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GENERALIZED MAGNETO-THERMOELASTICITY FOR AN INFINITE PERFECT CONDUCTING BODY WITH A CYLINDRICAL CAVITY

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Abstract. A model of the equations of generalized magneto-thermoelasticity for a perfect conducting isotropic thermoelastic media developed in [1] is given. This model is applied to solve a problem of an infinite body with a cylindrical cavity is considered in the presence of an axial uniform magnetic field. The boundary of the cavity is subjected to a combination of thermal and mechanical shock acting for a finite period of time. The solution is obtained by a direct approach by using the thermoelastic potential function. Laplace transform techniques are used to derive the solution in the Laplace transform domain. The inversion process is carried out using a numerical method based on Fourier series expansions. Numerical computations for the temperature, the displacement and the stress distributions as well as for the induced magnetic and electric fields are carried out and represented graphically. Comparisons are made with the results predicted by the generalizations, Lord-Shulman theory, and Green-Lindsay theory as well as to the coupled theory.

Nomenclature

ρ density;
t time;
λ,μ Lamé's constants ;
T absolute temperature;
T_0 reference temperature chosen so that $\frac{|T-T_0|}{T_0} << 1$;
u_i components of displacement vector;
σ_{ij} components of stress tensor;
e_{ij} components of strain deviator tensor;
ε_{ij} components of strain tensor;
e $=\varepsilon_{kk}$, dilatation;
δ_{ij} Delta Kronecker;
α_T coefficient of linear thermal expansion;
γ $= (3\lambda+2\mu)\,\alpha_T$;
k thermal conductivity;
C_E specific heat at constant strain;
τ_o, ν relaxation times;

μ_o magnetic permeability;

ε_o electric permeability;

α_o $=\left[\mu_o H_o^2/\rho\right]^{1/2}$, Alfven velocity;

c_1 $=\left[(\lambda+2\mu)/\rho\right]^{1/2}$, speed of propagation of isothermal elastic waves;

c $=1/[\mu_o\, \varepsilon_o]^{1/2}$, speed of light;

η $=\rho\, C_E/k$;

β $=\left[(\lambda+2\mu)/\mu\right]^{1/2}$;

b $=\dfrac{\gamma T_o}{\mu}$;

g $=\dfrac{\gamma}{k\eta}$.

1. Introduction

The classical uncoupled theory of thermoelasticity predicts two phenomena not compatible with physical observations. First, the equation of heat conduction of this theory does not contain any elastic terms; second, the heat equation is of a parabolic type, predicting infinite speeds of propagation for heat waves.

Biot [2] introduced the theory of coupled thermoelasticity to overcome the first shortcoming. The governing equations for this theory are coupled, eliminating the first paradox of the classical theory. However, both theories share the second shortcoming since the heat equation for the coupled theory is also parabolic.

Two generalizations to the coupled theory were introduced. The first is due to Lord and Shulman [3], who obtained a wave-type heat equation by postulating a new law of heat conduction to replace the classical Fourier’s law. Since the heat equation of this theory is of the wave-type, it automatically ensures finite speeds of propagation for heat and elastic waves. The remaining governing equations for this theory, namely, the equations of motion and constitutive relations, remain the same as those for the coupled and the uncoupled theories.

The second generalization to the coupled theory of elasticity is known as the theory of thermoelasticity with two relaxation times or the theory of temperature-rate-dependent thermoelasticity. Müller [4], in review of the thermodynamics of thermoelastic solids, proposed an entropy production inequality, with the help of which the considered restrictions on a class of constitutive equations. Green and Laws [5] proposed a generalization of this inequality. Green and Lindsay obtained an explicit version of the constitutive equations in [6]. These equations were also obtained independently by Şuhubi [7] and Ezzat [8] has obtained the fundamental solution for this theory. This theory contains two constants that act as relaxation times and modify all the equations of the coupled theory, not only the heat equation. The classical Fourier’s law of heat conduction is not violated if the medium under consideration has a center of symmetry.

An increasing attention is being devoted to the interaction between magnetic field and strain field in a thermoelastic solid due to its many applications in the fields of geophysics, plasma physics and related topics. In all papers quoted above it was assumed that the interactions between the two fields take place by means of the Lorentz forces appearing in the equations of motion and by means of a term entering Ohm’s law and describing the electric field produced by the velocity of a material particle, moving in a magnetic field. Usually, in these investigations the heat equation under consideration is taken as the uncoupled or the coupled [2] not the generalized one [3, 6]. This attitude is justified in many situations since the solutions obtained using any of these equations differ little quantitatively. However, when short time effects are considered, the full-generalized system of equations has to be used a great deal of accuracy is lost.

Among the authors who considered the generalized magneto-thermoelasticity equations are Nayfeh and Nasser [9] who studied the propagation of plane waves in a solid under the influence of an electromagnetic field. Choudhuri [10] extend these results to rotating media. Sherief [11] solved a problem for a solid cylinder, while Sherief and Ezzat [12] solved a thermal shock half-space problem using asymptotic expansions. Lately, Sherief and Ezzat [13] solved a problem for an infinitely long annular cylinder, while Ezzat [14-16] and Ezzat et al. [17-22] solved some problems for a perfect conducting media.

In this work we introduced a new model of the equations of generalized thermoelasticity for isotropic perfect conducting media in the presence a constant magnetic field. This model is applied to solve a problem of an infinite perfect conducting isotropic body with cylindrical cavity. The solution is obtained using a direct approach. The resulting formulation together with the Laplace transform technique is applied to a problem considered. The inversion of the Laplace transform is carried out using a numerical technique [23]. The results obtained are represented graphically.

