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ELASTO DYNAMIC WAVE PROPAGATION IN A TRANSVERSELY ISOTROPIC PIEZOELECTRIC CIRCULAR PLATE IMMERSED IN FLUID

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Abstract. The elasto dynamic wave propagation in a piezoelectric plate immersed in fluid is studied based on the three dimensional theory of linear elasticity. Three displacement potential functions are used to uncouple the equations of motion in radial, circumferential and axial directions. The frequency equations that include the interaction between the plate and fluid are obtained by the perfect-slip boundary conditions using the Bessel function solutions. The numerical calculations are carried out for the material PZT-4 and the computed non-dimensional frequency, phase velocity and attenuation coefficient are plotted as the dispersion curves for the immersed plate with different velocity ratio.

1. Introduction

To construct the sensors and transducers piezoelectric materials have been used extensively due to their direct and converse piezoelectricity effects. The direct piezoelectric effect is used in sensing applications, such as in force or displacement sensors. The converse piezoelectric effects are used in transduction applications, such as in motors and device that precisely control positioning, and in generating sonic and ultra sonic signals. In recent years, polymers piezoelectric materials have been used in numerous fields taking advantage of the flexible characteristics of these polymers. Some of the applications of these polymers include Audio device-microphones, high frequency speakers, tone generators and acoustic modems; Pressure switches – position switches, accelerometers, impact detectors, flow meters and load cells; Actuators- electronic fans and high shutters. Since piezoelectric polymers allow their use in a multitude of compositions and geometrical shapes for a large variety of applications from transducers in acoustics, ultrasonic’s and hydrophone applications to resonators in band pass filters, power supplies, delay lines, medical scans and some industrial non-destructive testing instruments.

Many studies have been devoted to transverse vibration, and the natural frequencies and mode shapes for transverse vibration have been well documented [1]. Studies by Tiersten [2] should be mentioned among the early notable contributions to the topic of the mechanics of piezoelectric solids. Rajapakse and Zhou [3] solved the coupled electroelastic equations for a long piezoceramic cylinder by applying Fourier integral transforms. Berg et al. [4] assumed electric field not to be constant over the thickness of piezoceramic cylindrical shells. Ebenezer and Ramesh [5] analyzed axially polarized piezoelectric cylinders with arbitrary boundary conditions on the flat surfaces using the Bessel series. Paper by Wang [6] should be mentioned among the studies of cylindrical shells with a piezoelectric coat. Shul’ga [7] studied the propagation of axisymmetric and non-axisymmetric waves in anisotropic piezoceramic cylinders with various pre polarization directions and boundary conditions. Kim and Lee [8] studied piezoelectric cylindrical transducers with radial polarization and compared their results with those obtained experimentally and numerically by the finite-element method. Recently, Ponnusamy and Selvamani [9, 10] have studied respectively, the three dimensional wave propagation of transversely isotropic magneto thermo elastic cylindrical panel and flexural vibration in a heat conducting cylindrical panel embedded in a Winkler elastic medium in the context of the linear theory of thermo elasticity.

Chan [11] studied the Lamb waves in highly attenuative plastic plate. The thermal deflection of an inverse thermo elastic problem in a thin isotropic circular plate was presented by Gaikward and Deshmukh [12]. A general plane-stress solution in cylindrical coordinates for a piezoelectric plate has been developed by Ashida [13]. Heyliger and Ramirez [14] analyzed the free vibration characteristics of laminated circular piezoelectric plates and discs by using a discrete-layer model of the weak form of the equations of periodic motion. Later, Ashida [15] analyzed the thermally induced wave propagation in a piezoelectric plate. Ahamed [16] discussed the guided waves in a transversely isotropic cylinder immersed in fluid. Nagy[17] investigated longitudinal guided wave propagation in a transversely isotropic rod immersed in fluid based on the superposition of partial waves.

Guo and Sun [18] discussed the propagation of Bleustein - Gulyaev wave in 6mm piezoelectric materials loaded with viscous liquid using the theory of continuum mechanics. Qian et al. [19] analyzed the propagation of Bleustein-Gulyaev waves in 6mm piezoelectric materials loaded with a viscous liquid layer of finite thickness. Ahamed et al. [20] discussed the guided waves in a transversely isotropic plate immersed in fluid.

The present article is aimed to study the elasto dynamic wave propagation in a piezoelectric circular plate immersed in fluid. The frequency equations are obtained from the perfect slip boundary conditions which describes the solid-fluid interface. The computed non-dimensional frequency, phase velocity and attenuation coefficient are plotted in the form of dispersion curves for various velocity ratio of the material PZT-4.

2. Modeling of the problem

We consider a homogeneous, transversely isotropic piezoelectric plate of radius R with uniform thickness h immersed in an in viscid fluid with density ρ_0 . The system displacements and stresses are defined in the polar coordinates r and θ for an arbitrary point inside the plate, with u denoting the displacement in the radial direction of r and v the displacement in the tangential direction of θ.

The linear constitutive equations of coupled elastic and electric field in a piezo electric medium are given by

$$\{\sigma\} = [C]\{e\} - [\eta]^T \{E\},$$

$$\{D\} = [\eta]\{e\} + [\varepsilon]^T \{E\}, \tag{1}$$

where the stress vector $\{\sigma\}$, the strain vector $\{e\}$, the electric field vector $\{E\}$ and the electric displacement vector $\{D\}$ are given in the cylindrical coordinate system (r,θ,z) by

$$\{\sigma\} = [\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{r\theta}, \sigma_{\theta z}, \sigma_{rz}]^T,$$

$$\{E\} = [E_r, E_\theta, E_z]^T,$$

$$\{e\} = [e_{rr}, e_{\theta\theta}, e_{r\theta}, e_{\theta z}, e_{rz}]^T,$$

$$\{D\} = [D_r, D_\theta, D_z]^T, \tag{2}$$

where $[C]$, $[\eta]$ and $[\varepsilon]$ denotes the matrices of elastic constants, piezoelectric constants and dielectric constants respectively. The matrices $[C]$, $[\eta]$ and $[\varepsilon]$ for the transversely isotropic material is given by

$$[C] = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix},$$

$$[\eta] = \begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{15} & 0 & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 \end{bmatrix}, \quad [\varepsilon] = \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{11} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix}. \tag{3}$$

