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GENERALIZED THERMOELASTIC WAVES IN A ROTATING RING SHAPED CIRCULAR PLATE IMMERSED IN AN INVISCID FLUID

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Abstract. In this paper, the generalized thermoelastic waves in a rotating ring shaped circular plate immersed in fluid are studied based on the Lord-Shulman (LS) and Green-Lindsay (GL) generalized two dimensional theory of thermoelasticity. Two displacement potential functions are introduced to uncouple the equations of motion. The frequency equations that include the interaction between the plate and fluid are obtained by the traction free boundary conditions using the Bessel function solutions. The numerical calculations are carried out for the material Zinc and the computed non-dimensional frequency, phase velocity, attenuation coefficient and relative frequency shift are plotted as the dispersion curves for the plate with thermally insulated and isothermal boundaries. The wave characteristics are found to be more dispersive and realistic in the presence of thermal relaxation time, fluid and the rotation parameter.

1. Introduction

The thermomechanical effects produced by the interaction of temperature and deformation fields are of particular importance in many modern designs including gas and steam turbines, jets and rockets, high speed airplanes and nuclear reactors. Hyperbolic heat transport has been receiving increasing attention both for theoretical and for analysis of some practical problems involving fast supply of thermal energy. The usual theory of thermal conduction, based on the Fourier's law implies an immediate response to a temperature gradient and leads to a parabolic differential equation for the evolution of the temperature. In contrast, when relaxation effects are taken in to account in the constitutive equation describing the heat flux, heat conduction equation becomes a hyperbolic equation, which implies a finite speed for heat transport. The analysis of thermally induced wave propagation of rotating circular plate immersed in an inviscid fluid medium is common place in the design of structures, atomic reactors, steam turbines, wave loading on submarine, the impact loading due to superfast train and jets and other devices operating at elevated temperature. When stress wave propagates along embedded structures, they are constrained between its geometric boundaries and they undergo multiple reflections. A complex mixture of constructive and destructive interferences arises from successive reflections, refractions and mode conversion due to the interaction between waves and embedding fluid medium. So, this type of study may be used in applications involving nondestructive testing (NDT), qualitative nondestructive evaluation (QNDE) of large diameter pipes and health monitoring of other ailing infrastructures in addition to check and verify the validity of FEM and BEM for such problems.

The generalized theory of thermoelasticity was developed by Lord-Shulman [1] involving one relaxation time for isotropic homogeneous media, which is called the first generalization to the coupled theory of elasticity. These equations determine the finite speeds

dimensional frequency, phase velocity, attenuation coefficient and relative frequency shift are plotted in dispersion curves for thermally insulated and isothermal boundary of the plate.

2. Formulation of the problem

We consider a thin homogeneous, isotropic, thermally conducting elastic plate of radius R with uniform thickness h and temperature T_0 in the undisturbed state. The system displacements and stresses are defined in polar coordinates (r, θ, z).

The two dimensional stress equations of motion in the absence of body force for a linearly elastic rotating medium are

$$\sigma_{rr,r} + r^{-1}\sigma_{r\theta,\theta} + \sigma_{rz,z} + r^{-1}(\sigma_{rr} - \sigma_{\theta\theta}) + \rho(\vec{\Omega}\times(\vec{\Omega}\times\vec{u}) + 2\vec{\Omega}\times\vec{u}_{,t}) = \rho u_{,tt},$$

$$\sigma_{r\theta,r} + r^{-1}\sigma_{\theta\theta,\theta} + \sigma_{,rz} + \sigma_{\theta z,z} + 2r^{-1}\sigma_{r\theta} = \rho v_{,tt}, \tag{1a}$$

$$\sigma_{rz,r} + r^{-1}\sigma_{\theta z,\theta} + r^{-1}\sigma_{r\theta} + \rho(\vec{\Omega}\times(\vec{\Omega}\times\vec{u}) + 2\vec{\Omega}\times\vec{u}_{,t}) = \rho w_{,tt}$$