2. Formulation of the problem

We shall consider a thermoelastic medium of perfect conductivity permeated by an initial magnetic field $\mathbf{H_o}$. Due to the effect of this magnetic field there arises in the conducting medium an induced magnetic field **h** and induced electric field **E**. Also, there arises a force **F** (the Lorentz force). Due to the effect of the force, points of the medium undergo a displacement **u**, which gives rise to a temperature. The linearized equations of electromagnetism for slowly moving media [14]

$$\text{Curl}\,\mathbf{h} = \mathbf{J} + \varepsilon_o \frac{\partial \mathbf{E}}{\partial t}, \tag{1}$$

$$\text{Curl}\,\mathbf{E} = -\mu_o \frac{\partial \mathbf{h}}{\partial t}, \tag{2}$$

$$\mathbf{E} = -\mu_o \frac{\partial \mathbf{u}}{\partial t} \wedge \mathbf{H_o}, \tag{3}$$

$$\text{div}\,\mathbf{h} = 0. \tag{4}$$

The above equations are supplemented by the displacement equations of the theory of generalized thermoelasticity, taking into account the external body force, which is here equal to the Lorentz force [15]

$$\rho \frac{\partial^2 u_i}{\partial t^2} = (\lambda + \mu) u_{j,ij} + \mu u_{i,jj} - \gamma (T + \nu \frac{\partial T}{\partial t})_{,i} + \mu_o (J \wedge H_o)_i, \tag{5}$$

and the generalized heat conduction equation [1]:

$$k T_{,ii} = \rho C_E \left(\frac{\partial T}{\partial t} + \tau_o \frac{\partial^2 T}{\partial t^2} \right) + \gamma T_o \left(\frac{\partial u_{i,i}}{\partial t} + n_o \tau_o \frac{\partial^2 u_{i,i}}{\partial t^2} \right) - \left(Q + n_o \tau_o \frac{\partial Q}{\partial t} \right), \tag{6}$$

where n_o is a constant.
The constitutive equation:

$$\sigma_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu e_{ij} - \gamma\left(T - T_o + \nu \frac{\partial T}{\partial t} \right), \tag{7}$$

and strain-displacement relations

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{8}$$

together with the previous equations, constitute a complete system of generalized magneto-thermoelasticity equations for a medium with a perfect electric conductivity. This model can be applied to both generalizations, Lord-Shulman theory ($n_0 = 1,\ \tau_0 > 0,\ \nu = 0$) and Green-Lindsay theory ($n_0 = 0,\ \tau_0 > 0,\ \nu > 0$), as well as to the coupled theory ($\tau_o = \nu = 0$).

Let (r, ψ, z) be cylindrical coordinates with the z-axis coinciding with the axis of a solid infinitely long elastic solid circular cylinder of a homogeneous, isotropic material with a perfect electric conductivity. Because of the cylindrical symmetry of the problem, and if there is no z-dependence of the field variables, all the considered functions will be functions of r and t only. The components of the displacement vector will be taken of the form

$u_r = u,\ u_\psi = u_z = 0.$

The strain tensor components are thus given by

$$e_{rr} = \frac{\partial u}{\partial r},\ e_{\psi\psi} = \frac{u}{r},\ e_{zz} = e_{rz} = e_{r\psi} = e_{\psi z} = 0. \tag{9}$$

It follows that the cubical dilatation e is of the form

$$e = \frac{\partial u}{\partial r} + \frac{u}{r} = \frac{1}{r}\frac{\partial ru}{\partial r}. \tag{10}$$

From Eq. (7) we obtain the components of the stress tensor as

$$\sigma_{rr} = (\lambda + 2\mu)\frac{\partial u}{\partial r} + \lambda \frac{u}{r} - \gamma\,(T - T_o + \nu \frac{\partial T}{\partial t}), \tag{11}$$

$$\sigma_{\Psi\Psi} = \lambda \frac{\partial u}{\partial r} + (\lambda + 2\mu)\frac{u}{r} - \gamma\,(T - T_o + \nu \frac{\partial T}{\partial t}), \tag{12}$$

$$\sigma_{zz} = \sigma_{rz} = \sigma_{z\Psi} = \sigma_{\Psi r} = 0. \tag{13}$$

Assume now that the initial magnetic field $\mathbf{H_o}$ acts in the direction of the z-axis and has the components (0, 0, H_o). The induced magnetic field **h** will have one component h in the z-direction, while the induced electric field **E** will have one component E in the ψ-direction.
Then, Eqs. (1)- (3) yield

$$J = -\left(\frac{\partial h}{\partial r} + \varepsilon_o \mu_o H_o \frac{\partial^2 u}{\partial t^2} \right), \tag{14}$$

$$h = -H_o e, \tag{15}$$

$$E = \mu_o H_o \frac{\partial u}{\partial t}. \tag{16}$$

Expressing the components of the vector **J** in terms of displacement, by eliminating from Eq. (14) the quantities h and E and introducing them into the displacement Eq. (5), we arrived at

$$\rho\,(1+\frac{\alpha_o^2}{c^2})\frac{\partial^2 u}{\partial t^2} = \rho(c_1^2+\alpha_o^2)\frac{\partial e}{\partial r} - \gamma\frac{\partial}{\partial r}\left(T+\nu\frac{\partial T}{\partial t}\right), \tag{17}$$

