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Static and dynamic analysis of trapezoidal cantilever plates

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ABSTRACT

Static and dynamic behaviour of trapezoidal cantilever plate are analysed in this work using ANSYS software. Static and dynamic analysis of trapezoidal cantilever plate has been carried out and studied the effect of change in taper ratio α_t, aspect ratio *a/b* and varying tip to root width ratio *c/b*. It has been observed that at α_t = 0.6 and *c/b* = 0.2, non-dimensional frequency of the trapezoidal cantilever plate is higher side, when both parameters varied. In bending analysis under uniformly distributed load when α_t = 0.8 and *c/b* = 0.8 then tip deflection and stresses will be maximum, whereas under edge load when α_t = 0.8 and *c/b* = 0.2 (smallest tip width) will have maximum deflection and stresses. Hence, present numerical results will be helpful for further researchers and designers to design safe thin-wall structures.

KEYWORDS

trapezoidal cantilever plate • modal analysis • buckling • bending • beam

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Introduction

Free vibration analysis of trapezoidal cantilever plates is a topic of interest in structural dynamics and engineering applications such as aircraft wings, turbine blades, balconies, etc. A trapezoidal cantilever plate is a plate with one end fixed and the other end free, and with a non-uniform cross-section that narrows from the fixed end to the free end. The free vibration of such a plate is the natural oscillation that occurs when the plate is subjected to an initial displacement or velocity and then left to vibrate on its own. The natural frequency and mode shape of the plate depend on its geometry, material properties, boundary conditions, and stiffness distribution. Several methods have been proposed in the literature to study the static and dynamic behaviour of trapezoidal cantilever plates, such as transformation of variables, finite element method, Rayleigh-Ritz method, and separation of variables.

Advantages of trapezoidal plate over other types of plates

A trapezoidal cantilever plate is a type of plate that has one end fixed and the other end free. The trapezoidal shape means that the width of the plate decreases from the fixed end to the free end, resulting in a tapered shape geometry. This shape has several advantages over a rectangular or uniform cantilever plate as discussed by [1,2]. Following are some advantages (i) reduced weight and material consumption (that will reduce the inertial forces, damping effects, stress concentration and fatigue failure on the plates;

save costs and resources, and also improve the performance and efficacy of structure); (ii) increased stiffness and strength (especially near the fixed end where the bending moment is maximum and prevent excessive deflection or deformation of the plate and also enhance its load-bearing capacity and durability moreover increased stiffness can also increase the natural frequency and resonance quality factor of the thin-walled structures, which can improve its sensitivity and selectivity for sensing or energy harvesting applications); (iii) improved energy harvesting (trapezoidal shape can improve the energy harvesting capability of the plates, especially when it is coupled with a piezoelectric material; these plates can increase the output voltage and efficiency of the energy harvester by creating a larger strain gradient along the length of the plates). Additionally, the trapezoidal shape can increase the bandwidth and adaptability of the energy harvester by tuning its natural frequency according to different vibration sources or environments.

Applications in different fields of engineering and industry

Trapezoidal cantilever plates have various applications in different fields of engineering such as automotive industry, construction industry, energy industry, biomedical industry, etc as discussed by [3–6]. A trapezoidal cantilever plate can improve the sensitivity and selectivity of this detection method by increasing its stiffness and natural frequency near its fixed end; applications of trapezoidal plate for aircraft wing is shown in Fig. 1. Also, it can also reduce the noise and interference from the environment or other sources by increasing its resonance quality factor and damping ratio.

