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FAILURE TIME OF ELASTIC MATERIALS SUBMITTED TO A CONSTANT LOAD

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Abstract. We present a model of delayed fracture in elastic materials, which takes into account the non-reversibility of the system. The results show reasonable agreement both with existing experimental data and 2-D numerical simulations on mode-I fracture.

Delayed fracture receives a lot of attention from the scientific community [1-5]. This interest is motivated by the practical benefits that a better knowledge on this subject would bring to several engineering domains, such as construction and mechanical technology. In a recent work [6], Pomeau has proposed a formula to predict the lifetime of an elastic sample submitted to a constant load:

$$\tau = \tau_0 \exp\left[\left(\frac{\sigma_0}{\sigma}\right)^{2d-2}\right], \tag{1}$$

where σ is the imposed load, d is the dimensionality of the system, and τ_0 and σ_0 are constants depending on the temperature, material and geometrical features. This model has been used to explain experimental results on gels [7], micro-crystals [8], and composite materials such as chipboard and fiberglass [9]. However, Eq. (1) is not completely satisfactory, in that it is unable to explain some experimental features. In Pomeau's model, the fracture is due to the nucleation of one pre-existing defect which is thermally activated. The defect is modelled as an elliptical hole, whose length L is fluctuating randomly due to thermal noise in a *reversible* way - that is the defect can be opened and closed. The fracture occurs when the amplitude of thermal fluctuation is big enough to make L larger than Griffith's critical length [10] L_c. The lifetime τ is calculated as the inverse of the probability to have such a rare fluctuation. If this was the case, one would expect that the lifetime follows Poisson's distribution, whereas it has been found that in micro-crystals, chipboard and fiberglass τ follows a normal distribution [9]. Moreover, other experimental observations [11] (acoustic emissions) have shown that in composites fracture is due to the nucleation and coalescence of several defects.

We think that the crucial point is that fracture is not a reversible phenomenon: once the defect has been opened by thermal noise, that is, the sample is damaged, it cannot be "repaired" by an opposite fluctuation. In order to take into account non-reversibility we propose to change the original model in the following way. Let σ be the imposed stress on the boundary of the sample. The stress σ_A on the tip of the defect is then proportional to $\sigma\sqrt{L}$ [12], that is :

$$\sigma_A(t) = A\sigma\sqrt{L} + \eta(t), \tag{2}$$

where $\eta(t)$ is a white gaussian term[1] due to thermal noise, and A is a constant which depends on the

[1] $\langle\eta\rangle = 0$ and $Var[\eta] = KT$, where K is Boltzmann's constant and T temperature.

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geometry and nature of the material. We assume that the length of the defect L increases by dL when σ_A is bigger than a threshold σ_c which is a material dependent constant. The mean nucleation time $d\tau$, that is, the expected time to go from L to $L + dL$, can be expressed as :

$$d\tau = \frac{dL}{\Pr ob\{\eta > \sigma_c - \langle\sigma_A\rangle\}} = \frac{2dL}{1-\mathrm{erf}\left(\frac{\sigma_c - \langle\sigma_A\rangle}{\sqrt{2KT}}\right)} \cong \sqrt{\frac{2\pi}{KT}}(\sigma_c - \langle\sigma_A\rangle)\exp\left(\frac{(\sigma_c - \langle\sigma_A\rangle)^2}{KT}\right)dL, \quad (3)$$

where $\langle\sigma_A\rangle = A\sigma\sqrt{L}$. The lifetime τ is then calculated by integration between the initial length of defect L_o and L_c:

$$\tau = \int_{L_0}^{L_c}\sqrt{\frac{2\pi}{KT}}(\sigma_c - \langle\sigma_A\rangle)\exp\left(\frac{(\sigma_c - \langle\sigma_A\rangle)^2}{2KT}\right)dL. \quad (4)$$

After change of variable $y = \frac{\sigma_c - \langle\sigma_A\rangle}{\sqrt{2KT}}$, one finds :

$$\tau = \frac{4\sqrt{2\pi KT}}{A^2\sigma^2}\int_0^{\frac{\sigma_c - A\sigma\sqrt{L_0}}{\sqrt{2KT}}}\left(\sigma_c - \sqrt{2kT}y\right)y\exp(y^2)dy. \quad (5)$$