Equation (6) is to be supplemented by the constitutive Eqs. (10) and the heat conduction equation

$$k\nabla^2 T = \rho C_E\left(\frac{\partial}{\partial t}+\tau_o\frac{\partial^2}{\partial t^2}\right)T + \gamma T_o\left(\frac{\partial}{\partial t}+n_o\tau_o\frac{\partial^2}{\partial t^2}\right)e - \left(1+n_o\tau_o\frac{\partial}{\partial t}\right)Q, \tag{18}$$

where ∇^2 is Laplace’s operator in cylindrical coordinates, given by

$$\nabla^2 = \frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \Psi^2}+\frac{\partial^2}{\partial z^2}.$$

In case of dependence on r only, this reduces to

$$\nabla^2 = \frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}.$$

We shall use the following non-dimensional variables

$$r' = c_1\eta r,\quad u' = c_1\eta r,\quad t' = c_1^2\eta t,\quad \nu' = c_1^2\eta\nu,\quad \tau_o' = c_1^2\eta\tau_o,\quad \sigma_{ij}' = \sigma_{ij}/\mu,\quad e' = e,$$

$$\theta = (T-T_o)/T_o,\quad Q' = Q/kT_o\eta^2 c_1^2,\quad h' = \frac{h}{H_o},\quad E' = \frac{E}{\mu_o H_o c_1},\quad J' = \frac{J}{H_o c_1\eta}.$$

Equations (11)-(18) take the following form (dropping the primes for convenience)

$$J = -\left(\frac{\partial h}{\partial r}+\frac{c_1^2}{c^2}\frac{\partial^2 u}{\partial t^2}\right), \tag{19}$$

$$h = -\,e, \tag{20}$$

$$E = \frac{\partial u}{\partial t}, \tag{21}$$

$$\alpha_1\frac{\partial^2 u}{\partial t^2} = \alpha_2\frac{\partial e}{\partial r} - b\frac{\partial}{\partial r}\left(\theta+\nu\frac{\partial\theta}{\partial t}\right), \tag{22}$$

$$\nabla^2\theta = \left(\frac{\partial}{\partial t}+\tau_o\frac{\partial^2}{\partial t^2}\right)\theta + g\left(\frac{\partial}{\partial t}+n_o\tau_o\frac{\partial^2}{\partial t^2}\right)e - \left(1+n_o\tau_o\frac{\partial}{\partial t}\right)Q, \tag{23}$$

$$\sigma_{rr} = \beta^2 \frac{\partial u}{\partial r} + (\beta^2 - 2)\frac{u}{r} - b\,(\theta + \nu \frac{\partial \theta}{\partial t}), \tag{24}$$

$$\sigma_{\psi\psi} = (\beta^2 - 2)\frac{\partial u}{\partial r} + \beta^2 \frac{u}{r} - b\,(\theta + \nu \frac{\partial \theta}{\partial t}), \tag{25}$$

$$\sigma_{zz} = \sigma_{rz} = \sigma_{z\Psi} = \sigma_{\Psi r} = 0, \tag{26}$$

where $\alpha_1 = \beta^2 \left(1 + \frac{\alpha_o^2}{c^2}\right)$, $\alpha_2 = \beta^2 \left(1 + \frac{\alpha_o^2}{c_1^2}\right)$.

3. Solution in the Laplace transform domain

Introducing the thermoelastic potential function Φ defined by the relation:

$$u = \frac{\partial \Phi(r,t)}{\partial r}. \tag{27}$$

Equations (19)-(25) give

$$J = \frac{\partial}{\partial r}\left(\nabla^2 \Phi - V^2 \frac{\partial^2 \Phi}{\partial t^2}\right), \tag{28}$$

$$h = -\nabla^2 \Phi, \tag{29}$$

$$E = \frac{\partial^2 \Phi}{\partial r\, \partial t}, \tag{30}$$

$$\alpha \frac{\partial^2 \Phi}{\partial t^2} = \nabla^2 \Phi - a\left(\theta + \nu \frac{\partial \theta}{\partial t}\right), \tag{31}$$

$$\nabla^2 \theta = \left(\frac{\partial}{\partial t} + \tau_o \frac{\partial^2}{\partial t^2}\right)\theta + g\, \nabla^2 \left(\frac{\partial}{\partial t} + n_o \tau_o \frac{\partial^2}{\partial t^2}\right)\Phi - \left(1 + n_o \tau_o \frac{\partial}{\partial t}\right) Q, \tag{32}$$

$$\sigma_{rr} = \beta^2 \nabla^2 \phi - \frac{2}{r}\frac{\partial \phi}{\partial r} - b\,(\theta + \nu \frac{\partial \theta}{\partial t}), \tag{33}$$

$$\sigma_{\psi\psi} = (\beta^2 - 2)\nabla^2 \phi + \frac{2}{r}\frac{\partial \phi}{\partial r} - b\,(\theta + \nu \frac{\partial \theta}{\partial t}), \tag{34}$$

where $\alpha = \frac{\alpha_1}{\alpha_2}$, $V = \frac{c_1}{c}$ and $a = \frac{b}{\beta^2}$.

From now we shall consider the medium without any heat sources.

