Mater.Phys.Mech. 4 (2001) 51-55

ANALYSIS OF SURFACE EFFECTS ON MECHANICAL PROPERTIES OF MICROCANTILEVERS

P. Lu[1], F. Shen[1], S.J. O'Shea[2], K. H. Lee[1] and T. Y. Ng[1]

[1]Institute of High Performance Computing, 89C Science Park Drive, #02-11/12, The Rutherford, Singapore 118261
[2]Institute of Materials Research & Engineering, 3 Research Link, Singapore 117602

Received: June 17, 2001

Abstract. In this paper, some problems concerning surface effects of microcantilever-based sensors are discussed. The problems include the influence on surface Q changes due to the variation of position and length of a coated film, and frequency changes induced by tensile or compressive surface stresses. Some useful results are obtained based on theoretical analysis, which are of interest in the design and fabrication of microcantilever based sensors.

1. INTRODUCTION

Microfabricated cantilever sensors based on the principle of resonance frequency change have found increasing applications in physical, chemical, and biological fields in recent years. Experimental results have indicated that surface effects have significant influence on the resonance character and sensitivity of the microcantilever devices. Therefore, a theoretical understanding of these influences is important to optimize the structural parameters and to improve the performance of the devices.

In this paper, two issues concerning the surface effects are considered. One is surface damping and Q factor of the microcantilevers caused by a coated layer. Another is surface stress induced frequency change. Although several papers have dealt with the issues [1, 2], there are still some problems to be discussed further. For example, to achieve high sensitivity, a high Q factor is desired and how Q varies with the position and length of a coated layer is of interest. For a resonator based cantilever sensor, understanding the influence of tensile and compressive surface stress on the resonance frequency of the cantilever is also important. Theoretical analyses on these problems are given in the paper, which will be useful for understanding the performance and optimising the cantilever structure for sensor applications.

2. *Q* FACTOR CHANGES OF A CANTILEVER CAUSED BY FULL OR PARTIAL COATING

Cantilever sensors are typically coated with a thin overlayer material (e.g. gold, polymer). As is known, coating on the surface of a cantilever will change the overall Q factor of the resonator. Fig. 1 shows the position and length of a coated layer on a lever, which is analysed in this paper.

In Fig. 1, l is the length of the base lever, x_1 and x_2 define the length and position of the thin film, and the thickness of the base lever and the coating thin film are defined by h_s and h_f respectively. The complex Young's modulus of the base lever and the thin film are given by E_s and E_f, respectively, as

$$E_s = E_{s1} + iE_{s2},$$
$$E_f = E_{f1} + iE_{f2}, \qquad (1)$$

where E_{s2} and E_{f2} are the conventional Young's modulus of the base lever and the thin film, respec-

Corresponding author: P.Lu, e-mail: lupin@ihpc.nus.edu.sg

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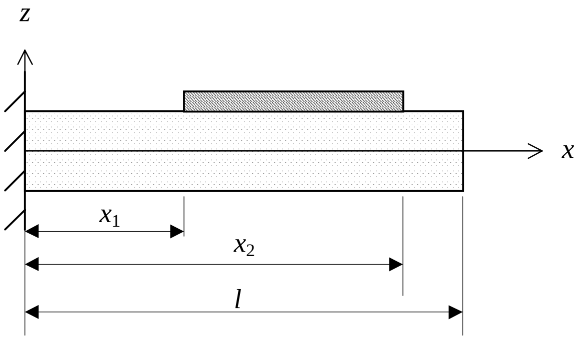

Fig. 1. Schematic view of a cantilever with coated thin film.

tively, E_{s2} and E_{f2} are the dissipative parts of the two layers, respectively.

Cantilever Q is defined as $Q=2\pi W_0/\Delta W$, where W_0 is the stored vibration energy and ΔW is the total energy lost per cycle of vibration. Generally, ΔW can be written as $\Delta W=\Sigma_i \Delta W_i$, where ΔW_i represents the energy lost due to the various dissipation mechanisms [1]. If the thickness of the coated layer is much smaller than the thickness of the base lever, i.e. $h_f << h_s$, the surface layer will not substantially change the stored energy in the cantilever, but it can significantly enhance the dissipated energy. Therefore, for a cantilever with harmonic vibration, the stored energy of the cantilever can be approximately expressed as [1]

$$_0 = \frac{1}{6} b h_{s1} E_{s1} \int_0^1 \varepsilon^2_{max}(x)dx \,, \tag{2}$$