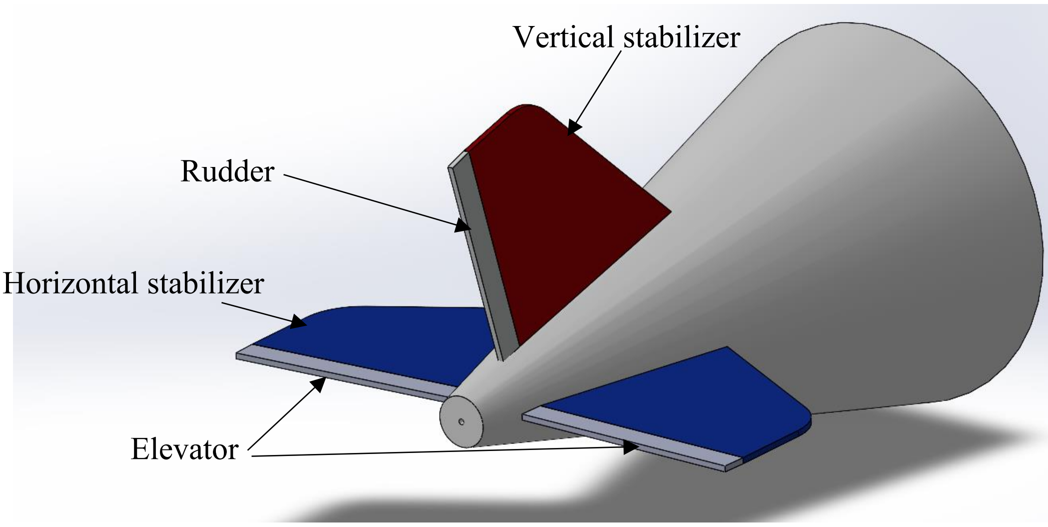

Fig. 1. Applications of trapezoidal plate i.e. plane fin / vertical stablizer

In this communication, authors reviewed the research papers on trapezoidal cantilever plates, focusing on their design, bending, buckling and vibration analysis thin-walled structures. Majidi et al. [3] analysed a cantilever CNT trapezoidal plate and modelled it using FSDT & Generalized differential quadrature method (GDQ). Authors derived the governing equations and boundary conditions using Hamilton’s principle. They studied the effect of geometrical parameters, volume fraction and distribution of

CNTs on the natural frequencies of the plate. Numerical results of study showed that adding CNTs to cantilever trapezoidal plates leads to considerable rise in all natural frequencies and in order to increase natural frequencies it is better to increase volume fraction of CNTs and using FG-X pattern for distribution of CNTs. Also, Numerical examples showed that increase in thickness of the plate leads to increase in natural frequencies but increase in width of the plate decreases all natural frequencies and may change sequence of modes.

Jena et al. [4] studied the vibration behaviour of trapezoidal cantilever plate-like composite beams made of a combination of fibre-reinforced polymer (FRP) composite and aluminium alloy using finite element method (FEM). In this study they examined the impact of various geometric parameters, such as taper angle, thickness, and width, as well as material properties on the modal frequencies and mode shapes of the composite beam. Additionally, the results indicated that the modal frequencies and mode shapes were influenced by the thickness and width of the beam, as well as the material properties. Zamani et al. [7] used the first-order shear deformation theory (FSDT) to derive the governing equations of motion for the laminated composites trapezoidal plates. Generalized differential quadrature (GDQ) method was employed for solution of governing equation and determined the natural frequencies and mode shapes of the plates. The effects of different boundary conditions, such as clamped, simply supported, and free, on the vibration behavior of the plates were also investigated.

Wang et al. [8] examined the vibration characteristics of triangular plates with different boundary conditions using finite element method. The findings of the study showed that the boundary conditions had a significant impact on the vibration characteristics of the triangular plates. Specifically, the natural frequencies of the plates decreased as the number of support points increased, and the vibration modes became more complex for plates with more support points. Torabi and Afshari [9] investigated the vibration characteristics of cantilevered trapezoidal thick plate with variable thickness through ANSYS. FSDT was used for kinetic and strain energy; Hamilton's principle used for governing equation and boundary condition. Natural frequency and mode shape are derived numerically using differential quadrature method. As value of the aspect ratio rises, the width of the plate grows which increase both stiffness and mass of the plate but value of the increase in mass is more than the increase in stiffness of the plate.

The analysis of hybrid metal-composite plates has gained significant attention in structural engineering due to their wide-ranging applications and superior mechanical properties. Shokrollahi and Shafaghat [10] introduced an approach for the free vibration analysis of hybrid plates with a trapezoidal platform by incorporating the first-order shear deformation plate theory (FSDT) and the global Ritz method. The proposed algorithm accurately considers the non-classic effects of transverse shear deformation and rotational inertia. Jiang et al. [11] investigated the nonlinear vibration characteristics of trapezoidal plates by incorporating von Karman's geometric nonlinearity through a finite element method. The authors employed Hamilton's principle to establish the equation of motion for each element of the trapezoidal plate, and by assembling these elements, they derived the equation of motion for the composite laminated trapezoidal plate. The study explored the effects of ply angle and length-height ratio on the nonlinear vibration frequency ratios of the composite laminated trapezoidal plates. Through numerical simulations and analysis of frequency-response curves for different ply angles and harmonic excitation forces, several conclusions are drawn. Overall, this study contributes to the understanding of nonlinear vibration characteristics in composite laminated trapezoidal plates; by utilizing FEM and Hamilton's principle, the authors establish the equation of motion and investigate the effects of ply angle and length-height ratio.