Taking the Laplace transform defined by the relation

$$\bar{f}(p) = \int_0^\infty e^{-pt} f(t)\, dt, \tag{35}$$

of both sides of Eqs. (29)-(35)

$$\bar{J} = \frac{\partial}{\partial r}\left(\nabla^2 \overline{\Phi} - p^2 V^2 \overline{\Phi}\right), \tag{36}$$

$$\bar{h} = -\nabla^2 \overline{\Phi}, \tag{37}$$

$$\bar{E} = p \frac{\partial \overline{\Phi}}{\partial r}, \tag{38}$$

$$(\nabla^2 - \alpha p^2)\overline{\Phi} = a(1 + \nu p)\bar{\theta}, \tag{39}$$

$$\nabla^2 \bar{\theta} = p(1 + \tau_o p)\bar{\theta} + g p(1 + n_o \tau_o p)\nabla^2 \overline{\Phi}, \tag{40}$$

$$\bar{\sigma}_{rr} = \beta^2 \nabla^2 \bar{\phi} - \frac{2}{r}\frac{\partial \bar{\phi}}{\partial r} - b(1 + \nu p)\bar{\theta}, \tag{41}$$

$$\bar{\sigma}_{\psi\psi} = (\beta^2 - 2)\nabla^2 \bar{\phi} + \frac{2}{r}\frac{\partial \bar{\phi}}{\partial r} - b(1 + \nu p)\theta, \tag{42}$$

Eliminating $\bar{\theta}$ between Eqs (35) and (36), we obtain the following fourth-order partial differential equation satisfied by $\overline{\Phi}$

$$\left\{\nabla^4 - [p(1+\varepsilon) + p^2(\alpha + \tau_o + \varepsilon\,\nu + \varepsilon\, n_o \tau_o (1 + \nu p))]\nabla^2 + \alpha p^2 (p + \tau_o p^2)\right\}\overline{\Phi} = 0 \, , \tag{43a}$$

where $\varepsilon = ga$.

This equation can be written in the form

$$(\nabla^2 - k_1^2)(\nabla^2 - k_2^2)\overline{\Phi} = 0, \tag{43b}$$

where k_1, k_2 are the positive roots of the characteristic equation

$$k^4 - p[1 + \varepsilon + p(\alpha + \tau_o + \varepsilon\,\nu + \varepsilon\, n_o \tau_o (1 + \nu p)]k^2 + \alpha p^3 (1 + \tau_o p) = 0. \tag{44}$$

The solution of the equation is given by

$$\bar{\theta} = \sum_{i=1}^{2} A_i K_o(k_i r), \tag{45}$$

$$\overline{\Phi} = \sum_{i=1}^{2} B_i K_o(k_i r) \tag{46}$$

where $K_0(*)$ is the modified Bessel function of the second kind of order zero

from equation (39), we can get the parameters B_i

$$B_i = \frac{a(1 + \upsilon p)}{k_i^2 - \alpha p^2} A_i \quad i = 1,2 \tag{47}$$

Finally, we get the following equations

$$\bar{\theta} = \sum_{i=1}^{2} A_i K_o(k_i r), \tag{48}$$

$$\bar{u} = -a(1+\upsilon p)\sum_{i=1}^{2}\frac{k_i}{k_i^2-\alpha p^2}A_i K_1(k_i r), \tag{49}$$

$$\bar{h} = -a(1+\upsilon p)\sum_{i=1}^{2}\frac{k_i^2}{k_i^2-\alpha p^2}A_i K_o(k_i r), \tag{50}$$

$$\bar{E} = -a p(1+\upsilon p)\sum_{i=1}^{2}\frac{k_i}{k_i^2-\alpha p^2}A_i K_1(k_i r), \tag{51}$$

$$\bar{\sigma}_{rr} = a(1+\upsilon p)\sum_{i=1}^{2}\frac{A_i}{k_i^2-\alpha p^2}\left[\alpha\beta^2 p^2 K_o(k_i r)+\frac{2k_i}{r}K_1(k_i r)\right], \tag{52}$$

$$\bar{\sigma}_{\psi\psi} = a(1+\upsilon p)\sum_{i=1}^{2}\frac{A_i}{k_i^2-\alpha p^2}\left[(\alpha\beta^2 p^2-2)K_o(k_i r)-\frac{2k_i}{r}K_1(k_i r)\right], \tag{53}$$

where $K_1(*)$ is the modified Bessel function of the second kind of order one.

It is now possible to solve a board class of problems of generalized magneto-thermoelasticity on perfectly conducting medium. Furthermore, it should be noted that the corresponding expressions in the absence of magnetic field can be deduced by sitting $\alpha_o = 0$.

4. Application

Thermo-mechanical shock problem under three theories

We will consider a thermal shock on the boundary r = R, i. e. $\theta(R,t) = \theta_o H(t)$ or, we have

$$\bar{\theta}(R,p) = \frac{\theta_o}{p}. \tag{54}$$

We will also consider a mechanical shock on the boundary r = R, i. e. $\sigma(R,t) = \sigma_o H(t)$ or, we have

$$\bar{\sigma}(R,p) = \frac{\sigma_o}{p}. \tag{55}$$

Using the above conditions, we get the following system of linear equations where the unknowns are the parameters A_i

$$\sum_{i=1}^{2} A_i K_o(k_i R) = \frac{\theta_o}{p}, \tag{56}$$

$$\sum_{i=1}^{2}\frac{A_i}{k_i^2-\alpha p^2}\left[\alpha\beta^2 p^2 K_o(k_i R)+\frac{2k_i}{R}K_1(k_i R)\right] = \frac{\sigma_o}{p}. \tag{57}$$