The elastic, the piezoelectric, and dielectric matrices of the 6mm crystal class, the piezoelectric relations are

$$\sigma_{rr} = c_{11}e_{rr} + c_{12}e_{\theta\theta} - e_{31}E_z, \quad \sigma_{\theta\theta} = c_{12}e_{rr} + c_{11}e_{\theta\theta} - e_{31}E_z, \quad \sigma_{zz} = c_{13}e_{rr} + c_{13}e_{\theta\theta} - e_{33}E_z,$$

$$\sigma_{r\theta} = c_{66}e_{r\theta}, \quad \sigma_{rz} = 2c_{44}e_{rz} - e_{15}E_r, \quad \sigma_{\theta z} = c_{44}e_{\theta z} - e_{15}E_\theta,$$

$$D_r = e_{15}e_{rz} + \varepsilon_{11}E_r, \quad D_\theta = e_{15}e_{\theta z} + \varepsilon_{11}E_\theta \quad D_z = e_{31}(e_{rr} + e_{\theta\theta}) + \varepsilon_{33}E_z, \tag{4}$$

where $\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}, \sigma_{r\theta}, \sigma_{\theta z}, \sigma_{rz}$ are the stress components, $e_{rr}, e_{\theta\theta}, e_{zz}, e_{r\theta}, e_{\theta z}, e_{rz}$ are the strain components, $c_{11}, c_{12}, c_{13}, c_{33}, c_{44}$ and $c_{66} = (c_{11} - c_{12})/2$ are the five elastic constants, e_{31}, e_{15}, e_{33} are the piezoelectric constants, $\varepsilon_{11}, \varepsilon_{33}$ are the dielectric constants, ρ is the mass density. The comma in the subscripts denotes the partial differentiation with respect to the variables.

The strain e_{ij} are related to the displacements are given by

$$e_{rr} = u_{r,r}, \; e_{\theta\theta} = r^{-1}(u_r + u_{\theta,\theta}) \; e_{zz} = u_{z,z},$$

$$e_{r\theta} = u_{\theta,r} + r^{-1}(u_{r,\theta} - u_\theta), \quad e_{z\theta} = (u_{\theta,z} + r^{-1}u_{z,\theta}), \quad e_{rz} = u_{z,r} + u_{r,z}. \tag{5}$$

The comma in the subscripts denotes the partial differentiation with respect to the variables.

The two dimensional stress equations of motion in the absence of body force for a linearly elastic plate are obtained by the in-plane vibration as

$$\sigma_{rr,r} + r^{-1}\sigma_{r\theta,\theta} + \sigma_{rz,z} + r^{-1}(\sigma_{rr} - \sigma_{\theta\theta}) = \rho u_{r,tt},$$

$$\sigma_{r\theta,r} + r^{-1}\sigma_{\theta\theta,\theta} + \sigma_{\theta z,z} + 2r^{-1}\sigma_{r\theta} = \rho u_{\theta,tt},$$

$$\sigma_{rz,r} + r^{-1}\sigma_{\theta z,\theta} + r^{-1}\sigma_{rz} = \rho u_{z,tt} . \tag{6}$$

The charge equilibrium equation of electrostatics is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rD_r\right) + \frac{1}{r}\frac{\partial D_\theta}{\partial \theta} + \frac{\partial D_z}{\partial r} = 0 . \tag{7}$$

Substituting the Eqs.(4) and (5) in the Eqs.(6) and (7), results in the following three-dimensional equations of motion, electric conductions are obtained as follows:

$$c_{11}\left(u_{rr,r} + r^{-1}u_{r,r} - r^{-2}u_r\right) - r^{-2}\left(c_{11}+c_{66}\right)u_{\theta,\theta} + r^{-2}c_{66}u_{r,\theta\theta} + c_{44}u_{r,zz} + \left(c_{44}+c_{13}\right)u_{z,rz}$$

$$+r^{-1}\left(c_{66}+c_{12}\right)u_{\theta,r\theta} + \left(e_{31}+e_{15}\right)V_{,rz} = \rho u_{r,tt} ,$$

$$r^{-1}\left(c_{12}+c_{66}\right)u_{r,r\theta} + r^{-2}\left(c_{66}+c_{11}\right)u_{r,\theta} + c_{66}\left(u_{\theta,rr} + r^{-1}u_{\theta,r} - r^{-2}u_\theta\right) + r^{-2}c_{11}u_{\theta,\theta\theta} + c_{44}u_{\theta,zz}$$

$$+r^{-1}\left(c_{44}+c_{13}\right)u_{z,\theta z} + \left(e_{31}+e_{15}\right)V_{,\theta z} = \rho u_{\theta,tt} ,$$

$$c_{44}\left(u_{z,rr} + r^{-1}u_{z,r} + r^{-2}u_{z,\theta\theta}\right) + r^{-1}\left(c_{44}+c_{13}\right)\left(u_{r,z} + u_{\theta,\theta z}\right) + \left(c_{44}+c_{13}\right)u_{r,rz} + e_{33}V_{,zz}$$

$$+e_{15}\left(V_{,rr} + r^{-1}V_{,r} + r^{-2}V_{,\theta\theta}\right) = \rho u_{z,tt} ,$$

$$e_{15}\left(u_{z,rr} + r^{-1}u_{z,r} + r^{-2}u_{z,\theta\theta}\right) + \left(e_{31}+e_{15}\right)\left(u_{r,zr} + r^{-1}u_{r,z} + r^{-1}u_{\theta,z\theta}\right) + e_{33}u_{z,zz} - \varepsilon_{33}V_{,zz}$$

$$-\varepsilon_{11}\left(V_{,rr} + r^{-1}V_{,r} + r^{-2}V_{,\theta\theta}\right) = 0 . \tag{8}$$

In an inviscid fluid-solid interface, the perfect-slip boundary condition allows discontinuity in planar displacement components. That is, the radial component of displacement of the fluid and solid must be equal and the circumferential, longitudinal components are discontinuous at the interface.