Huang et al. [12] studied the free vibration behaviour of cantilever trapezoidal plates using experimental and numerical methods. They utilize the amplitude-fluctuation electronic speckle pattern interferometry (AF-ESPI) technique with an out-of-plane setup for non-contact and full-field measurement of plate vibrations. Twenty different plate configurations, including triangular and trapezoidal plates, are analysed to measure their first seven vibration modes. The AF-ESPI method enables the determination of resonant frequencies and mode shapes without the need for contact sensors.

Majidi et al. [13] investigated the effect of carbon nanotube (CNT) reinforcements on the flutter boundaries of cantilever trapezoidal plates exposed to yawed supersonic fluid flow. The research utilized the first-order shear deformation theory (FSDT) to model the plate structure and calculates the effective mechanical properties using the extended rule of mixture. The aerodynamic pressure was estimated through the piston theory, and the governing equations and boundary conditions are derived using Hamilton's principle. To obtain numerical solutions for natural frequencies, mode shapes, critical speed, and flutter frequency, the generalized differential quadrature method (GDQM) was employed. The findings reveal that incorporating CNTs enhances the critical speed at which flutter occurs and increases the flutter frequency. It is observed that placing the CNTs away from the middle layer of the plate expands the range of speeds at which flutter is minimized. Additionally, decreasing the width of the plate near the outer edge and adjusting certain angles contribute to improved resistance against flutter. Emelyanov and Kislov [14] determined the state of stresses and lifespan of thin-walled structures under mechanical load in the presence of hydro-chemical medium. Chernyshov et al. [15] considered the three different boundary conditions for rectangular bar to study displacement response and stresses in it and optimized the boundary conditions for rectangular bar. Ropalekar et al. [16] investigated the fatigue strength of composite materials under different loads. Large amplitude flexural vibration behaviour of trapezoidal panels was studied by Kumari and Lal [17] using finite element method. Recently, static and dynamic behaviour of trapezoidal flat and curved panels under various loading and boundary conditions were studied [18–22]. From literature review it is noticed that further research work required to understand the static and dynamic behaviour of cantilever laminated composite trapezoidal plates. Hence, in this article authors investigated the bending, buckling and vibration characteristics of cantilever trapezoidal plates.

Problem formulation

Schematic representation of trapezoidal plate is shown in Fig. 2 with variable thickness. The finite element method based commercial software ANSYS 18.1 is used investigate the bending, buckling and vibration characteristics of trapezoidal panels under different loading and boundary conditions. Eight-node shell 281 is used here to discretize the trapezoidal panel having six degrees of freedom, in which three translational displacement u_x, u_y, u_z and three rotational θ_x, θ_y and θ_z along and about *x*-axis, *y*-axis and *z*-axis, respectively.

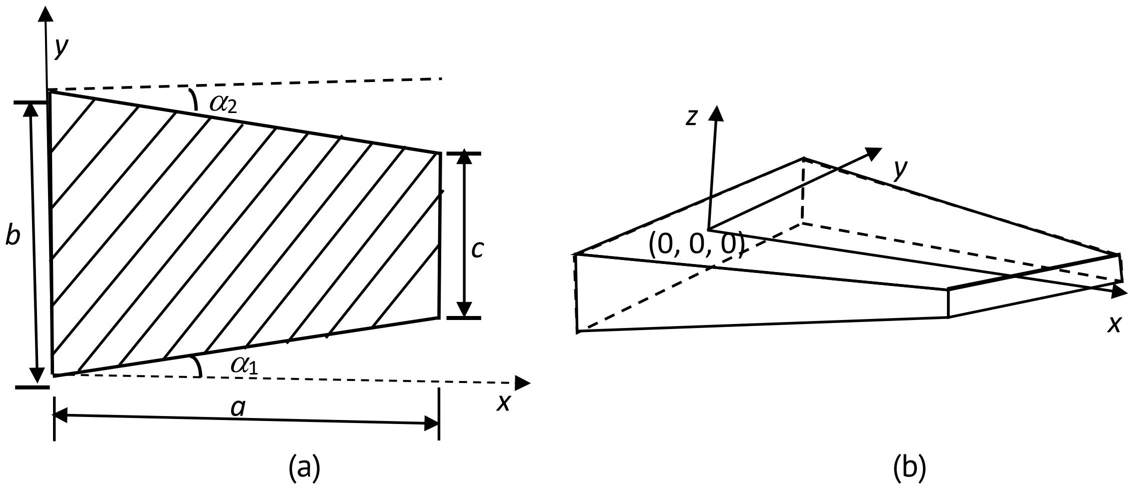

Fig. 2. Geometry of (a) symmetric trapezoidal plate and (b) trapezoidal plate with variable thickness

