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QUANTUM COMPUTING IN SEMICONDUCTOR STRUCTURES WITH 0.1 μm SEPARATION OF NUCLEAR-SPIN QUBITS

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Abstract. Nuclear-spin qubit interactions in a heterostructure can be mediated via the bound outer electrons of impurity atoms whose nuclear spins 1/2 are the qubits. These outer electrons, in turn, interact via the two-dimensional electron gas in the quantum Hall effect regime. We devised a quantum computing scheme, based on this mechanism, with qubit separation of order 0.1 μm, attainable with the present-day semiconductor device technologies, and retaining all the gate-control and measurement aspects of earlier nuclear-spin quantum computing proposals.

1. INTRODUCTION

The general schematic of a semiconductor-heterostructure quantum computer is shown in Fig. 1. Spin or quantum-dot qubits are positioned with precision of several nanometers in a heterostructure. One must design how to effect and control single-qubit interactions, two-qubit interactions, and explore how this controlled dynamics compares to decoherence and relaxation. The proposal must include ideas for implementation of initialization, read-out, and gate functions.

Here, we outline results for schemes of quantum computing with nuclear spins as qubits, with their pair-wise coupling mediated by the two-dimensional electron gas in the integer quantum Hall regime [1-3]. In strong magnetic fields, the spatial states of the electrons confined in the two-dimensional layer in which the qubits are placed, are quantized by the field to have properties similar to free-space Landau levels. The lattice potential and the impurities cause formation of narrow bands instead of the sharp levels, separated by localized states. As a result, for ranges of magnetic field, the localized states fill up while the extended states resemble completely filled integer number of Landau levels. These states are further Zeeman split owing to the electron spin. At low temperatures, one can find ranges of field values such that only one Zeeman sublevel is completely filled.

The electronic state in such systems, that show the quantum Hall effect in conductivity, is highly correlated and nondissipative. If nuclear spins are used as qubits, i.e., atoms with nuclear spin 1/2 are sparsely positioned in the zero-nuclear spin host, such as the zero-nuclear-spin isotope 28 of Si, which constitutes 92% of natural silicone, then their zero-temperature relaxation will be slowed down, $T_1 = O(10^3)$ sec [1].

Localized spins, nuclear and electronic, interact by exchanges of spin excitons – spin waves consisting of a superposition of bound electron-hole pairs. The spectrum of these excitations [4,5], observed experimentally in [6], has a gap at zero momentum, corresponding to the Zeeman splitting. This gap is the cause of slow relaxation and decoherence. The exchange of virtual spin excitons mediates the qubit-qubit interaction and also, via scattering of virtual excitons by impurity potentials, relaxation and decoherence of qubits.

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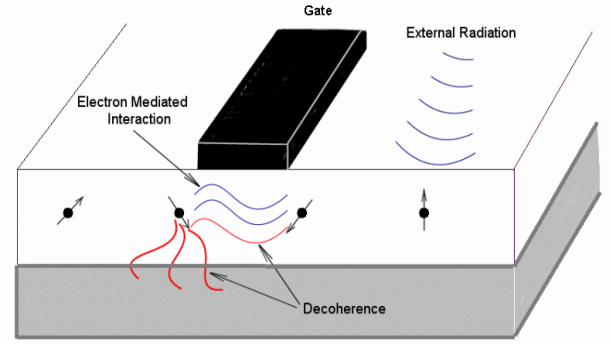

Fig. 1. Schematic of a semiconductor heterostructure quantum information processor. The qubits are spins 1/2 of nuclei or localized electrons. Individual control of the temporal evolution of these spins can be achieved with the use of external electromagnetic radiation, i.e., NMR or ESR pulses. The spins are also coupled with each other via interaction mediated by the two-dimensional electron gas in the heterostructure, or by other means. The external and internal interactions can be controlled by electrostatic gates. The external environment, that includes crystal lattice, electron gas, defects, impurity potentials, causes relaxation and decoherence of the qubits.