Unfortunately the above integral admits no analytical primitive. However, we can find a lower and an upper bound for lifetime. First, suppose that $y << 1$ so that the term $\sqrt{2KT}y^2$ can be neglected (this is the case if $L \approx L_c$). We are able to solve exactly Eq. (5) and get an upper bound for τ:

$$\tau < \frac{2\sqrt{2\pi KT}}{A\sigma}\frac{\sigma_c}{A\sigma}\left[\exp\left(\frac{(\sigma_c - A\sigma\sqrt{L_0})^2}{2KT}\right)-1\right]. \quad (6)$$

Now suppose that $y >> 1$, that is $L << L_c$. We can get a lower bound for τ using the following argument:

$$\int_0^{y_0} y^2\exp(y^2)dy = \frac{y_0}{2}\exp(y_0^2) - \int_0^{y_0}\exp(y^2)dy > \frac{y_0}{2}\exp(y_0^2).$$

Note that, as big values of y_0 lead to small relative errors, one can hope to get a tight approximation of τ if $\frac{\sigma_c - A\sigma\sqrt{L_0}}{\sqrt{2KT}} >> 1$. This seems to be the case for reasonable values of the physical parameters and the temperature T. After some calculations one finds :

$$\frac{2\sqrt{2\pi KT}}{A\sigma}\sqrt{L}\exp\left(\frac{(\sigma_c - A\sigma\sqrt{L_0})^2}{2KT}\right) < \tau. \quad (7)$$

In both Eq. (6) and Eq. (7), τ is clearly dominated by the same exponential term, so that one can write:

$$\tau = \tau_0\exp\left(\alpha\frac{(\sigma_0 - \sigma)^2}{KT}\right), \quad (8)$$

where $\alpha = \frac{A^2L_0}{2}$, $\sigma_0 = \frac{\sigma_c}{A\sqrt{L_0}}$ and τ_o depends weakly on σ and other physical parameters. Following a more detailed derivation, it is possible to obtain full growth dynamic of a single crack in addition to the previous result [13].

To verify the validity of our model we have compared predictions given by Eq. (8) and Pomeau's one Eq. (1) with existing experimental data and a new 2-D numerical simulation based on a well known electro-mechanical analogy [14]. We model the sample with a 2-D fuse network [15, 16] (see Fig. 2). A constant current I (the load) is injected into the net by the upper busbar, it flows through the fuse net, and exits by the lower busbar. The voltage V in each node is calculated by the Kirchhoff's laws. Thermal fluctuations are modelled by a current noise [17]. Each fuse burns (i.e. its conductivity goes to zero) when its current i is greater than a threshold value i_c. The net fails when the current is unable to pass through.

In Fig. 1 we plot existing experimental data relative to micro-crystals [8] (Fig. 1a) and chipboard [9] (Fig. 1b) submitted to a constant load. In Fig. 3 we plot the data obtained by 2-D simulations. The points represent measured lifetimes; the lines represent a fit to Eq. (8) (solid line) and a fit to Eq. (1) (dashed line). One sees that in the case of micro-crystals and simulations our model seems to give better predictions than Pomeau's one. In the case of chipboard one can hardly distinguish which model works better. This is because the measure range of load σ is too small. However, using the χ^2 test we found that the prediction of Eq. (8) is slightly more accurate that the one in Eq. (1).