For a vibrating thin / moderately thick trapezoidal plate as shown in Fig. 2, the strain energy *U* and kinetic energy *T* are expressed as:

$$U=\int_0^a\int_0^b\frac{D}{2}\left[\left(\frac{\partial^2\delta}{\partial x^2}\right)^2+\left(\frac{\partial^2\delta}{\partial y^2}\right)^2+2\nu\left(\frac{\partial^2\delta}{\partial x^2}\right)\left(\frac{\partial^2\delta}{\partial y^2}\right)+2(1-\nu)\left(\frac{\partial^2\delta}{\partial x\partial y}\right)^2\right]dxdy, \quad (1)$$

$$T=\int_0^a\int_0^b\frac{\rho h}{2}(\dot{\delta})^2dxdy, \quad (2)$$

where, δ is the displacement vector, $\dot{\delta}$ is the velocity vector, ρ is the mass per unit area, h_0 is the plate thickness, *D* is the plate flexural rigidity, ν is the Poisson's ratio.

For the dynamic analysis equations of motion is expressed by Hamilton's principle:

$$\int_{t_1}^{t_2}(\delta T-\delta U+\delta W_{NC})\,dt=0. \quad (3)$$

Here, δT is a first variation in kinetic energy, δU is a first variation in strain energy of conservative force fields, δW is the virtual work of non-conservative force fields.

Bending analysis

Firstly, carried out the bending analysis of trapezoidal cantilever plate under pure compression, pure shear and pure moment and combination of shear and moment using following governing equation:

$$[K_L]\{\delta\}=\{F\}, \quad (4)$$

where, $[K_L]$ is the linear stiffness matrix, $\{\delta\}$ is the displacement vector and $\{F\}$ is the force vector.

Static analysis of trapezoidal cantilever panels under various loading conditions has to be conducted to investigate the bending deformation, normal (σ_{xx}, σ_{yy}, σ_{zz}), shear (τ_{xy}, τ_{yz}, τ_{zx}) and principal stresses (σ_{11}, σ_{22}, σ_{33}) in it.

Dynamic analysis

Assumptions:

(a) Quasi-static analysis – inertia forces are negligible: if the smallest time period of forcing function is significantly greater than the largest natural time period of the structure, then dynamics analysis is carried out.

(b) Dynamic analysis – inertia forces are significant, hence included in the analysis: If the smallest time period of forcing function is comparable to the largest natural time period of the structure.

Kinetic energy is:

$$T = \frac{1}{2}\int_V (\dot{u}^2 + \dot{v}^2 + \dot{w}^2)\rho dV. \quad (5)$$

For the free vibration analysis of structures, the governing equation of motion might be written as:

$$[M]\{\ddot{\delta}\} + [K_L]\{\delta\} = \{0\}, \quad (6)$$

where, $[M]$ is the mass matrix and $\{\ddot{\delta}\}$is the acceleration vector. Equation (6) is used to calculate the eigenvalues or vibration frequencies, and eigenvector to plot mode shapes as given by [23,24].

Results and Discussion

Bending analysis of trapezoidal cantilever plate

In bending analysis of the trapezoidal cantilever plate (schematic geometry of trapezoidal plate is shown in Fig. 3) carried out to find the tip deflection, normal stress, shear stress and von-mises stress under uniformly distributed load (UDL) and edge load. Here first validation of the bending results done with trapezoidal cantilever plate, then bending results with varying thickness and tip to root width ratio obtained. Loading of the structure is done under uniformly distributed load over the upper face and concentrated loading at the free edge of the cantilever plate.