The above coupled partial differential equations are also subjected to the following non-dimensional boundary conditions at the surfaces $r = a,\ b$

(i).Stress free inner boundary conditions

$$\left(\sigma_{rr} + p_1^{\ f}\right) = \left(\sigma_{r\theta}\right) = \sigma_{rz} = V = \left(u - u^f\right) = 0 . \tag{9a}$$

(ii).Stress free outer boundary conditions

$$\left(\sigma_{rr} + p_2^{\ f}\right) = \left(\sigma_{r\theta}\right) = \sigma_{rz} = V = \left(u - u^f\right) = 0 . \tag{9b}$$

3. Method of solution of solid

To obtain the propagation of harmonic waves in piezoelectric circular plate, we assume the solutions of the displacement components to be expressed in terms of derivatives of potentials as follows

$$u_r\left(r,\theta,z,t\right) = \left(\phi_{,r} + r^{-1}\psi_{,\theta}\right)\exp\left\{i(kz+\omega t)\right\} ,$$

$$u_\theta\left(r,\theta,z,t\right) = \left(r^{-1}\phi_{,\theta} - \psi_{,r}\right)\exp\left\{i(kz+\omega t)\right\} ,$$

$$u_z\left(r,\theta,z,t\right) = \left(\frac{i}{a}\right)W\exp\left\{i(kz+\omega t)\right\} ,$$

$$V\left(r,\theta,z,t\right)=iV\exp\left\{i(kz+\omega t)\right\},$$

$$E_r\left(r,\theta,z,t\right)=-E_{,r}\exp\left\{i(kz+\omega t)\right\},$$

$$E_\theta\left(r,\theta,z,t\right)=-r^{-1}E_{,\theta}\exp\left\{i(kz+\omega t)\right\},$$

$$E_z\left(r,\theta,z,t\right)=E_{,z}\exp\left\{i(kz+\omega t)\right\}, \tag{10}$$

where $i=\sqrt{-1}$, k is the wave number, ω is the angular frequency, $\phi\left(r,\theta\right),W\left(r,\theta\right)$, $\psi\left(r,\theta\right)$ and $E\left(r,\theta\right)$ are the displacement potentials and $V\left(r,\theta\right)$ is the electric potentials and a is the geometrical parameter of the plate.

By introducing the dimensionless quantities such as $x=r/a$, $\zeta=ka$, $\Omega^2=\rho\omega^2a^2/c_1^2$, $\bar{c}_{11}=c_{11}/c_{44}$, $\bar{c}_{13}=c_{13}/c_{44}$, $\bar{c}_{33}=c_{33}/c_{44}$, $\bar{c}_{66}=c_{66}/c_{44}$, $c_1^2=c_{44}/\rho$, $T_a=\sqrt{c_{11}/\rho}/a$, $\bar{z}=z/a$ and substituting Eq.(7) in Eq.(6), we obtain

$$\left(\bar{c}_{11}\nabla^2+\left(\Omega^2-\zeta^2\right)\right)\phi-\zeta\left(1+\bar{c}_{13}\right)W-\zeta\left(\bar{e}_{31}+\bar{e}_{15}\right)V=0,$$

$$\zeta\left(1+\bar{c}_{13}\right)\nabla^2\phi+\left(\nabla^2+\left(\Omega^2-\zeta^2\bar{c}_{33}\right)\right)W+\left(\bar{e}_{15}\nabla^2-\zeta^2\right)V=0, \tag{11}$$

$$\zeta\left(\bar{e}_{31}+\bar{e}_{15}\right)\nabla^2\phi+\left(\bar{e}_{15}\nabla^2-\zeta^2\right)W+\left(\zeta^2\bar{\varepsilon}_{33}-\bar{\varepsilon}_{11}\nabla^2\right)V=0,$$

and

$$\left(\bar{c}_{66}\nabla^2+\left(\Omega^2-\zeta^2\right)\right)\psi=0, \tag{12}$$

where $\nabla^2=\dfrac{\partial^2}{\partial x^2}+x^{-1}\dfrac{\partial}{\partial x}+x^{-2}\dfrac{\partial^2}{\partial\theta^2}$.

The Eq. (11) can be written as

$$\begin{vmatrix}\left(\bar{c}_{11}\nabla^2+\left(\Omega^2-\zeta^2\right)\right) & -\zeta\left(1+\bar{c}_{13}\right) & -\zeta\left(\bar{e}_{31}+\bar{e}_{15}\right)\\ \zeta\left(1+\bar{c}_{13}\right)\nabla^2 & \left(\nabla^2+\left(\Omega^2-\zeta^2\bar{c}_{33}\right)\right) & \left(\bar{e}_{15}\nabla^2-\zeta^2\right)\\ \zeta\left(\bar{e}_{31}+\bar{e}_{15}\right)\nabla^2 & \left(\bar{e}_{15}\nabla^2-\zeta^2\right) & \left(\zeta^2\bar{\varepsilon}_{33}-\bar{\varepsilon}_{11}\nabla^2\right)\end{vmatrix}\left(\phi,W,V\right)=0. \tag{13}$$

Evaluating the determinant given in Eq.(13), we obtain a partial differential equation of the form

$$\left(A\nabla^6+B\nabla^4+C\nabla^2+D\right)\left(\phi,W,V\right)=0, \tag{14}$$

where

$$A=c_{11}\left(\bar{e}_{15}^{2}+\varepsilon_{11}\right),$$

$$B=\left[\left(1+\bar{c}_{11}\right)\bar{\varepsilon}_{11}+\bar{e}_{15}^{2}\right]\Omega^2+\left\{2\left(\bar{e}_{31}+\bar{e}_{15}\right)\bar{c}_{13}\bar{e}_{15}-\left(1+\bar{\varepsilon}_{11}\bar{c}_{33}\right)\bar{c}_{11}+\bar{c}_{13}^{2}\bar{\varepsilon}_{11}+2\bar{c}_{13}\bar{\varepsilon}_{11}-2\bar{e}_{15}\bar{c}_{11}+2\bar{e}_{13}^{2}\right\}\varsigma^2,$$

$$C=\bar{\varepsilon}_{11}\Omega^4-\left[\left(1+\bar{c}_{13}\right)\bar{\varepsilon}_{11}+\left(1+\bar{c}_{11}\right)+\left(\bar{e}_{31}+\bar{e}_{15}\right)+2\bar{e}_{15}\right]\varsigma^2\Omega^2+\{\bar{c}_{11}\left(1+\bar{c}_{33}\bar{\varepsilon}_{33}\right)-\left[\left(\bar{e}_{31}+\bar{e}_{15}\right)^2+\bar{\varepsilon}_{11}\right]$$

$$-2\bar{e}_{31}\left(1+\bar{c}_{13}\right)-\bar{c}_{13}\bar{\varepsilon}_{33}\left(\bar{c}_{33}+\bar{c}_{13}\right)+2\bar{e}_{15}\}\varsigma^4,$$

$$D=-\left\{\left(1+\bar{c}_{33}\right)\varsigma^6-\left[2\left(1+\bar{c}_{33}\right)\bar{\varepsilon}_{33}+1\right]\varsigma^4\Omega^2+\bar{\varepsilon}_{33}\varsigma^2\Omega^4\right\}. \tag{15}$$