By solving the above equations we get

$$A_1 = \frac{1}{\omega}[a_{11}\sigma_o + a_{12}\theta_o], \quad A_2 = \frac{1}{\omega}[a_{21}\sigma_o + a_{22}\theta_o],$$

where

$$a_{11} = -K_0(k_2R)(k_2^2 - \alpha p^2)(k_1^2 - \alpha p^2)p,$$

$$a_{12} = (k_1^2 - \alpha p^2)(\alpha\beta^2 p^3 K_0(k_2R) + 2K_1(k_2R)k_2),$$

$$a_{21} = K_0(k_1R)(k_1^2 - \alpha p^2)(k_2^2 - \alpha p^2)p,$$

$$a_{22} = -(k_2^2 - \alpha p^2)(\alpha\beta^2 p^3 K_0(k_1R) + 2K_1(k_1R)k_1),$$

$$\omega = \alpha\beta^2 K_0(k_1R)K_0(k_2R)p^4(k_1^2 - k_2^2) + 2K_0(k_1R)K_1(k_2R)k_2p(k_1^2 - \alpha p^2)$$

$$-2K_0(k_2R)K_1(k_1R)k_1p(k_2^2 - \alpha p^2).$$

Finally, we have the solution in the Laplace transform domain in the following forms

$$\bar{\theta} = \frac{1}{\omega}\sum_{i=1}^{2}[a_{i1}\sigma_o + a_{i2}\theta_o]K_o(k_i r), \tag{58}$$

$$\bar{u} = \frac{-a(1+\upsilon p)}{\omega}\sum_{i=1}^{2}\frac{k_i[a_{i1}\sigma_o + a_{i2}\theta_o]}{k_i^2 - \alpha p^2}K_1(k_i r), \tag{59}$$

$$\bar{h} = -\frac{a(1+\upsilon p)}{\omega}\sum_{i=1}^{2}\frac{k_i^2[a_{i1}\sigma_o + a_{i2}\theta_o]}{k_i^2 - \alpha p^2}K_o(k_i r), \tag{60}$$

$$\bar{E} = \frac{-ap(1+\upsilon p)}{\omega}\sum_{i=1}^{2}\frac{k_i[a_{i1}\sigma_o + a_{i2}\theta_o]}{k_i^2 - \alpha p^2}K_1(k_i r), \tag{61}$$

$$\bar{\sigma}_{rr} = \frac{a(1+\upsilon p)}{\omega}\sum_{i=1}^{2}\frac{[a_{i1}\sigma_o + a_{i2}\theta_o]}{k_i^2 - \alpha p^2}\left[\alpha\beta^2p^2K_o(k_i r) + \frac{2k_i}{r}K_1(k_i r)\right], \tag{62}$$

$$\bar{\sigma}_{\psi\psi} = \frac{a(1+\upsilon p)}{\omega}\sum_{i=1}^{2}\frac{[a_{i1}\sigma_o + a_{i2}\theta_o]}{k_i^2 - \alpha p^2}\left[(\alpha\beta^2p^2 - 2)K_o(k_i r) - \frac{2k_i}{r}K_1(k_i r)\right], \tag{63}$$

5. Inversion of the Laplace transform

We shall now outline the method used to invert the Laplace transform in the above equations.

Let $\bar{f}(s)$ be the Laplace transform of a function f(t). The inversion formula for Laplace transform can be written as

$$f(t) = \frac{1}{2\pi i}\int_{d-i\infty}^{d+i\infty} e^{st}\,\bar{f}(s)\,ds,$$

where d is an arbitrary real number greater than all the real parts of the singularities of $\bar{f}(s)$. Taking s = d + i y , the above integral takes the form

$$f(t) = \frac{e^{dt}}{2\pi}\int_{-\infty}^{\infty} e^{ity}\,\bar{f}(d + iy)dy.$$

Expanding the function h(t) = exp(-dt) f(t) in a Fourier series in the interval [0, 2L], we obtain the approximate formula [23]:

$f(t) = f_\infty(t) + ED$,

where

$$f_\infty(t) = \tfrac{1}{2} c_0 + \sum_{k=1}^{\infty} c_k, \quad \text{for } 0 \le t \le 2L, \tag{64}$$

and

$$c_k = \frac{e^{dt}}{L} \operatorname{Re}\left[e^{ik\pi t/L} \bar{f}(d + ik\pi/L)\right], \tag{65}$$

ED, the discretization error, can be made arbitrarily small by choosing d large enough [20].
As the infinite series in (64) can only be summed up to a finite number N of terms, the approximate value of f(t) becomes

$$f_N(t) = \tfrac{1}{2} c_0 + \sum_{k=1}^{N} c_k, \quad \text{for } 0 \le t \le 2L. \tag{66}$$

Using the above formula to evaluate f(t), we introduce a truncation error ET that must be added to the discretization error to produce the total approximation error.
Two methods are used to reduce the total error. First, the 'Korrecktur' method is used to reduce the discretization error. Next, the ε-algorithm is used to reduce the truncation error and therefore to accelerate convergence.
The Korrecktur-method uses the following formula to evaluate the function f(t)

$f(t) = f_\infty(t) - e^{-2dL} f_\infty(2L+t) + E'D$,

where the discretization error $|E'D| << |ED|$ [17]. Thus, the approximate value of f(t) becomes

$$f_N K(t) = f_N(t) - e^{-2dL} f_{N'}(2L+t)\ . \tag{67}$$

N' is an integer such that N' < N.
We shall now describe the ε-algorithm that is used to accelerate the convergence of the series in Eq. (66). Let N be an odd natural number and let

$$s_m = \sum_{k=1}^{m} c_k,$$

be the sequence of partial sums of (66). We define the ε-sequence by

$$\varepsilon_{0,m} = 0, \varepsilon_{1,m} = s_m, \quad m = 1, 2, 3, \ldots$$

and

$$\varepsilon_{n+1,m} = \varepsilon_{n-1,m+1} + \frac{1}{\varepsilon_{n,m+1} - \varepsilon_{n,m}}, \quad n, m = 1, 2, 3, \ldots$$

It can be shown [17] that the sequence

$\varepsilon_{1,1}, \ \varepsilon_{3,1}, \ldots, \ \varepsilon_{N,1}$

converges to $f(t) + ED - c_0/2$ faster than the sequence of partial sums

s_m , m =1,2,3, ...