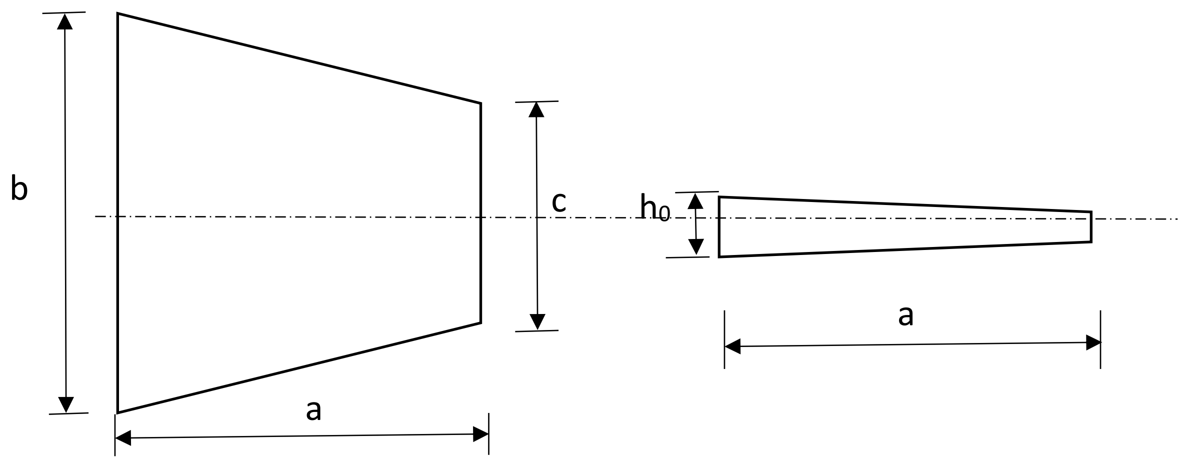

Fig. 3. Trapezoidal plate geometry with varying thickness

Validation of bending results

For validation of the results obtained for *CFCF* ($u = v = w = \theta_x = \theta_y = \theta_z = 0$, along $x = 0, a$) trapezoidal plate made of isotropic material ($\nu = 0.3$) subjected to uniformly distributed load ($q_0 = 30 \times 10^3$ N/m^2) is compared with Zhao et al. [25] and Liew and Han [26] as given in Table 1. It is noticed that percentage of error is 13.95 and 11.29 % with Zhao et al. and Liew and Han, respectively. Here, percentage of error is higher because three-dimensional numerical results are compared with two-dimensional results.

Table 1. Comparison of central deflection of a homogeneous CFCF trapezoidal plate (E = 206 GPa, ρ = 7800 kg/m^3 and v = 0.3. a/b = 1, c/b = 0.7, h_0 = 0.2 m) UDL of 30000 N/m^2

Reference	Boundary condition	Deflection, m
Present (24 × 28 × 6)	*CFCF*	8.9314×10^{-7}
Zhao et al. [25]	*CFCF*	7.8379×10^{-7}
Liew and Han [26]	*CFCF*	8.0252×10^{-7}

Bending of trapezoidal cantilever plate having constant *c/b* ratio and variable thickness in *x*-direction

Next, analysed trapezoidal cantilever plate (*CFFF* such as $u = v = w = \theta_x = \theta_y = \theta_z = 0$, along $x = 0$) having variable thickness of the plate in x-direction ($h = h_0(1 - \alpha_t(x/a))$, α_t is taper ratio) under uniformly distributed load ($q_0 = 10 \times 10^3$ N/m^2) and edge load (q_0 = 100 kN). By considering following geometric and material properties: a/h_0 = 15, h_0 = 0.5, c/b = 0.6, v = 0.3, $E = 2\times 10^5$ MPa, ρ = 7800 kg/m^3, (a) square cantilever plate: a/b = 1 and (b) rectangular cantilever plate: a/b = 3.