Solving the Eq.(11), we get solutions for a circular cylinder as

$$\phi=\sum_{i=1}^{3}\left[A_iJ_n\left(\alpha_iax\right)+B_iY_n\left(\alpha_iax\right)\right]\cos n\theta,$$

$$W=\sum_{i=1}^{3}a_i\left[A_iJ_n\left(\alpha_iax\right)+B_iY_n\left(\alpha_iax\right)\right]\cos n\theta,$$

$$V=\sum_{i=1}^{3}b_i\left[A_iJ_n\left(\alpha_iax\right)+B_iY_n\left(\alpha_iax\right)\right]\cos n\theta, \tag{16}$$

Here $\left(\alpha_ia\right)^2>0,\quad \left(i=1,2,3\right)$ are the roots of the algebraic equation

$$A\left(\alpha a\right)^6-B\left(\alpha a\right)^4+C\left(\alpha a\right)^2+D=0. \tag{17}$$

The solutions corresponding to the root $\left(\alpha_ia\right)^2=0$ is not considered here, since $J_n\left(0\right)$ is zero, except for $n=0$. The Bessel function J_n is used when the roots $\left(\alpha_ia\right)^2,\quad \left(i=1,2,3\right)$ are real or complex and the modified Bessel function I_n is used when the roots $\left(\alpha_ia\right)^2,\quad \left(i=1,2,3\right)$ are imaginary.

The constants a_i, b_i defined in the Eq.(12) can be calculated from the equations

$$\left(1+\bar{c}_{13}\right)\varsigma a_i+\left(\bar{e}_{31}+\bar{e}_{15}\right)\varsigma b_i=-\left(\bar{c}_{11}\left(\alpha_ia\right)^2-\Omega^2+\varsigma^2\right),$$

$$\left(\left(\alpha_ia\right)^2-\Omega^2+\varsigma^2\bar{c}_{33}\right)a_i+\left(\bar{e}_{15}\left(\alpha_ia\right)^2+\varsigma^2\right)b_i=-\left(\bar{c}_{13}+1\right)\varsigma\left(\alpha_ia\right)^2, \tag{18}$$

Solving the Eq.(12), we obtain

$$\psi=\left[A_4J_n\left(\alpha_4ax\right)+B_4Y_n\left(\alpha_4ax\right)\right]\sin n\theta, \tag{19}$$

where $\left(\alpha_4a\right)^2=\Omega^2-\zeta^2$. If $\left(\alpha_4a\right)^2<0$, the Bessel function J_n is replaced by the modified Bessel function.

4. Method of solution of the fluid

In cylindrical coordinates, the acoustic pressure and radial displacement equation of motion for an in viscid fluid are of the form

$$p^f=-B^f\left(u^f{}_{,r}+r^{-1}\left(u^f+v^f{}_{,\theta}\right)\right), \tag{20}$$

and

$$c_f^{-2} u^f{}_{,tt} = \Delta_{,r}\,, \tag{21}$$

respectively, where $\left(u^f, v^f\right)$ is the displacement vector, B^f is the adiabatic bulk modulus, $c_f = \sqrt{B^f / \rho^f}$ is the acoustic phase velocity of the fluid in which ρ^f is the density of the fluid and

$$\Delta = \left(u^f{}_{,r} + r^{-1}\left(u^f + v^f{}_{,\theta}\right)\right), \tag{22}$$

substituting $u^f = \phi^f{}_{,r}$ and $v^f = r^{-1}\phi^f{}_{,\theta}$ and seeking the solution of Eq.(14) in the form

$$\bar{\phi}^f\left(r,\theta,t\right) = \phi^f \cos n\theta \exp\left\{i\left(\varsigma \bar{z} + \Omega T_a\right)\right\}, \tag{23}$$

where

$$\phi^f = A_4 J_n^1\left(\alpha_5 a x\right) \tag{24}$$

for inner fluid. In Eq.(17), $\left(\alpha_5 a\right)^2 = \Omega^2 / \bar{\rho}\,\overline{B^f}$ in which $\bar{\rho} = \rho / \rho^f$, $\overline{B^f} = B^f / \mu$, J_n^1 is the Bessel function of the first kind. If $\left(\alpha_5 a\right)^2 < 0$, the Bessel function of first kind is to be replaced by the modified Bessel function of second kind K_n. Similarly,

$$\bar{\phi}^f\left(r,\theta,t\right) = \phi^f \cos n\theta \exp\left\{i\left(\varsigma \bar{z} + \Omega T_a\right)\right\}, \tag{25}$$

where

$$\phi^f = A_5 H_n^1\left(\alpha_5 a x\right) \tag{26}$$

for outer fluid. In Eq.(17) $\left(\alpha_5 a\right)^2 = \Omega^2 / \bar{\rho}\,\bar{B}^f$. H_n^1 is the Hankel function of the first kind. If $\left(\alpha_5 a\right)^2 < 0$, then the Hankel function of first kind is to be replaced by K_n, where K_n is the modified Bessel functioning of second kind.

By substituting the expression of the displacement vector in terms of ϕ^f and the equations (21) and (23) in Eq.(20), we could express the acoustic pressure both inner and outer surface of the plate as

$$p_1{}^f = A_4 \Omega^2 \bar{\rho} J_n^1\left(\alpha_5 a x\right) \cos n\theta \exp\left\{i\left(\varsigma \bar{z} + \Omega T_a\right)\right\} \tag{27}$$

for inner fluid and

$$p_2{}^f = A_5 \Omega^2 \bar{\rho} H_n^1\left(\alpha_5 a x\right) \cos n\theta \exp\left\{i\left(\varsigma \bar{z} + \Omega T_a\right)\right\} \tag{28}$$

for outer fluid.

5. Frequency equations

Substituting the solutions given in the Eqs.(16), (19) and (27) in the boundary condition in the Eqs. (9), we obtain a system of five linear algebraic equation as follows:

$$[A]\{X\}=\{0\},\tag{29}$$

where $[A]$ is a 5×5 matrix of unknown wave amplitudes, and $\{X\}$ is an 5×1 column vector of the unknown amplitude coefficients $A_{1,}\ A_{2,}\ A_3,\ A_{4,}\ A_5$. The solution of Eq.(29) is nontrivial when the determinant of the coefficient of the wave amplitudes $\{X\}$ vanishes, that is

$$|A|=0.\tag{30}$$

The elements in the determinant are obtained as follows

$$a_{11}=2\bar{c}_{66}\left\{n(n-1)-\bar{c}_{11}(\alpha_1 a)^2-\varsigma\left(\bar{c}_{13}a_1+\bar{e}_{31}b_1\right)\right\}J_n(\alpha_1 a)+2\bar{c}_{66}(\alpha_1 a)J_{n+1}(\alpha_1 a),$$