All constants are given in SI units.
The actual procedure used to invert the Laplace transforms consists of using equation (67) together with the ε-algorithm. The values of d and L are chosen according the criteria outlined in [23].

6. Numerical results

The copper material was chosen for purposes of numerical evaluations. The computations were carried out for one value of time, namely for t=0.15 and we have considered $\sigma_o = \theta_o = 1$. The Figures 1, 2 and 3 represent the solutions for thermo-mechanical problem while, Figs 4, 5 and 6 represent the solution for thermal shock problem. The temperature distributions are shown in Figs. 1 and 4, the displacement distributions are shown in Figs. 2 and 5 and the radial stress components distributions are shown in Figs. 3 and 6. In all these figures dotted lines represent the values predicted by the coupled theory, solid lines represent the values predicted by the L-S theory and dashed lines represent the values predicted by the G-L theory. The constants of the problem are given in the Table 1.

Table 1. Values of the constants.

k= 386 N/K sec	α_T= 1.78 10^{-5} K^{-1}	C_E= 383.1 m^2/K sec^2	η= 8886.73 m/sec^2
μ= 3.86 10^{10} N/m^2	λ= 7.76 10^{10} N/m^2	c_1= 4.158 10^3 m/sec	ρ= 8954 kg/m^3
ε_o= $10^{-9}/(36\pi)$ C^2/N m^2	μ_o= 4π 10^{-7} N m sec^2 C^{-2}	H_o= 1 C/m sec	τ_o=0.02
υ= 0.03	b= 0.042	g= 1.61	β^2= 4
V= 1.39 10^{-5}	T_o= 293 K	α_1= 1.005	α_2= 1.001

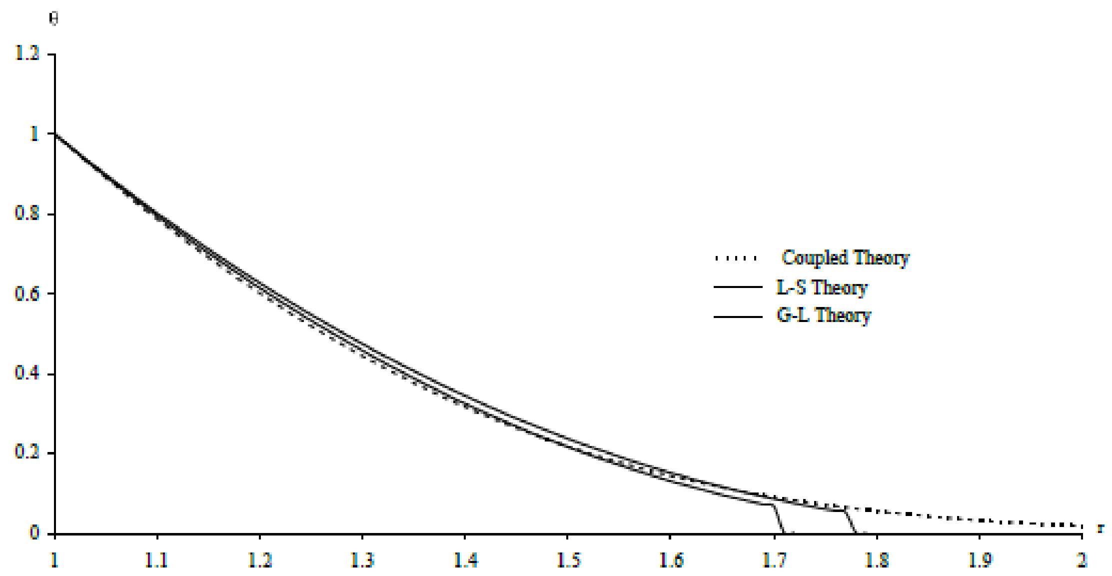

Fig. 1. Temperature distribution due to thermo-mechanical shock.

It is clear from all figures that the results for generalized thermoelasticity are distinctly different from those of coupled thermoelasticity. The solution of any of the considered function for the generalized theories vanishes identically outside a bounded region of space. This demonstrates clearly the difference between the coupled and the generalized theories of thermoelasticity. In the first and older theory the waves propagate with infinite speeds, so the value of any function is not identically zero (though it may be very small) for any large value of r. In the generalized theories the response to the thermal and mechanical effects does not reach infinity instantaneously but remains in a bounded region of the surface.