where b is the width of the cantilever, and ε_{max} is the strain that occurs along the top of the cantilever. The energy lost per cycle caused by the surface coating can be obtained as

$$\Delta W_f = \pi \int_V E_{f2} \varepsilon^2_{max}(\boldsymbol{r})dV \,, \tag{3}$$

where the volume integral is over the volume of the thin film. Since the film is very thin, it is assumed that the strain in the thin film is same as the strain along the surface of the base lever. Therefore, relation (3) can be further written as

$$\Delta W_f = \pi E_{f2} \int_V \frac{4z^2}{h_s^2} \varepsilon^2_{max}(x)dV =$$

$$\pi E_{f2} \int_{x1}^{x2} \varepsilon^2_{max}(x)dx \int_{-b/2}^{b/2} dy \int_{h_s/2}^{h_s/2+h_f} \frac{4z^2}{h_s^2} dz \approx \tag{4}$$

$$\pi b h_f E_{s2} \int_{x_1}^{x_2} \varepsilon^2_{max}(x)dx.$$

In the above result, the higher order terms with h_f^2 and h_f^3 have been neglected. Therefore, the Q factor due to the partially coated surface layer is given by

$$Q_{surface} = 2\pi \frac{W_0}{\Delta W_f} =$$

$$\frac{1}{3} \frac{h_s}{h_f} \frac{E_{1s}}{E_{2f}} \frac{\int_0^l \varepsilon^2_{max}(x)dx}{\int_{x_1}^{x_2} \varepsilon^2_{max}(x)dx}. \tag{5}$$

Now, the problem has been reduced to determine the strain $\varepsilon_{max}(x)$ along the top surface of the cantilever. According to beam bending theory, we have

$$\varepsilon_{max}(x) = -\frac{h_s}{2} \frac{d^2 Y(x)}{dx^2}, \tag{6}$$

where $Y(x)$ is the vibration mode shape of the cantilever. Since the cantilever usually works at the first order vibration mode shape, $Y(x)$ is given by [3]

$$(x) = \cosh kx - \cos kx -$$

$$\frac{\sinh kl - \sin kl}{\cosh kl + \cos kl}(\sinh kx - \sin kx), \tag{7}$$

where kl = 1.875. By substituting (5) and (4) into (3), the surface Q changes due to the different position and length of the coated gold film can be investigated.

Defining a parameter

$$\alpha(x_1 / l, x_2 / l) = \frac{\int_0^l \varepsilon^2_{max}(x)dx}{\int_{x_1}^{x_2} \varepsilon^2_{max}(x)dx}. \tag{8}$$

Generally, the parameter α varies with x_1 and x_2, and α =1 indicates that the cantilever is fully coated. By using α, (5) can be written as

$$Q_{surface} = \frac{1}{3} \frac{h_s}{h_f} \frac{E_{1s}}{E_{2f}} \alpha. \tag{9}$$

The changes of $Q_{surface}$ due to the different position and length of the coated film can be investigated through the parameter $\alpha(x_1/l, x_2/l)$.

Fig. 2 shows the variation of α with the changes of x_2/l when x_1/l is fixed to be zero. Fig. 3 shows the variation of α with the changes of x_1/l when x_2/l=0. It

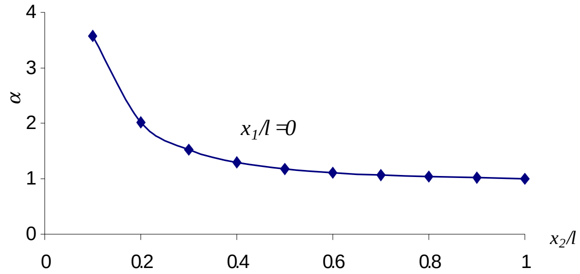

Fig. 2. Variation of α with x_2/l when $x_1/l = 0$. This shows the variation in α as the length of the film changes with respect to the clamped end of the cantilever.