$$a_{13}=2\bar{c}_{66}\left\{n(n-1)-\bar{c}_{11}(\alpha_2 a)^2-\varsigma\left(\bar{c}_{13}a_2+\bar{e}_{31}b_2\right)\right\}J_n(\alpha_2 a)+2\bar{c}_{66}(\alpha_2 a)J_{n+1}(\alpha_2 a),$$

$$a_{15}=2\bar{c}_{66}\left\{n(n-1)-\bar{c}_{11}(\alpha_3 a)^2-\varsigma\left(\bar{c}_{13}a_3+\bar{e}_{31}b_3\right)\right\}J_n(\alpha_3 a)+2\bar{c}_{66}(\alpha_3 a)J_{n+1}(\alpha_3 a),$$

$$a_{17}=2\bar{c}_{66}n\left\{(n-1)J_n(\alpha_4 a)-(\alpha_4 a)J_{n+1}(\alpha_4 a)\right\},$$

$$a_{19}=\rho^f\Omega^2 H_n^{(1)}(\alpha_5 a),$$

$$a_{21}=2n\left\{(n-1)J_n(\alpha_1 a)+(\alpha_1 a)J_{n+1}(\alpha_1 a)\right\},$$

$$a_{23}=2n\left\{(n-1)J_n(\alpha_2 a)+(\alpha_2 a)J_{n+1}(\alpha_2 a)\right\},$$

$$a_{25}=2n\left\{(n-1)J_n(\alpha_3 a)+(\alpha_3 a)J_{n+1}(\alpha_3 a)\right\},$$

$$a_{27}=\left\{\left[(\alpha_4 a)^2-2n(n-1)\right]J_n(\alpha_4 a)-2(\alpha_4 a)J_{n+1}(\alpha_4 a)\right\},$$

$$a_{29}=0\ E_{27}=nJ_n(\delta ax)-(\delta ax)J_{n+1}(\delta ax),$$

$$a_{31}=\left((\varsigma+a_1)+\bar{e}_{15}b_1\right)\left\{nJ_n(\alpha_1 a)-(\alpha_1 a)J_{n+1}(\alpha_1 a)\right\},$$

$$a_{33}=\left((\varsigma+a_2)+\bar{e}_{15}b_2\right)\left\{nJ_n(\alpha_2 a)-(\alpha_2 a)J_{n+1}(\alpha_2 a)\right\},$$

$$a_{35}=\left((\varsigma+a_3)+\bar{e}_{15}b_3\right)\left\{nJ_n(\alpha_3 a)-(\alpha_3 a)J_{n+1}(\alpha_3 a)\right\},$$

$$a_{37}=n\varsigma J_n(\alpha_4 a),\qquad a_{39}=0,\qquad a_{41}=b_1 J_n(\alpha_1 a),\qquad a_{43}=b_2 J_n(\alpha_2 a),$$

$$a_{45}=b_3 J_n(\alpha_3 a),\qquad a_{47}=0,\qquad a_{49}=0,$$

$$a_{51}=\left\{nH_n^{(1)}(\alpha_1 a)-(\alpha_1 a)H_{n+1}^{(1)}(\alpha_1 a)\right\},$$

$$a_{53}=\left\{nH_n^{(1)}(\alpha_2 a)-(\alpha_2 a)H_{n+1}^{(1)}(\alpha_2 a)\right\},$$

$$a_{55}=\left\{nH_n^{(1)}(\alpha_3 a)-(\alpha_3 a)H_{n+1}^{(1)}(\alpha_3 a)\right\},$$

$a_{57} = nJ_n\left(\alpha_4 a\right)$,

$a_{59} = -\left\{nH_n^{(1)}\left(\alpha_5 a\right) - \left(\alpha_5 a\right)H_{n+1}^{(1)}\left(\alpha_5 a\right)\right\}$,

where $E_{ij}\ \left(j = 2, 4, 6, 8, 10\right)$ can be obtained by just replacing the Bessel functions of the first kind in $E_{ij}\ \left(i = 1, 3\,5, 7, 9\right)$ with those of the second kind, respectively, while $E_{ij}\ \left(i = 6, 7, 8, 9, 10\right)$ can be obtained by just replacing a in $E_{ij}\ \left(i = 1, 2, 3, 4, 5\right)$ with *b*. Now we consider the coupled free vibration problem.

6. Numerical results and discussion

The coupled free wave propagation in a homogenous isotropic generalized thermo elastic cylindrical plate immersed in water is numerically solved for the PZT-4 material. The material properties of PZT-4 are given as follows and for the purpose of numerical computation the liquid is taken as water. The material properties of PZT-4 is taken form Berlincourt et al. [21] are used for the numerical calculation is given below:
$c_{11} = 13.9\times10^{10}\,N\,m^{-2}$, $c_{12} = 7.78\times10^{10}\,N\,m^{-2}$, $c_{13} = 7.43\times10^{10}\,N\,m^{-2}$, $c_{33} = 11.5\times10^{10}\,N\,m^{-2}$, $c_{44} = 2.56\times10^{10}\,N\,m^{-2}$, $c_{66} = 3.06\times10^{10}\,N\,m^{-2}$, $e_{31} = -5.2\,C\,m^{-2}$, $e_{33} = 15.1\,C\,m^{-2}$, $e_{15} = 12.7\,C\,m^{-2}$, $\varepsilon_{11} = 6.46\times10^{-9}\,C^2N^{-1}m^{-2}$, $\varepsilon_{33} = 5.62\times10^{-9}\,C^2N^{-1}m^{-2}$, $\rho = 7500\,Kg\,m^{-2}$ and for fluid the density $\rho^f = 1000\,Kg\,m^{-3}$, phase velocity $c_f = 1500\,m\sec^{-1}$. The velocity ratio between the fluid and solid is defined as $v_R = c_1/c_f$.

The roots of the algebraic equation in Eq.(14) were calculated using a combination of the Birge-Vita method and Newton-Raphson method. For the present case, the simple Birge-Vita method does not work for finding the root of the algebraic equation. After obtaining the roots of the algebraic equation using the Birge-Vita method, the roots are corrected for the desired accuracy using the Newton-Raphson method. Such a combination can overcome the difficulties encountered in finding the roots of the algebraic equations of the governing equations.

The complex secular Eq.(30) contains complete information regarding wave number, phase velocity and attenuation coefficient and other propagation characteristics of the considered surface waves. In order to solve this equation we take

$$c^{-1} = v^{-1} + i\omega^{-1}q\,, \tag{31}$$

where $k = R + iq$, $R = \dfrac{\omega}{v}$ and R, q are real numbers. Here it may be noted that v and q respectively represent the phase velocity and attenuation coefficient of the waves. Upon using the representation (31) in equation (30) and various relevant relations, the complex roots $\lambda_i^2\ (i = 1, 2, 3, 4)$ of the quadratic equation (14) can be computed with the help of Secant method. The characteristics roots $\lambda_i^2\ (i = 1, 2, 3, 4)$ are further used to solve the equation (30) to obtain phase velocity (v) and attenuation coefficient (q) by using the functional iteration numerical technique as given below.