The heat conduction equation is

$$k\left(T_{,rr} + r^{-1}T_{,r} + r^{-2}T_{,\theta\theta}\right) - \rho c_v\left(T_{,t} + \tau_0 T_{,tt}\right) = \beta T_0\left(\frac{\partial}{\partial t} + \tau_0\delta_{1k}\frac{\partial^2}{\partial t^2}\right)[e_{rr} + e_{\theta\theta}], \tag{1b}$$

where ρ is the mass density, c_v is the specific heat capacity, k is the thermal conductivity, T_0 is the reference temperature, the displacement equation of motion has the additional terms with a time dependent centripetal acceleration $\vec{\Omega}\times(\vec{\Omega}\times\vec{u})$ and $2\vec{\Omega}\times\vec{u}_{,t}$, where $\vec{u} = (u, 0, w)$ is the displacement vector and $\vec{\Omega} = (0, \Omega, 0)$ is the angular velocity, the comma notation used in the subscript denotes the partial differentiation with respect to the variables

The stress strain relation is given by

$$\sigma_{rr} = \lambda(e_{rr} + e_{\theta\theta}) + 2\mu e_{rr} - \beta(T + \delta_{2k}\tau_1 T_{,t}),$$

$$\sigma_{\theta\theta} = \lambda(e_{rr} + e_{\theta\theta}) + 2\mu e_{\theta\theta} - \beta(T + \delta_{2k}\tau_1 T_{,t}),$$

$$\sigma_{r\theta} = 2\mu e_{r\theta},$$

$$\sigma_{rz} = 2\mu e_{rz}, \tag{2}$$

where e_{ij} are the strain components, $\beta = (3\lambda + 2\mu)\alpha_T$ is the thermal stress coefficient, α_T is the coefficient of linear thermal expansion, T is the temperature, t is the time, λ and μ are Lame' constants. τ_0 and τ_1 are the thermal relaxation times and the comma notation is used for spatial derivatives. Here δ_{ij} is the Kronecker delta function. In addition, we can replace $k = 1$ for LS theory and $k = 2$ for GL theory. The thermal relaxation times τ_0 and τ_1 satisfies the inequalities $\tau_0 \geq \tau_1 \geq 0$ for GL theory only.

The strain e_{ij} are related to the displacements are given by

$$e_{rr} = u_{,r},\ e_{\theta\theta} = r^{-1}(u + v_{,\theta}), \quad 2e_{r\theta} = v_{,r} - r^{-1}(v - u_{,\theta}), \quad 2e_{zr} = (u_{,z} + w_{,r}), \tag{3}$$

in which u and v are the displacement components along radial and circumferential

The stress –strain relation is replaced by

$$\sigma_{rr} = \lambda\left(e_{rr} + e_{\theta\theta}\right) + 2\mu e_{\theta\theta} - \beta\left(T\right),$$

$$\sigma_{\theta\theta} = \lambda\left(e_{rr} + e_{\theta\theta}\right) + 2\mu e_{\theta\theta} - \beta\left(T\right), \quad (6b)$$

$$\sigma_{r\theta} = 2\mu e_{r\theta}.$$

Upon using these relations in Eq.1 we can get the following displacement equation

$$\left(\lambda + 2\mu\right)\left(u_{,rr} + r^{-1}u_{,r} - r^{-2}u\right) + r^{-2}\mu u_{,\theta\theta} + r^{-1}\left(\lambda + \mu\right)v_{,r\theta} + r^{-2}\left(\lambda + 3\mu\right)v_{,\theta} - \beta\left(T\right) = \rho u_{,tt},$$

$$\left(\mu\right)\left(v_{,rr} + r^{-1}v_{,r} - r^{-2}v\right) + r^{-1}\left(\lambda + \mu\right)u_{,r\theta} + r^{-2}\left(\lambda + 3\mu\right)u_{,\theta} + r^{-2}\left(\lambda + 3\mu\right)v_{,\theta}$$

$$+ r^{-2}\left(\lambda + 2\mu\right)v_{,\theta\theta} - \beta\left(T\right) = \rho v_{,tt} \quad ,$$

$$\left(\lambda + \mu\right)u_{,rz}\mu + r^{-1}\left(\lambda + \mu\right)v_{,\theta z} + \mu\left(w_{,rr} + r^{-1}w_{,r} + r^{-2}w_{,\theta\theta}\right)$$

$$-\beta\left(T\right) + \rho\left(\Omega^2 w + 2\Omega u_{,t}\right) = \rho w_{,tt}, \quad (6c)$$

The symbols and notations having the same meaning are defined in earlier sections. Since the heat conduction equation in this theory is of hyperbolic wave type, it is automatically ensures that the finite speeds of propagation for heat and elastic waves.