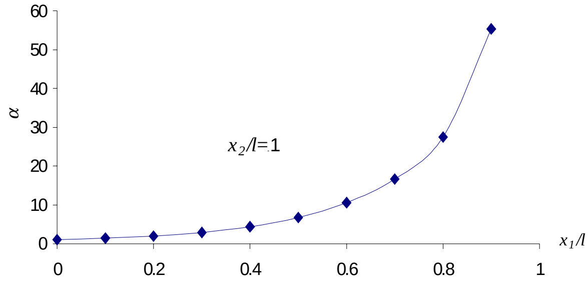

Fig. 3. Variation of α with x_1/l when x_2/l =0. This shows the variation in α as the length of the film changes with respect to the free end of the cantilever.

can be seen that the position and length of the coated film have a great influence on α, i.e. $Q_{surface}$. The shorter the length of the coated film, the higher the value of α and Q. Also placing the coated film near the free end of the cantilever will give much higher Q than that placed close to the clamped end of the lever. In Fig. 4, the length of the coated film has been fixed to be $x_2/l - x_1/l = 0.5$, and the position of the film has been changed from the clamped end to the free end. This represents a case which would be of interest for a realistic sensor. It can be seen that α increases significantly when the coated film moves to the free end. Therefore, it is suggested that if the cantilever is partially coated, the coated film should be deposited close to the free end of the cantilever to get higher Q factor.

3. SURFACE STRESS INDUCED RESONANT FREQUENCY CHANGE

Surface stresses arising from the adsorption or deposition of material onto a surface can be very large, even for a single mololayer [4]. Consider a microcantilever under adsorption–induced surface stress s, which is defined as force per unit length with units Nm^{-1}. To investigate how the surface stress affects the stiffness of the cantilever, the surface stress can be expressed as an equivalent force F and moment M acting at the free end of the cantilever as shown in Fig. 5 [2], which are given by

$$F = sl, \quad M = slt/2, \tag{10}$$

where l and t are the length and thickness of the cantilever, respectively. In this way, the problem is converted to an axial force F acting on the beam, similar to a string under compression and tension. With this model, the influence of the surface stress on the transverse vibration frequencies of the cantilever can be studied.

The free transverse vibration equation of the beam with axial force can be written as [3]

$$EI\frac{\partial^4 w}{\partial x^4} - F\frac{\partial^2 w}{\partial x^2} = -\rho A\frac{\partial^2 w}{\partial t^2}. \tag{11}$$

By assuming harmonic motion

$$w = Y(A\cos\omega t + B\sin\omega t), \tag{12}$$

Eq. (11) is reduced to

$$\frac{d^4 Y}{dx^4} - S\frac{d^2 Y}{dx^2} - k^4 Y = 0, \tag{13}$$

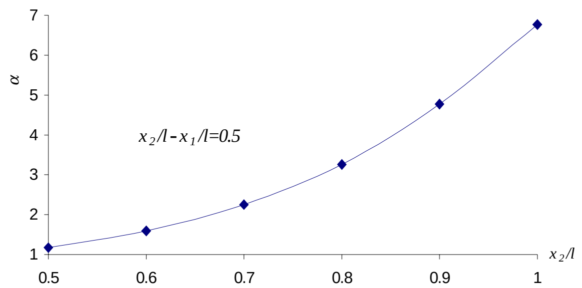

Fig. 4. Variation of α with x_2/l when $(x_2/l - x_1/l) = 0.5$. This shows the variation in α for a film of fixed length as the position of the film along the cantilever changes.

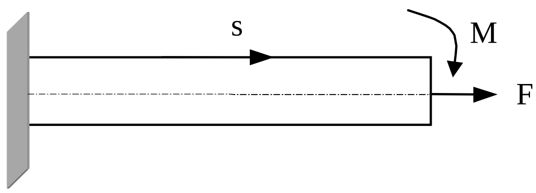

Fig. 5. Schematic view of a cantilever with surface stress.

where

$$k^4 = \frac{\rho A}{EI}\omega^2, \quad S = \frac{F}{EI}. \tag{14}$$

The solution of Eq. (13), satisfying the prescribed boundary conditions, gives the corresponding frequency equation and normal functions, which can be used to investigate the influence of the surface stresses. The general solution of Eq. (13) is given by

$$= C_1 \cosh pt + C_2 \sinh pt + C_3 \cos qt + C_4 \sin qt, \tag{15}$$

where

$$p = \sqrt{\frac{\sqrt{S^2 + 4k^4} + S}{2}}, \quad q = \sqrt{\frac{\sqrt{S^2 + 4k^4} - S}{2}}. \tag{16}$$

From relations (16), we also have the following useful expressions

$$k^2 = pq, \quad p^2 - q^2 = S. \tag{17}$$

To determine the constants C_i in (15), proper boundary conditions of the beam should be defined. We consider a cantilever to investigate the changes in frequency due to surface stresses.