The equation (30) is of the form $F(C) = 0$ which upon using representation (31) leads to a system of two real equations $f(v,q) = 0$ and $g(v,q) = 0$. In order to apply functional iteration method, we write $v = f^*(v,q)$ and $q = g^*(v,q)$, where the functions f^* and g^* are selected in such a way that they satisfies the conditions

$$\left|\frac{\partial f^*}{\partial v}\right| + \left|\frac{\partial g^*}{\partial q}\right| < 1, \left|\frac{\partial g^*}{\partial v}\right| + \left|\frac{\partial f^*}{\partial q}\right| < 1. \tag{32}$$

For all v, q in the neighborhood of the roots. If (v_0, q_0) be the initial approximation of the root, then we construct a successive approximation according to the formulae

$$\begin{aligned}
&v_1 = f^*(v_0, q_0), && q_1 = g^*(v_1, q_0),\\
&v_2 = f^*(v_1, q_1), && q_2 = g^*(v_2, q_1),\\
&v_3 = f^*(v_2, q_2), && q_3 = g^*(v_3, q_2),\\
&\ldots\ldots\ldots\ldots\ldots\ldots, && \ldots\ldots\ldots\ldots\ldots\ldots,\\
&v_n = f^*(v_n, q_n), && q_n = g^*(v_{n+1}, q_n).
\end{aligned} \tag{33}$$

The sequence $\{v_n, q_n\}$ of approximation to the root will converge to the actual value (v_0, q_0) of the root provided (v_0, q_0) lie in the neighborhood of the actual root. For the initial value $c = c_0 = (v_0, q_0)$, the roots α_i $(i = 1, 2, 3)$ are computed from the equation (14) by using Secant method for each value of the wave number k, for assigned frequency. The values of α_i $(i = 1, 2, 3)$ so obtained are then used in the equation (30) to obtained the current values of v and q each time which are further used to generate the sequence (33). This process is terminated as and when the condition $|v_{n+1} - v_n| < \varepsilon$, ε being arbitrary small number to be selected at random to achieve the accuracy level, is satisfied. The procedure is continually repeated for different values of the wave number (k) to obtain the corresponding values of the phase velocity (c) and attenuation coefficient (q).

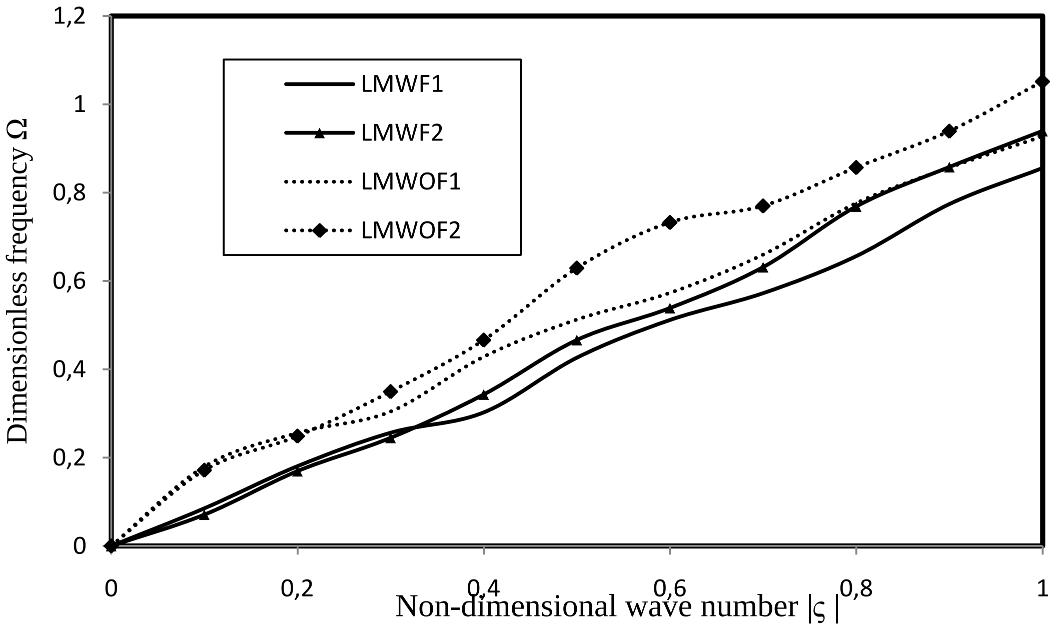

Fig. 1. Dispersion of dimensionless frequency with non-dimensional wave number of cylindrical plate for $v_R = 0.5$.

The notations used in the figures namely, LMWF denotes the longitudinal mode with fluid and LMWOF denotes the longitudinal mode without fluid, 1 refer to the first mode and 2 refer the second mode in all the dispersion curves. In Figs. 1 and 2, the dispersion of dimensionless frequency Ω with the non-dimensional wave number $|\varsigma|$ is studied for the two values of the velocity ratio $v_R = 0.5$ and $v_R = 1.0$.