2.2. Green-Lindsay (GL) theory. The second generalization to the coupled thermoelasticity with two relaxation times called Green-Lindsay theory of thermoelasticity is obtained by setting k =2 in the heat conduction Eq.(1b)

The heat conduction equation is simplified as

$$k\left(T_{,rr} + \frac{1}{r}T_{,r} + \frac{1}{r^2}T_{,\theta\theta}\right) = \rho C_v\left[T_{,t} + \tau_0 T_{,tt}\right] + \beta T_0 \frac{\partial}{\partial t}\left(e_{rr} + e_{\theta\theta}\right). \quad (7a)$$

The stress –strain relation is replaced by

$$\sigma_{rr} = \lambda\left(e_{rr} + e_{\theta\theta}\right) + 2\mu e_{\theta\theta} - \beta\left(T + \tau_1 T_{,t}\right),$$

$$\sigma_{\theta\theta} = \lambda\left(e_{rr} + e_{\theta\theta}\right) + 2\mu e_{\theta\theta} - \beta\left(T + \tau_1 T_{,t}\right), \quad (7b)$$

$$\sigma_{r\theta} = 2\mu e_{r\theta}.$$

Substituting these relations in Eq.1, the displacement equation is reduced as

$$\left(\lambda + 2\mu\right)\left(u_{,rr} + r^{-1}u_{,r} - r^{-2}u\right) + r^{-2}\mu u_{,\theta\theta} + r^{-1}\left(\lambda + \mu\right)v_{,r\theta} + r^{-2}\left(\lambda + 3\mu\right)v_{,\theta}$$

$$-\beta\left(T_{,r} + \tau_1 T_{,rt}\right) = \rho u_{,tt},$$

$$\left(\mu\right)\left(v_{,rr} + r^{-1}v_{,r} - r^{-2}v\right) + r^{-1}\left(\lambda + \mu\right)u_{,r\theta} + r^{-2}\left(\lambda + 3\mu\right)u_{,\theta} + r^{-2}\left(\lambda + 3\mu\right)v_{,\theta}$$

$$+ r^{-2}\left(\lambda + 2\mu\right)v_{,\theta\theta} - \beta\left(T_{,\theta} + \tau_1 T_{,\theta t}\right) = \rho v_{,tt},$$

$$\left(\nabla^2+\left(2+\bar{\lambda}\right)\varpi^2-\varsigma^2+\Gamma\right)\ \psi_n=0, \tag{11}$$

where $\nabla^2 \equiv \partial^2/\partial x^2 + x^{-1}\,\partial/\partial x + x^{-2}\,\partial^2/\partial\theta^2$.

The parameters defined in Eq. (10) namely, ε_1 couples the equations corresponding to the elastic wave propagation and the heat conduction which is called the coupling factor; the coefficient ε_2, which is introduced by the theory of generalized thermoelasticity, may render the governing system of equations hyperbolic. The parameter ε_3 is the coefficient of the term indicating the difference between empirical and thermodynamic temperatures.

Solving the partial differential Eq. (6), the solutions for symmetric mode is obtained as

$$\phi_n=\sum_{i=1}^{2}\left[A_{in}J_n\left(\alpha_i ax\right)+B_{in}Y_n\left(\alpha_i ax\right)\right]\cos n\theta, \tag{12a}$$

$$T_n=\sum_{i=1}^{2}d_i\left[A_{in}J_n\left(\alpha_i ax\right)+B_{in}Y_n\left(\alpha_i ax\right)\right]\cos n\theta, \tag{12b}$$

and the solution for the anti symmetric mode $\bar{\phi}_n$ is obtained by replacing $\cos n\theta$ by $\sin n\theta$ in Eqs. (12), we get

$$\bar{\phi}_n=\sum_{i=1}^{2}\left[\bar{A}_{in}J_n\left(\alpha_i ax\right)+\bar{B}_{in}Y_n\left(\alpha_i ax\right)\right]\sin n\theta, \tag{13a}$$

$$\bar{T}_n=\sum_{i=1}^{2}d_i\left[\bar{A}_{in}J_n\left(\alpha_i ax\right)+\bar{B}_{in}J_n\left(\alpha_i ax\right)\right]\sin n\theta, \tag{13b}$$