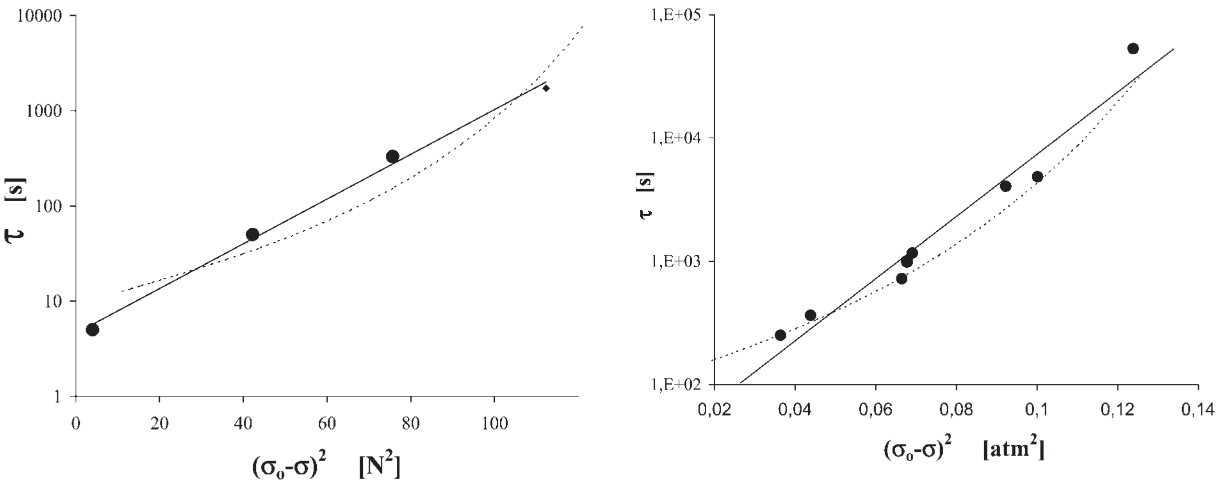

Fig. 1. Micro-crystals (a) and chipboard samples (b) have been submitted to a constant load σ. The lifetime τ is plotted on a semi logarithmic scale as a function of $(\sigma_0 - \sigma)^2$. The value of the critical load σ_0 has been extrapolated from the experimental data [8, 9]. The dashed line represents the original fit with Eq. (1) proposed by authors. The solid line represents the fit with Eq. (8) predicted by our model.

In conclusion, we propose a prediction for delayed fracture in elastic solids. We assume that the failure is due to a step by step irreversible growth of a defect. Every growth-step *dL* is due to a thermal nucleation process. According to Griffith's criterion, the failure will occur when the length of the defect *L* reaches the critical length L_c. This model seems to be quite general, in that it is in agreement with both experimental data on different materials (micro-crystals [8], chipboard and fiberglass [9]) and 2-D numerical simulations. Remarkably, the same functional dependence of the lifetime τ on the load σ is also found in simple simulations on the DFBM [17].

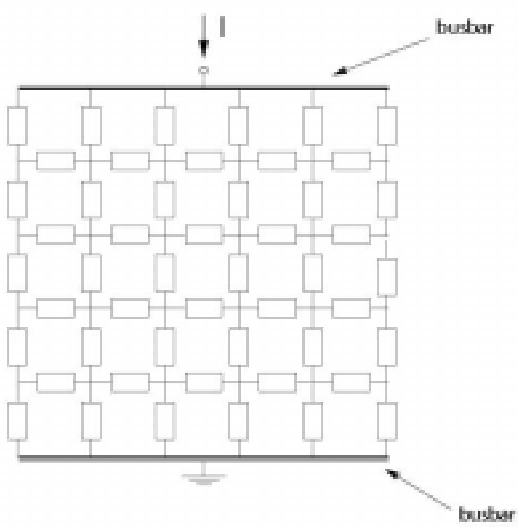

Fig. 2. Sketch of the 2-D fuse network model. The net is composed of fuses which form a grid of *N*x*N* squares. The current *I* is injected into the net by the upper busbar, it flows through the net, and exit from the lower busbar. At the beginning all the fuses have the same resistance *R*. When a fuse fails the current cannot flow through (i.e. $R = \infty$), therefore it is redistributed over the other fuses according to Kirchhoff's law.

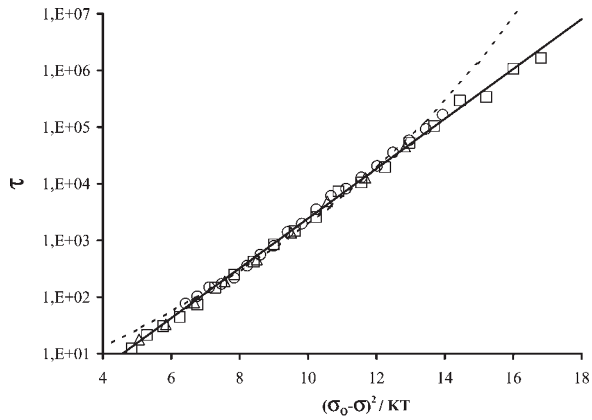

Fig. 3. Numerical data obtained from numerical simulation on a 10 x 10 fuse network. The critical current $i_c = 1$ is the same for all the fuses (homogeneous net). The lifetime τ is plotted on a semi logarithmic scale as a function of $(\sigma_0 - \sigma)^2 = KT$ for different values of the parameter *KT* ((∘) *KT* = 0.15,(Δ) *KT* = 0.12, ( ) *KT* = 0.1). The dashed line represents the initial fit with Eq. (1) as proposed by some authors. The solid line represents the fit with Eq. (8) predicted by our model.

Several aspects of the model need to be more deeply investigated. First, disorder is expected to play an important role [9, 17]. Second, it would be interesting to generalize Eq. (8) to the case of a time dependent load. Finally, this model could be useful to check statistical properties of fracture [11, 18-20] which remain little understood.

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