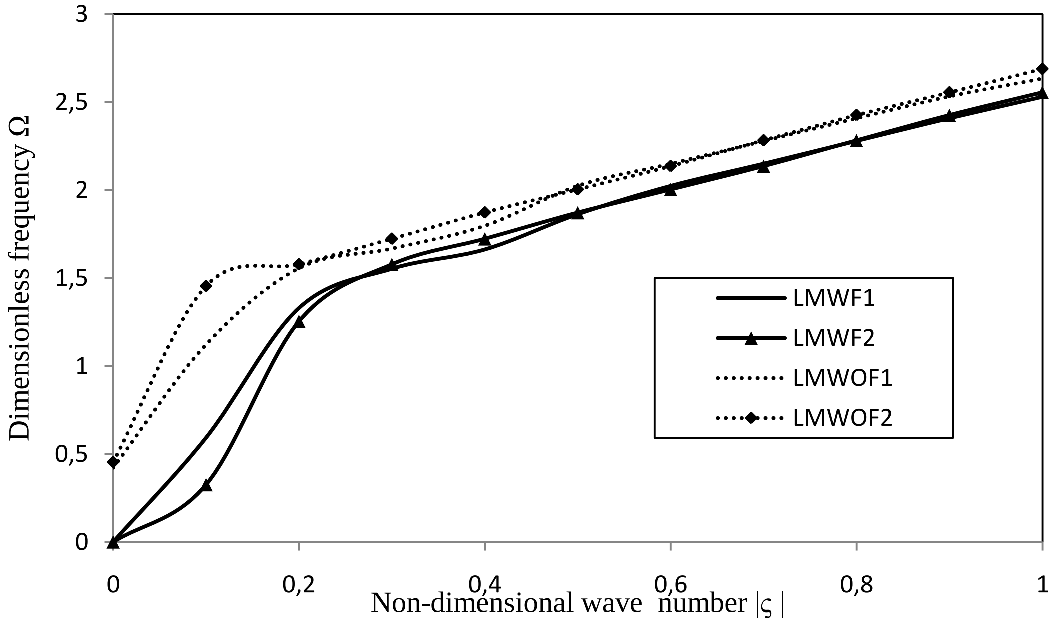

Fig. 2. Dispersion of dimensionless frequency with non-dimensional wave number of cylindrical plate for $v_R = 1.0$.

From Fig. 2, it is observed that the frequency increases linearly with increasing wave number for the low velocity ratio. There is a small deviation from the linear behavior in the current range of wave numbers in Fig. 3 for the larger velocity ratio. As the wave number increases, the longitudinal modes getting more energy than the longitudinal mode with fluid due to the dissipating nature of embedding fluid.

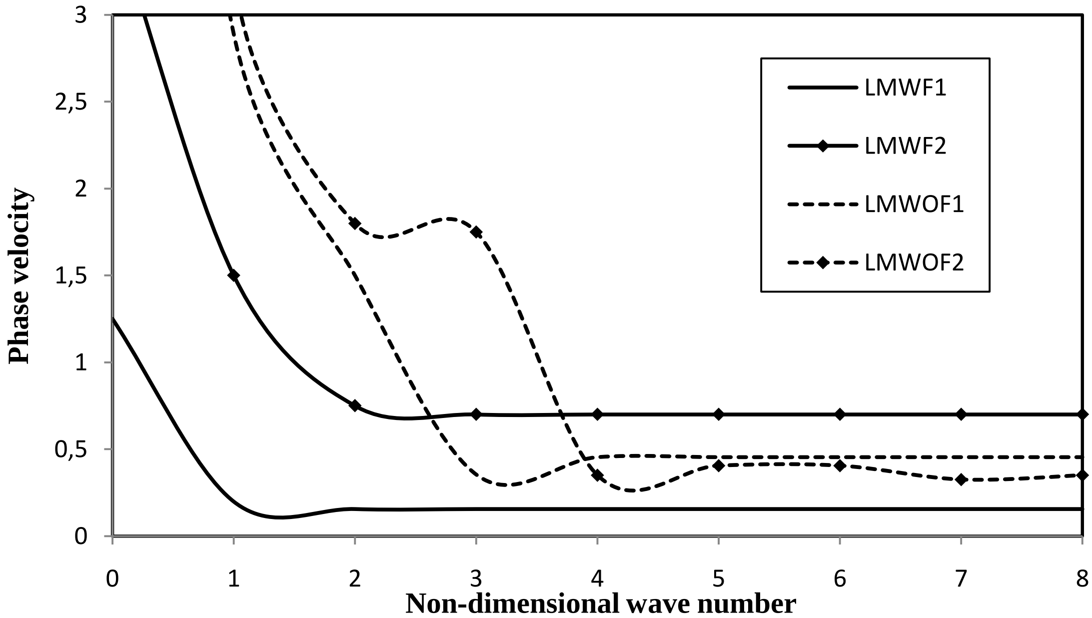

Fig. 3. Dispersion of phase velocity c with non-dimensional wave number $|\varsigma|$ of cylindrical plate for $v_R = 0.5$.

The variation of phase velocities c with the non-dimensional wave number $|\varsigma|$ is discussed in Figs. 3 and 4 for the two values of velocity ratio $v_R = 0.5$ and $v_R = 1.0$ I. Same trends is observed in the energy reduction of the phase velocity from the two figures. Lower wave number getting high reduction of phase velocity and also, there is an energy transformation between the fluid and solid at higher velocity ratio. The longitudinal mode without fluid attains maximum energy compared with the mode with fluid due to the surrounding fluid.

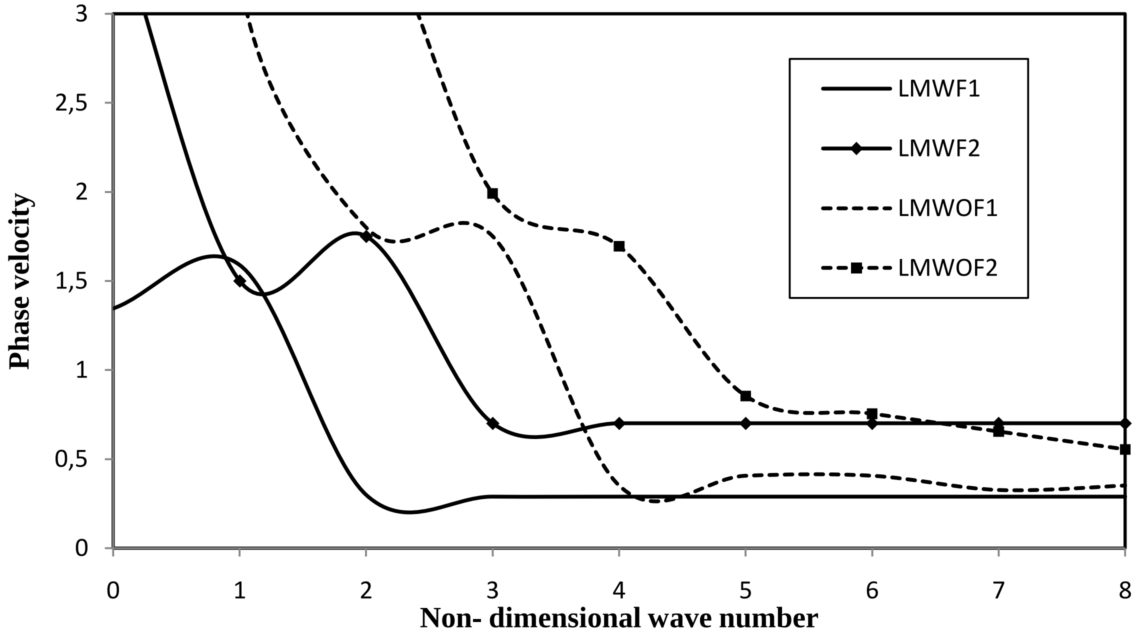

Fig. 4. Dispersion of phase velocity c with non-dimensional wave number $|\varsigma|$ of cylindrical plate for $v_R = 0.5$.