Equation (11) is a Bessel equation with its possible solution as

$$\bar{\psi}=\begin{cases}A_3J_n(\alpha_3ax)+B_3Y_n(\alpha_3ax) & \alpha_3ax>0\\ A_3a^n+B_3a^{-n} & \alpha_3ax=0\\ A_3I_n(\alpha_3ax)+B_3K_n(\alpha_3ax) & \alpha_3ax<0\end{cases}, \tag{14}$$

where J_n and Y_n are Bessel functions of the first and second kinds respectively while, I_n and K_n are modified Bessel functions of first and second kinds respectively. $\left(A_i,B_i\right)\ \ i=1,2,3$ are the arbitrary constants. Generally $\alpha_3ax\neq 0$, so that the situation $\alpha_3ax\neq 0$ will not be discussed in the following. For convenience, we will pay attention only to the case of $\alpha_3ax>0$ in what follows, and the derivation for the case of $\alpha_3ax<0$ is similar.

Solving Eq. (11), we obtain

$$\psi_n=\left[A_{3n}J_n\left(\alpha_3ax\right)+B_{3n}Y_n\left(\alpha_3ax\right)\right]\sin n\theta \tag{15a}$$

for symmetric mode, and for the antisymmetric mode $\bar{\psi}_n$ is obtained from Eq. (15a) by replacing $\sin n\theta$ by $\cos n\theta$.

$$\bar{\psi}_n=\left[\bar{A}_{3n}J_n\left(\alpha_3ax\right)+\bar{B}_{3n}J_n\left(\alpha_3ax\right)\right]\cos n\theta, \tag{15b}$$

where $(\alpha_3a)^2=(2+\bar{\lambda})\varpi^2-\varsigma^2+\Gamma$. If $(\alpha_ia)^2<0\ \ (i=1,2,3)$, then the Bessel functions J_n and

Eqs. (20) and (22) in Eq. (17), we could express the acoustic pressure both inner and outer surface of the ring as

$$p_1^{\,f} = A_4 \varpi^2 \overline{\rho} J_n(\delta a x) \exp\{i(p\theta - \omega t)\} \tag{24}$$

for inner fluid and

$$p_2^{\,f} = A_5 \varpi^2 \overline{\rho} H_n(\delta a x) \exp\{i(p\theta - \omega t)\} \tag{25}$$

for outer fluid.

4.1. Flexural mode. Expanded expressions for the stress and displacement at any point can be derived in terms of potential functions. Upon using the potential functions defined in Eqs. (8) in to Eqs. (5) results in the following equations

$$\left|E_{ij}\right| = 0 \qquad i, j = 1, 2.....8. \tag{26}$$

Equation (26) represents the frequency equation for flexural wave propagating along an isotropic circular plate immersed in fluid.

4.2. Longitudinal mode. Longitudinal waves are axially symmetric waves characterized by the presence of displacement component in the radial and axial directions. That is, for the longitudinal wave travelling along the axis, the displacement field is independent of θ coordinate and is of the form $(u,0,0)$. This mode of wave propagation corresponds to $n = 0$ in Eqs. (12)-(13) resulting

$$\det(E_{ij}) = \begin{vmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{vmatrix} = 0. \tag{27}$$

Equation (27) is the frequency equation for longitudinal waves propagating along an isotropic circular plate immersed in fluid.

5. Frequency equations

In this section we shall derive the secular equation for the three dimensional vibrations circular plate subjected to stress free boundary conditions at the upper and lower surfaces at $r = a, b$:

$$\left|E_{ij}\right| = 0 \qquad i, j = 1, 2.....8 \tag{28}$$

$$E_{11} = (2+\overline{\lambda})(ip)\Big((nJ_n(\alpha_1 ax) + (\alpha_1 ax)J_{n+1}(\alpha_1 ax)) - (ip)((\alpha_1 ax)^2 R^2 - n^2)J_n(\alpha_1 ax)\Big)$$
$$+\overline{\lambda}\Big((ip)n(n-1)\big(Jn(\alpha_1 ax) - (\alpha_1 ax)J_{\delta+1}(\alpha_1 ax)\big)\Big) - \beta T(i\omega)\eta_2 d_1 (ax)^2,$$