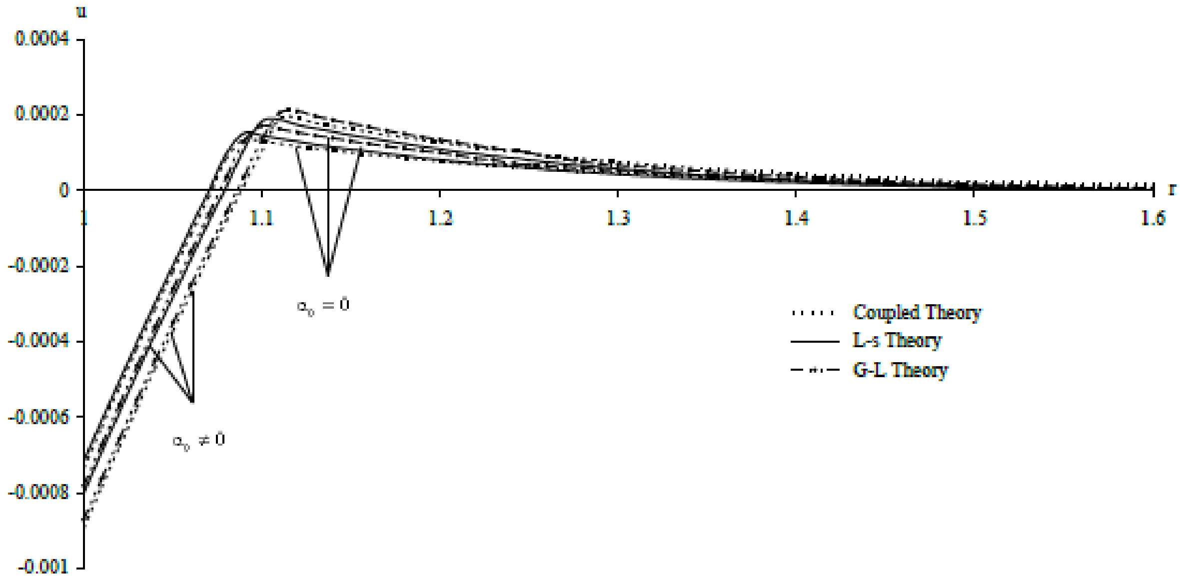

Fig. 2. Displacement distribution due to thermo-mechanical shock.

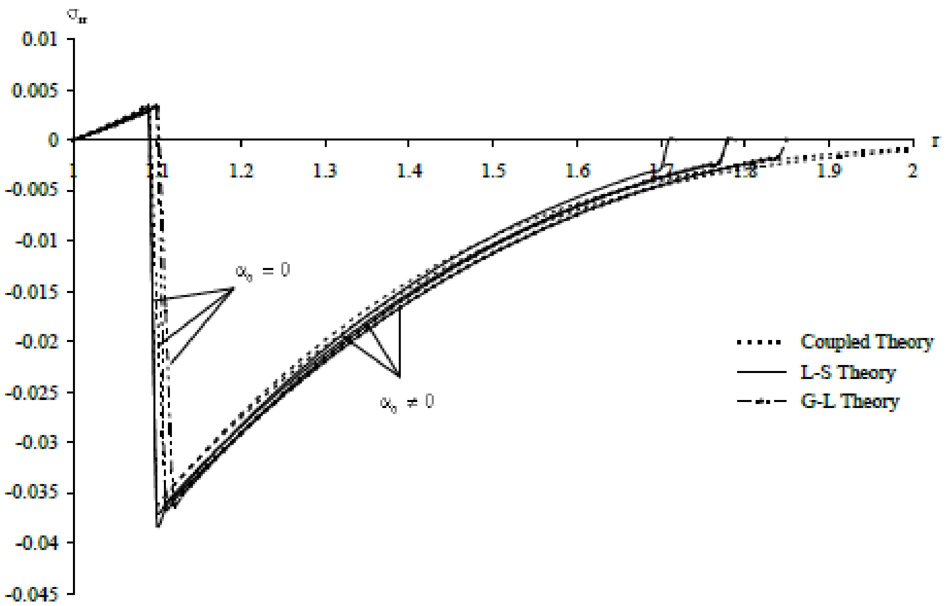

Fig. 3. Radial stress distribution due to thermo-mechanical shock.

The presence of the magnetic field which acts to the perfect conducting elastic medium raises the velocity of the dilatational elastic waves from c_1 to $c_o = \left(c_1^2 + \alpha_o^2\right)^{1/2}$, the modified electromagnetic elastic wave is propagated with velocity c_o, and that is, with the same velocity as the modified elastic wave that produces a jump in stress.

It is seen from Figs. 3 and 6 that the magnetic field acts to decrease the magnitude of thermal stress. This is usually known as the magnetic damping.

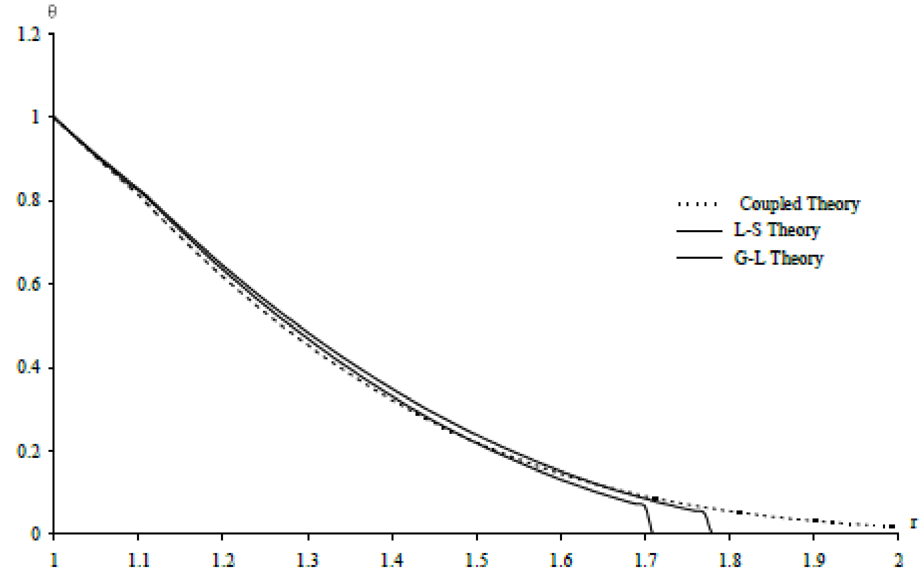

Fig. 4. Temperature distribution due to thermal shock.

