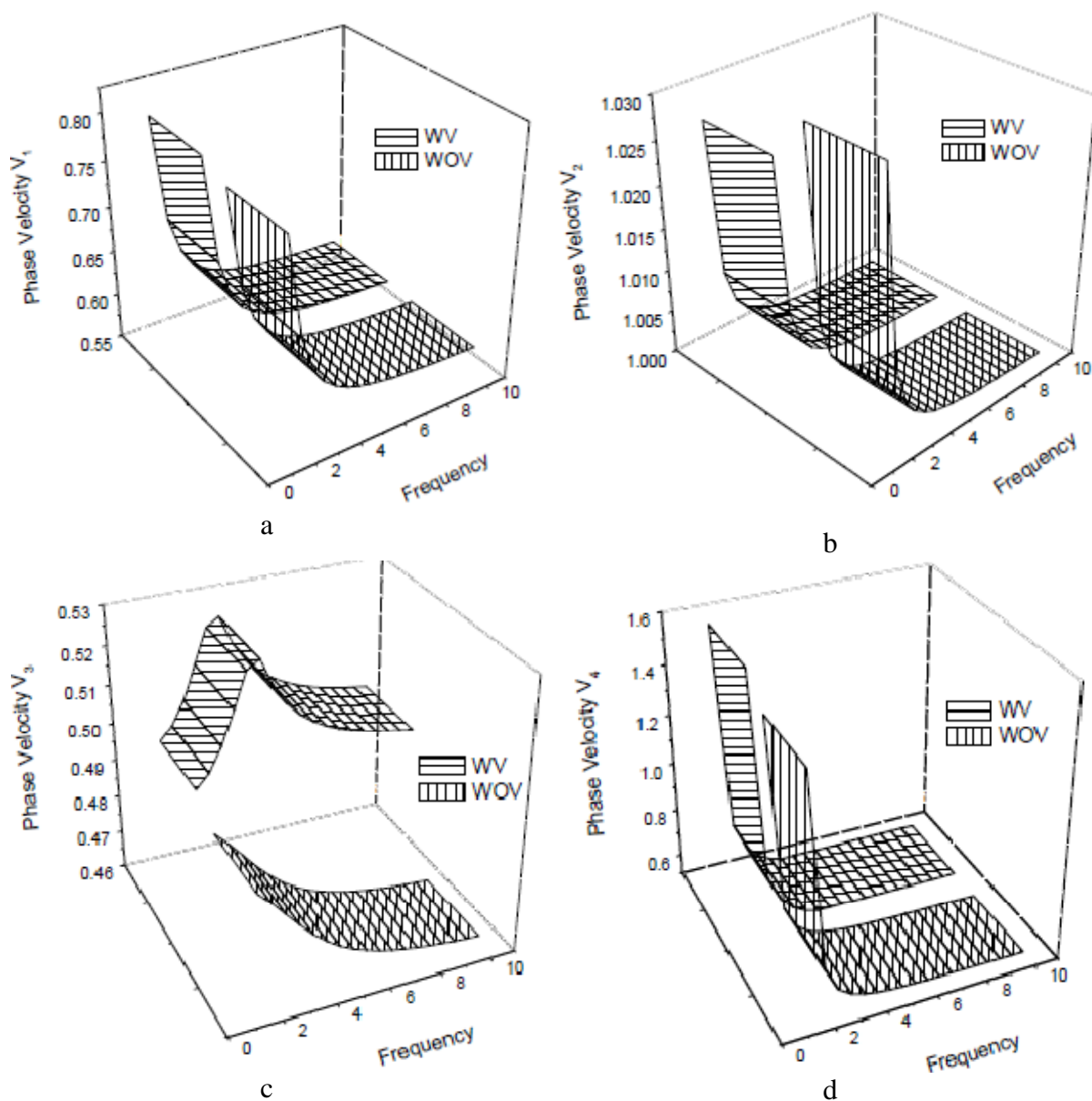


Fig. 1. Variations of phase velocities $V_1, V_2, V_3,$ and V_4 w.r.t. frequency ω .

The software Matlab 7.0.4 has been used to determine the values of phase velocities, attenuation coefficients, specific losses and penetration depth of plane waves i.e. LD, LM, CD-I, and CD-II. The variations of resulting quantities with respect to frequency have been shown in Figs. 1-4, respectively. In all the Figures, WV and WOV correspond to electro-microstretch viscoelastic solids and microstretch elastic solids respectively.

Figures 1(a)-(d) show the variation of phase velocities, the curves with \square and the curves with \square show the electro-microstretch viscoelastic solid & electro-microstretch elastic solid.