$$E_{13} = (2+\overline{\lambda})(ip)\Big((nJ_n(\alpha_2 ax) + (\alpha_2 ax)J_{n+1}(\alpha_2 ax)) - (ip)((\alpha_2 ax)^2 R^2 - n^2)J_n(\alpha_2 ax)\Big)$$
$$+\overline{\lambda}(ip)\Big(n(n-1)\big(Jn(\alpha_2 ax) - (\alpha_2 ax)J_{\delta+1}(\alpha_2 ax)\big)\Big) - \beta T(i\omega)\eta_2 d_2 (ax)^2,$$

$$E_{15} = (2+\overline{\lambda})\big((n(n-1)J_n(\alpha_3 ax) - (\alpha_3 ax)J_{n+1}(\alpha_3 ax)\big) + \overline{\lambda}\big(n(n-1)J_n(\alpha_3 ax) - (\alpha_3 ax)J_{n+1}(\alpha_3 ax)\big),$$

$$E_{16} = \varpi^2 \overline{\rho}(ax)^2 \big(nJ_n(\delta ax) - (\delta ax)J_{n+1}(\delta ax)\big),$$

where $k = R + iq$, $R = \frac{\omega}{v}$ and R, q are real numbers. Here it may be noted that v and q respectively represent the phase velocity and attenuation coefficient of the waves. Upon using the representation (29) in Eq. (28) and various relevant relations, the complex roots $\alpha_i^2 (i = 1, 2, 3)$ of the quadratic Eq. (9) can be computed with the help of Secant method. The characteristics roots $\alpha_i^2 (i = 1, 2, 3)$ are further used to solve the Eq. (28) to obtain phase velocity (v) and attenuation coefficient (q) by using the functional iteration numerical technique as given below.

The Eq. (28) is of the form $F(C) = 0$ which upon using representation (29) leads to a system of two real equations $f(v,q) = 0$ and $g(v,q) = 0$. In order to apply functional iteration method, we write $v = f^*(v,q)$ and $q = g^*(v,q)$, where the functions f^* and g^* are selected in such a way that they satisfies the conditions

$$\left|\frac{\partial f^*}{\partial v}\right| + \left|\frac{\partial g^*}{\partial q}\right| < 1, \left|\frac{\partial g^*}{\partial v}\right| + \left|\frac{\partial f^*}{\partial q}\right| < 1 \qquad (30)$$

for all v, q in the neighborhood of the roots. If (v_0, q_0) is the initial approximation of the root, then we construct a successive approximation according to the formulae

$$v_1 = f^*(v_0, q_0) \qquad q_1 = g^*(v_1, q_0)$$

$$v_2 = f^*(v_1, q_1) \qquad q_2 = g^*(v_2, q_1)$$

$$v_3 = f^*(v_2, q_2) \qquad q_3 = g^*(v_3, q_2) \qquad (31)$$

$$\ldots\ldots\ldots\ldots\ldots.. \qquad \ldots\ldots\ldots\ldots\ldots\ldots.$$

$$v_n = f^*(v_n, q_n) \qquad q_n = g^*(v_{n+1}, q_n)$$

The sequence $\{v_n, q_n\}$ of approximation to the root will converge to the actual value (v_0, q_0) of the root provided, (v_0, q_0) lies in the neighborhood of the actual root. For the initial value $c = c_0 = (v_0, q_0)$, the roots α_i $(i = 1, 2, 3)$ are computed from the Eq. (9) by using Secant method for each value of the wave number k for assigned frequency. The values of $\alpha_i (i = 1, 2, 3)$ so obtained are then used in the Eq. (28) to obtained the current values of v and q each time which are further used to generate the Eq. (31). This process is terminated as and when the condition $|v_{n+1} - v_n| < \varepsilon$, ε being arbitrary small number to be selected at random to achieve the accuracy level, is satisfied. The procedure is continually repeated for different values of the wave number (k) to obtain the corresponding values of the phase velocity (c) and attenuation coefficient (q).

The dispersion curves of non-dimensional frequency, phase velocity, attenuation and relative frequency shift of a generalized thermoelastic circular plate immersed fluid and in space with wave number are plotted for thermally insulated and isothermal boundaries. The notations used in figures like FM and FSM represent the flexural mode and flexural symmetric mode, respectively.