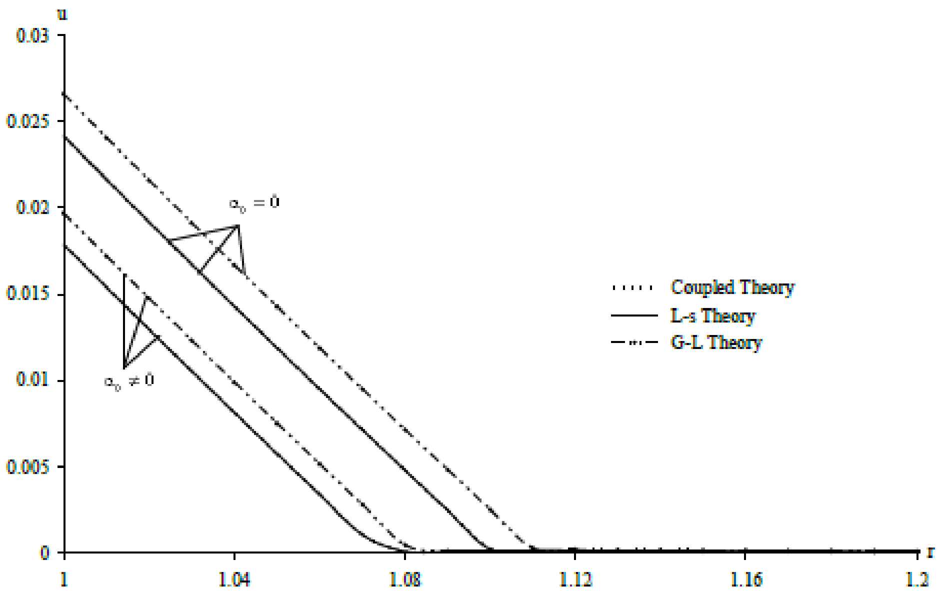

Fig. 5. Displacement distribution due to thermal shock.

7. Appendix

For the thermal shock problem, we can get the solution by substitution $\sigma_o = 0$ in the equations (58)-(63), we get

$$\bar{\theta} = \frac{\theta_o}{\omega} \sum_{i=1}^{2} a_{i2} K_o(k_i\, r)\,, \tag{68}$$

$$\bar{u} = \frac{-a\theta_o (1+\upsilon p)}{\omega} \sum_{i=1}^{2} \frac{k_i\, a_{i2}}{k_i^2 - \alpha\, p^2} K_1(k_i\, r), \tag{69}$$

$$\bar{h} = -\frac{a\,\theta_o (1+\upsilon p)}{\omega} \sum_{i=1}^{2} \frac{k_i^2 a_{i2}}{k_i^2 - \alpha\, p^2} K_o(k_i\, r), \tag{70}$$

$$\bar{E} = \frac{-a\,\theta_o\,p(1+\upsilon p)}{\omega}\sum_{i=1}^{2}\frac{k_i a_{i2}}{k_i^2-\alpha p^2}K_1(k_i\,r),\tag{71}$$

$$\bar{\sigma}_{rr} = \frac{a\,\theta_o\,(1+\upsilon p)}{\omega}\sum_{i=1}^{2}\frac{a_{i2}}{k_i^2-\alpha p^2}\left[\alpha\beta^2 p^2 K_o(k_i\,r)+\frac{2k_i}{r}K_1(k_i\,r)\right],\tag{72}$$

$$\bar{\sigma}_{\psi\psi} = \frac{a\,\theta_o(1+\upsilon p)}{\omega}\sum_{i=1}^{2}\frac{a_{i2}}{k_i^2-\alpha p^2}\left[(\alpha\beta^2 p^2-2)K_o(k_i\,r)-\frac{2k_i}{r}K_1(k_i\,r)\right],\tag{73}$$

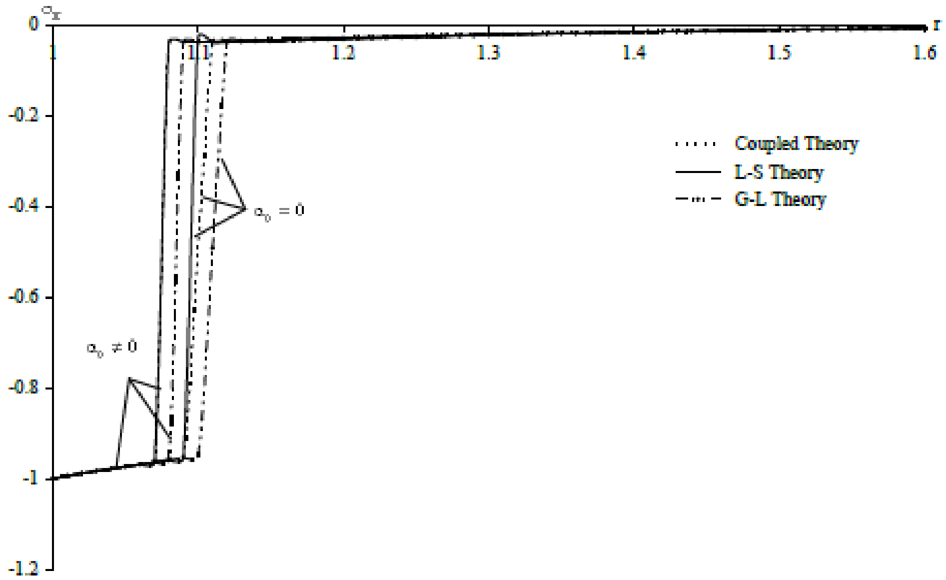

Fig. 6. Radial stress distribution due to thermal shock.

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