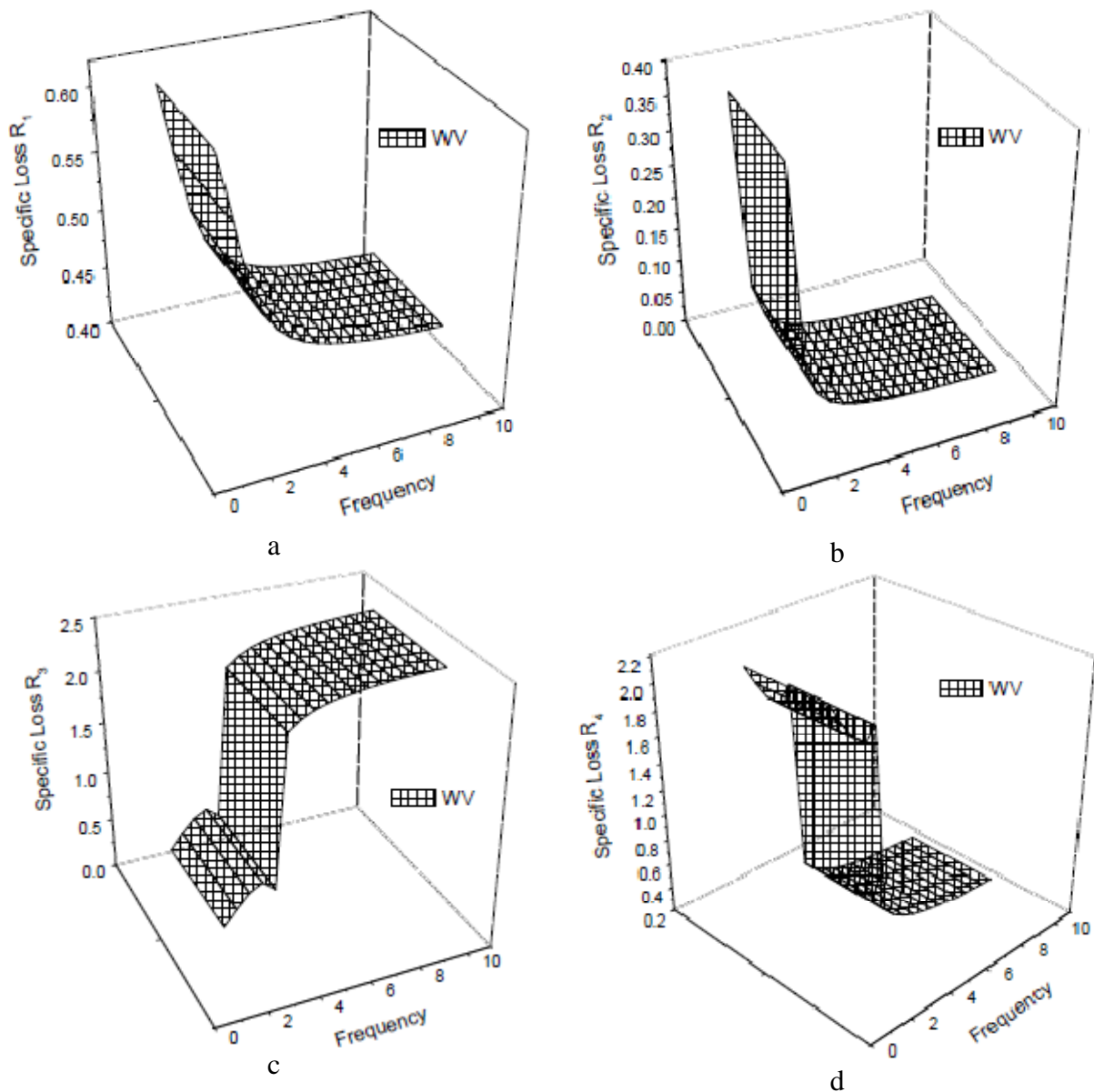


Fig. 3. Variation of specific losses R_1 , R_2 , R_3 , and R_4 w.r.t. frequency ω .

Figures 2, 3 and 4 show the variation of attenuation coefficients, specific losses and penetration depth with respect to frequency, respectively.

Figures 1(a)-(d) show the variation of phase velocities V_1 , V_2 , V_3 , and V_4 with respect to ω with and without viscous effect. Figures 1(a) and 1(b) show that phase velocities V_1 and V_2 decrease smoothly with frequency ω and attain minimum value for higher values of ω . V_2 attains higher values in comparison to V_1 . Due to the effect of viscosity V_1 remains more

$\omega \leq 4$ and show small variation and appear to be constant for other values of ω . Figure 3(c) depicts that R_3 first increases for $\omega \leq 4$ and then shows small variation attenuation coefficient Q_3 but increases sharply for higher values of ω . Figure 3(d) shows that behavior and variation of R_4 are opposite to that of R_3 but with different magnitude values.

Figures 4(a)-(d) show the variation of penetration depth $S_1, S_2, S_3,$ and S_4 with respect to ω with viscous effect. Figure 4(a) shows that penetration depth S_1 decreases smoothly for all values of ω . It is clear from Fig. 4(b) that S_2 increases sharply for all ω . The values of S_2 are demagnified by dividing the original values by 10^2 . Figure 4(c) depicts that S_3 decreases sharply for $\omega \leq 4$, but becomes constant with small magnitude values for $\omega > 4$. Figure 4(d) shows that S_4 fluctuates for $\omega < 4$ and then decreases smoothly for $\omega \geq 4$.

9. Conclusion

The propagation of plane waves and representation of fundamental solution in a homogeneous, isotropic electro-microstretch viscoelastic solid medium has been studied. For two dimensional model, there exist two coupled longitudinal waves, namely longitudinal displacement wave (LD-wave), longitudinal microstretch wave (LM-wave) and two coupled transverse waves viz. (CD-I and CD-II waves) affected by viscosity. Appreciable viscosity effect has been observed on phase velocity and attenuation coefficients for LD, LM, CD-I and CD-II waves. The fundamental solution $G(x)$ of the system of equations (4.2) makes it possible to investigate three- dimensional boundary value problems in the theory of electro-microstretch viscoelastic solids by potential method [31].

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