6.1. Frequency. In Figs.1 and 2 the dispersion of frequency with wave number is studied for both thermally insulated and isothermal boundary of a circular plate immersed in fluid and in space. The first two longitudinal and flexural symmetric modes of vibration are

is non-dispersive and decreases rapidly in the presence of liquid and thermal relaxation times with increasing wave number.

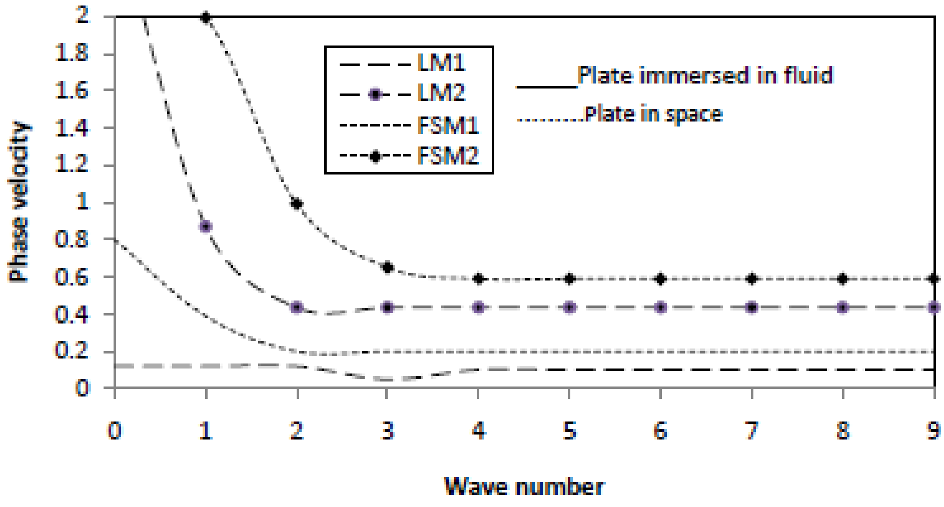

Fig. 3. Variation of non-dimensional phase velocity of thermally insulated circular plate with wave number.

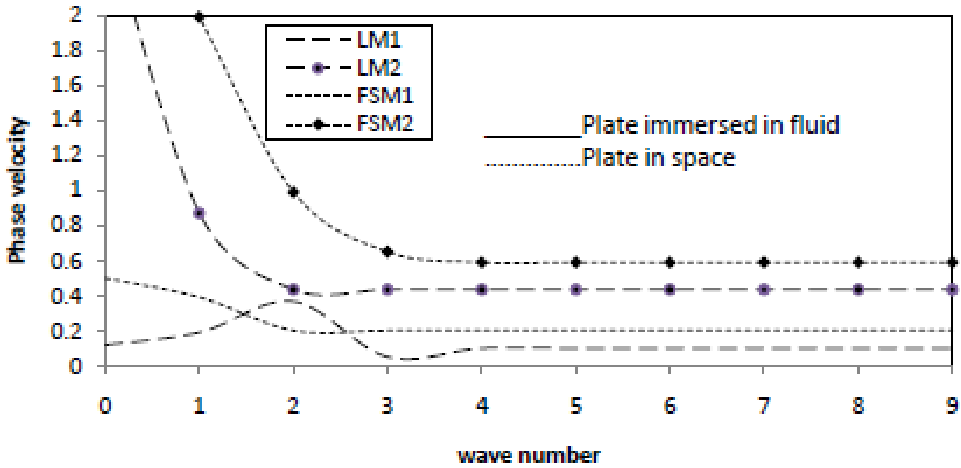

Fig. 4. Variation of non-dimensional phase velocity of isothermal circular plate with wave number.

6.3. Attenuation coefficient. In Fig. 5 the variation of attenuation coefficient with respect to wave number of circular plate with and without fluid is discussed for thermally insulated boundary. The magnitude of the attenuation coefficient increases monotonically to attain maximum value between 0.1 and 0.3 for first two modes of longitudinal and flexural (symmetric) vibration and slashes down to became asymptotically linear in the remaining range of wave number. The variation of attenuation coefficient with respect to wave number of isothermal circular plate is discussed in Fig. 6, here the attenuation coefficient attain maximum value in 0.1 and 0.4 with a small oscillation in the starting wave number and decreases to become linear due relaxation times. From Figs. 5 and 6, it is clear that the effects of stress free thermally insulated and isothermal boundaries of the plate are quite pertinent due to the combine effect of thermal relaxation times and damping effect of fluid medium.