For a cantilever, the boundary conditions are described as

$$\left.\right|_{x=0} = 0, \quad \left.\frac{dY}{dx}\right|_{x=0} = 0, \quad \left.\frac{d^3Y}{dx^3} - S\frac{dY}{dx}\right|_{x=l} = 0, \quad \left.\frac{d^2Y}{dx^2}\right|_{x=l} = 0. \tag{18}$$

Substituting (18) into (15), we obtain the corresponding frequency equation as

$$\frac{q \sin ql + p \sinh pl}{q^2 \cos ql + p^2 \cosh pl} = -\frac{p^2 \cos ql + q^2 \cosh pl}{qp^2 \sin ql - pq^2 \sinh pl}. \tag{19}$$

By letting F be zero, the above equation reduces to the frequency equation of a cantilever without axial force

$$\cos kl \cosh kl = -1. \tag{20}$$

With an axial force F, the related eigenvalues can be obtained from (19). The frequencies of the beam will be

$$\omega_n = \frac{(k_n l)^2}{l^2}\sqrt{\frac{EI}{\rho A}}. \tag{21}$$

In our problem, the values of $k_n l$ depend on the axial force F. Fig. 6 gives the variation of the first order eigenvalue $k_1 l$ with the dimensionless axial force $\bar{s}=Fl^2/FI$. It can be seen from the figure that when the surface stress is tensile ($\bar{s}$ positive), the natural frequencies increase, and for a compressive surface stress, the frequencies of the transverse vibrations decrease. Although the frequency changes due to tensile surface stress have been reported [2], analysis of a general surface stress on the frequency shift has not been dealt with in literature.

According to the above analysis, we find that surface stress can increase or decrease the resonance frequency of a microcantilever, dependant on whether the surface stress is tensile or compressive. Since

$$\omega \propto \sqrt{K/M}, \tag{22}$$

where K is the stiffness and M the effective mass of the cantilever, adsorption induced changes of both K and M can occur to change the resonance frequencies of the cantilever. Generally, surface stress mainly causes the change in stiffness, while mass loading causes the change of the effective mass. To understand what is the main cause of frequency shift of a cantilever sensor will depend on whether the frequency shift is dominated by surface stress, mass loading or both. For example, for very thin adsorbed films (e. g. bio-molecule-film), considerable surface stress can occur while the contribution of mass loading is small [2].

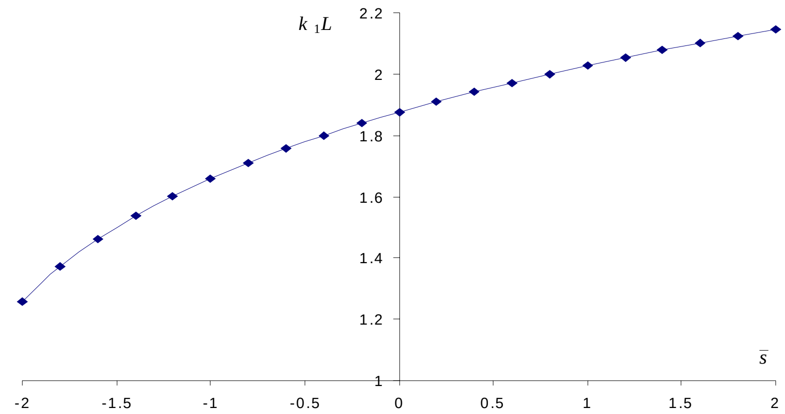

Fig. 6. Variation of $k_1 l$ with dimensionless axial force $\bar{s}$.

4. CONCLUSIONS

Two issues concerning surface effects and microcantilever resonance frequency are considered in this paper. One is surface damping and *Q* factor of the microcantilevers caused by a coated layer. Energy dissipation and quality factor of a cantilever with fully and partially coated thin film are studied. The influence on the surface *Q* changes due to the variation of position and length of the coated film is discussed. It is found that for partially coated cantilever, higher *Q* factor can be obtained if the coating is close to the free end of the lever.

Another issue discussed is surface stress induced frequency change. For cantilever-based biosensors, molecular adsorbing on the surface of the cantilever may induce surface stress, which results in resonance frequency variation of the cantilever. The induced surface stress can be tensile or compressive. The results show that the tensile surface stress will increase and the compressive stress will decrease the resonance frequency of the cantilever. The results are important for the understanding of the performance of cantilever sensors.

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