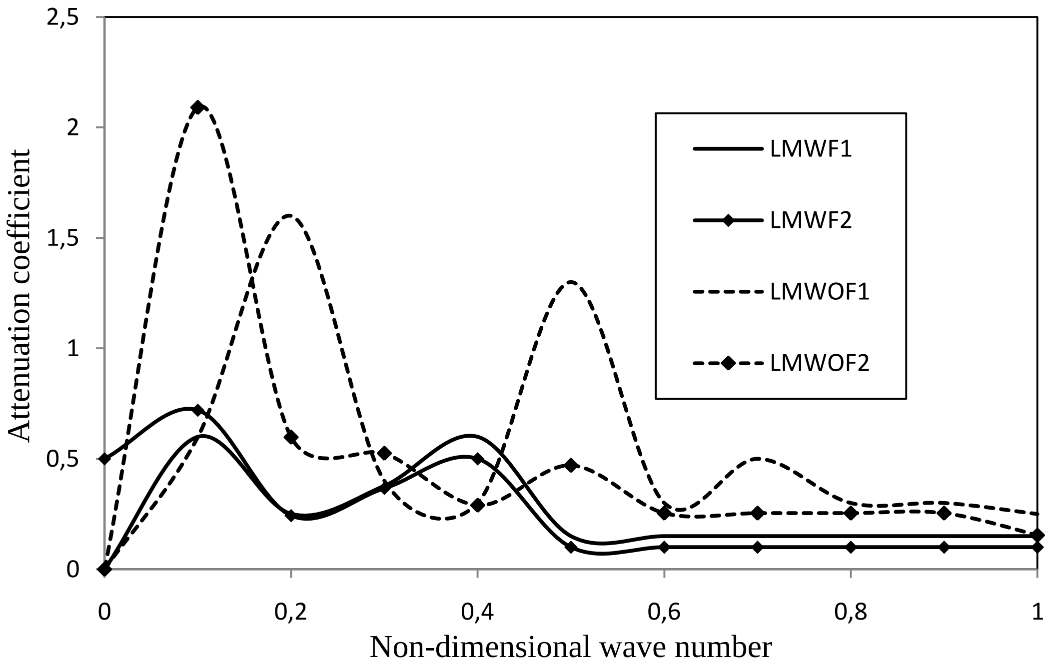

Fig. 5. Dispersion of attenuation coefficient q with non-dimensional wave number $|\varsigma|$ of cylindrical plate for $v_R = 0.5$.

The dispersion of attenuation coefficient q with respect to the wave number $|\varsigma|$ is discussed for the two velocity ratio $v_R = 0.5$ and $v_R = 1.0$ in Figs. 5 and 6. The amplitude of displacement of the attenuation coefficient increases monotonically to attain maximum value in lower wave number and slashes down to became asymptotically linear in the remaining range of the wave number in Fig. 5. The variation of attenuation coefficient for increasing velocity ratio of longitudinal modes with and without fluid is oscillating in the maximum range of wave number as shown in Fig. 6. From Fig. 5 and Fig. 6 it is clear that the attenuation profiles exhibits high oscillating nature in the higher velocity ratio The cross over points in the vibrational modes indicates the energy transfer between the solid and fluid medium.

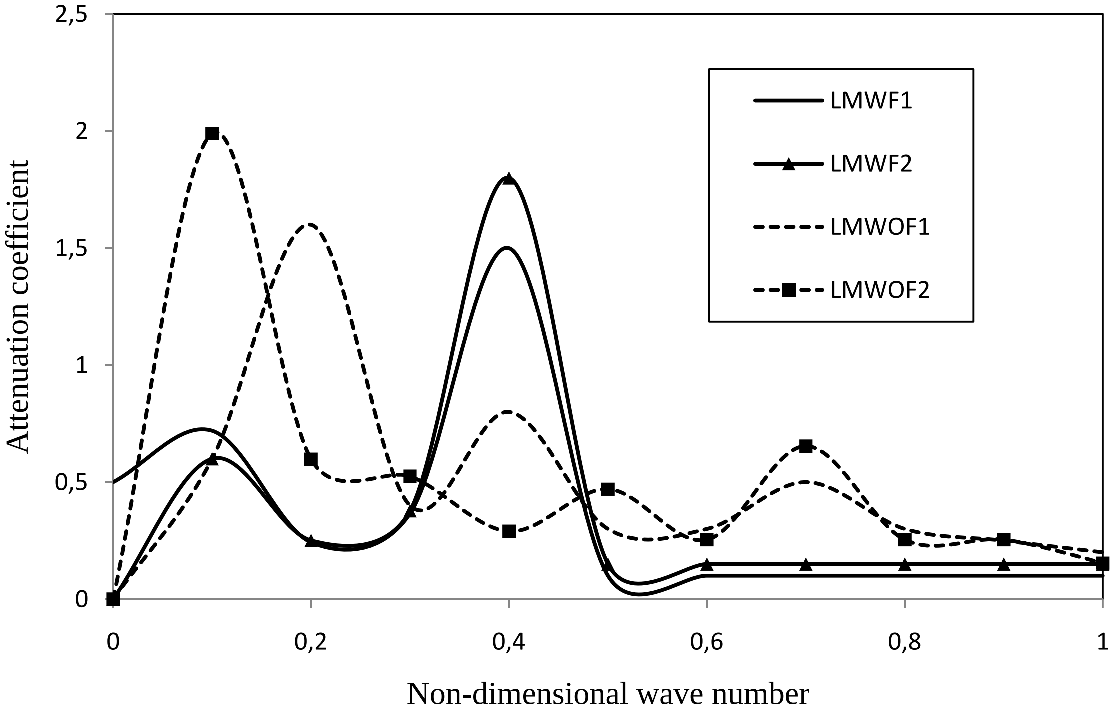

Fig. 6. Dispersion of attenuation coefficient q with non-dimensional wave number $|\varsigma|$ of cylindrical plate for $v_R = 1.0$.

7. Conclusions

The three-dimensional elasto dynamic wave propagation of a homogeneous transversely isotropic piezoelectric cylindrical plate immersed in fluid is investigated in this paper. For this problem, the governing equations of three-dimensional linear theory of elasticity have been employed with electrostatic equation and solved by the Bessel function solutions with complex arguments. The effects of the frequency, attenuation coefficient and phase velocity with respect to the non-dimensional wave number of a PZT-4 cylindrical plate is studied and are plotted as dispersion curves.

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