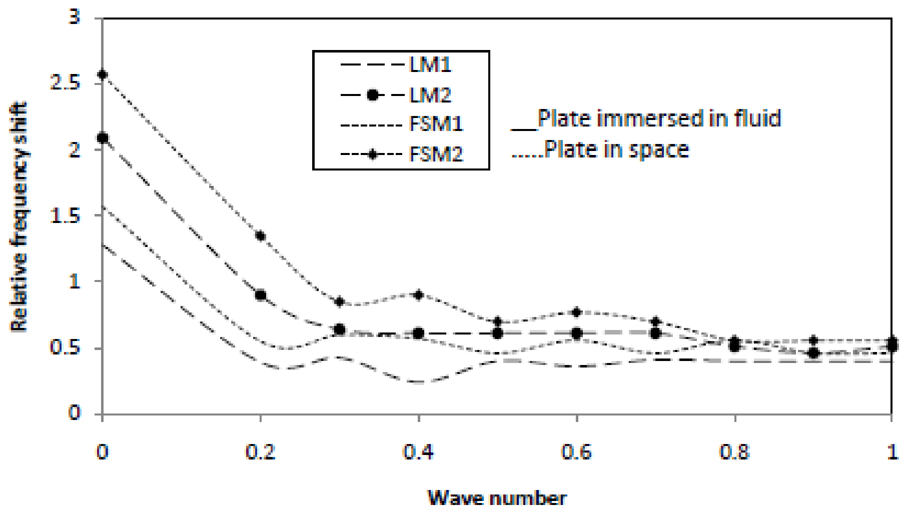

Fig. 7. Variation of relative frequency shift with wave number for thermally insulated circular plate.

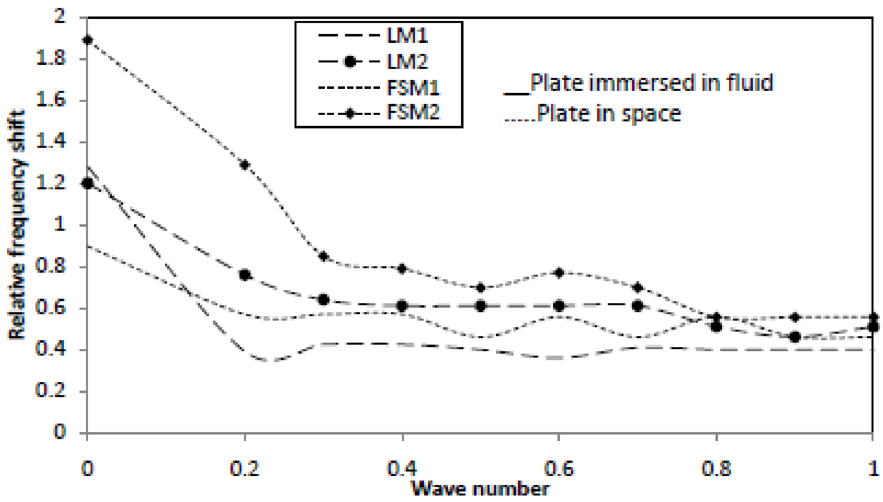

Fig. 8. Variation of relative frequency shift with wave number for isothermal circular plate.

7. Conclusions

The generalized thermoelastic waves in the rotating circular plate immersed in an inviscid fluid are studied based on the Lord-Shulman (LS) and Green-Lindsay (GL) generalized two dimensional theory of thermoelasticity. Two displacement potential functions are introduced to uncouple the equations of motion. The frequency equations that include the interaction between the plate and fluid are obtained by the traction free boundary conditions using the Bessel function solutions. The numerical calculations are carried out for the material Zinc and the computed non-dimensional frequency, phase velocity, attenuation coefficient and relative frequency shift are plotted as the dispersion curves for the plate with thermally insulated and isothermal boundaries. The wave characteristics are found to be more dispersive and realistic in the presence of thermal relaxation times, fluid and the rotation parameter.

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