The original proposal to use nuclear spin qubits directly coupled by the two-dimensional electron gas [3], required positioning the qubits at distances comparable to few magnetic lengths. The latter is of order 10 nm for magnetic fields of several Tesla. The qubit-qubit interaction decays exponentially on this length scale. Recently, we proposed a new improved model [2] in which the qubit interactions are mediated by coupling of the two-dimensional electron gas to the outer impurity electrons. This applies if the atoms, whose nuclear spins are the qubits, are single-electron donors such as the isotope 31 of P. These phosphorous impurities were originally utilized in the proposal by Kane [7] where they must be positioned at separations of about 4 nm for the wavefunctions of the outer electrons, which are bound at low temperatures, to overlap significantly.

2. THE NEW IMPROVED MODEL

In our new scheme [2], the nuclear spins are coupled to the outer bound electrons which, in turn, interact via the two-dimensional electron gas. This interaction is stronger and/or of a much longer range than the couplings utilized in earlier approaches [1,3,7]: the qubit separation can be of order 100 nm. Another advantage is that gate control of the individual qubits and of qubit-qubit interactions is possible, as in [7]. We have carried out extensive perturbative many-body calculations [1-3,8] allowing estimation of T_{int} and T_2, defined in Table 1, for both the original two-dimensional-electron-gas-involving quantum-computing proposal [3] and its new, improved version [2], where the main improvement is in the possibility of the gate control along the lines of [7]. The “clock speed" of the improved model is also faster by about two orders of magnitude.

The results are summarized in Table 1. We show estimates of all four relevant time scales for the two models introduced earlier. The "original" model [3] corresponds to nuclear spins 1/2, used at qubits, of atoms without an outer loosely bound electron. The "improved" model corresponds to the case when the outer electron is present and its interaction with the nuclear spin and the two-dimensional electron gas dominates the dynamics.

The data shown in Table 1 were obtained assuming typical parameters for the standard heterojunctions utilized in quantum-Hall-effect experiments today, and qubit separation of 65 nm. Thus, the parameter values taken [2,3] were more appropriate for the GaAs system than for Si, even though the main isotopes of gallium and arsenic

Table 1. Time scales of the qubit dynamics for the original [3] and improved [2] versions of the nuclear spin quantum computer with interactions mediated by the two-dimensional electron gas.

	The original model	*The improved model*	*Definition of the time scale*
T_{ext}	O(10^{-5}) sec	O(10^{-5}) sec	Single-qubit external NMR-radiation control time
T_{int}	O(1) sec	O(10^{-2}) sec	Time scale defined by the two-qubit interactions
T_1	O(10^{3}) sec	O(10) sec	Time scale associated with energy relaxation
T_2	O(10) sec	O(10^{-1}) sec	Intrinsic quantum-mechanical decoherence time

have nuclear spin 3/2 and cannot serve as spin-zero hosts. The reason for using these values has been that experimental verification of some of the numbers might be possible in the available materials before cleaner and different composition materials needed for quantum computing are produced.

Our estimates, see Table 1, indicate that the desired [9] quality factor $Q = T_{int}/T_2 = 10^{-5}$ is not obtained for the present system. Actually, no solid-state quantum computing proposal to date, satisfies the 10^{-5} quality-factor criterion. The values range from 10^{-1} to 10^{-2}. The resolution could come from development of better error-correction algorithms or from improving the physical system.

In our estimation of the decoherence time scale, we used parameters typical of a standard, "dirty" heterostructure with large spatial fluctuations of the impurity potential. These heterostructures have been suitable for standard experiments because they provide wider quantum-Hall plateaus, *i.e.*, ranges of magnetic field for which all the extended states of a Zeeman sublevel are filled. Much cleaner, ultra-high mobility structures can be obtained by placing the ionized impurity layer at a larger distance from the two-dimensional gas or by injecting conduction electrons into the heterostructure by other means. Thus, our quantum-computing proposals [2,3] are unique not only in the large qubit separation allowed but also in that there is a clear direction of exploration to allow physical, rather than algorithmic, resolution of the quality factor problem. This possibility should be further explored both experimentally and theoretically.

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