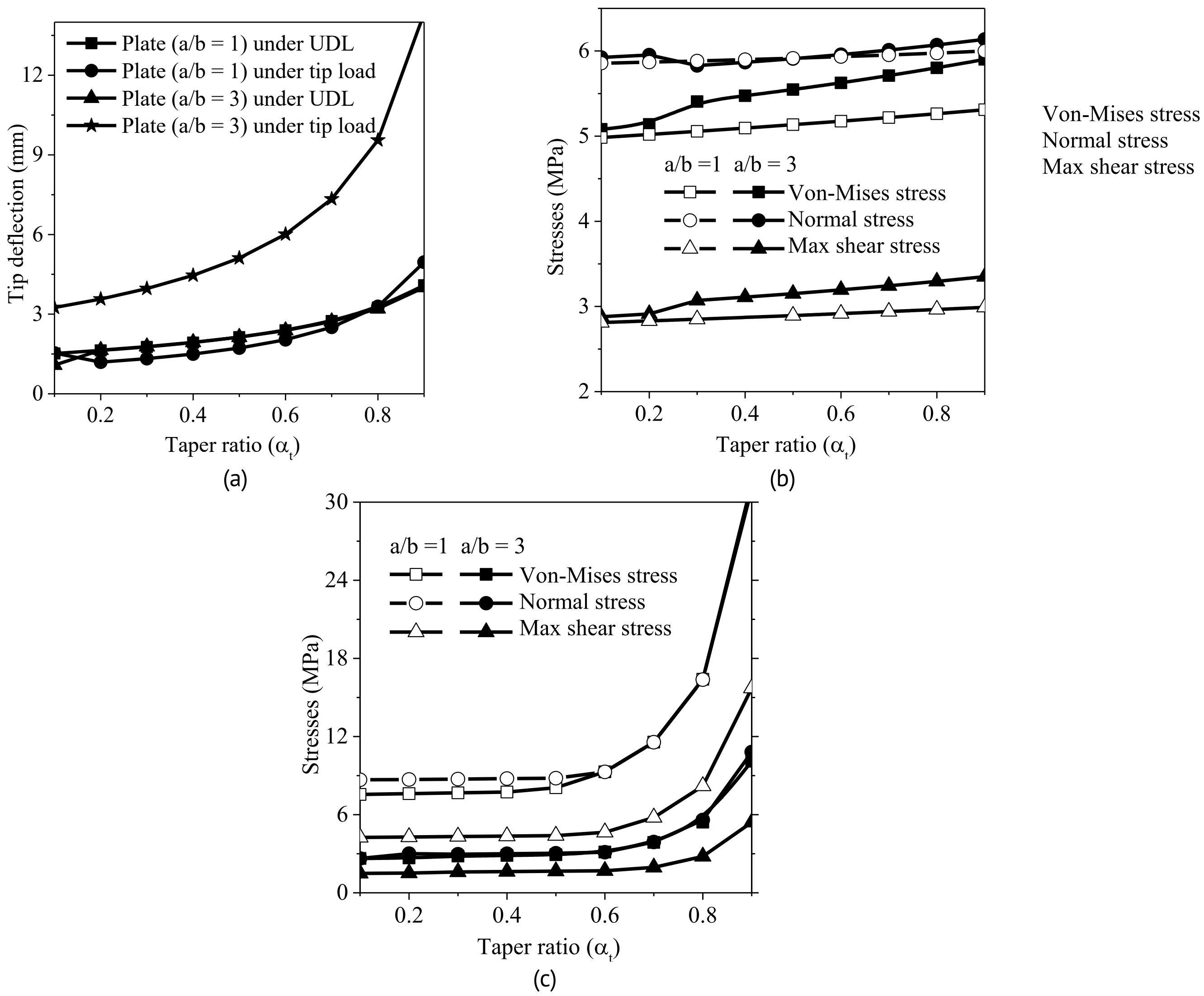

Fig. 4. Tip deflection and stresses of trapezoidal cantilever plate with variable thickness function linearly tapered along x-direction $h = h_0(1 - \alpha_t(x/a))$, under uniformly distributed load (q_0 = 10000 N/m^2) and edge load (q_0 = 100 kN): (a) tip deflection under UDL and tip load; (b) stresses under UDL; (c) stresses under tip load

The responses of tip deflection, and stresses (von-mises stress; normal stress and maximum shear stress) of square (*a/b* = 1) and rectangular (*a/b* = 3) cantilever trapezoidal plates under UDL and edge load are plotted in Fig. 4, as taper ratio increases max deflection of the plate increases exponentially whereas all other stresses increases in linear manner under UDL as shown in Fig. 4(b). Under Edge load deflection as well as stresses increase in similar manner. When aspect ratio of the plate increases from 1 to 3 then under UDL there is no major change in the deflection and stresses but under edge load when aspect ratio is 3 then deflection and all stresses (von-mises, normal, max shear) increases significantly as can be seen from the Fig. 4(a,c).

Bending results of trapezoidal cantilever plate having variable thickness in *x*-direction ($h = h_0(1 - \alpha_t(x/a))$) and variable *c/b* ratio

After analysing plate under separately for keeping one parameter constant and other variable, now here both of the parameter taper ratio (α_t) and tip cord to root width ratio (*c/b*) varied together and analysed the bending response of trapezoidal cantilever plate.

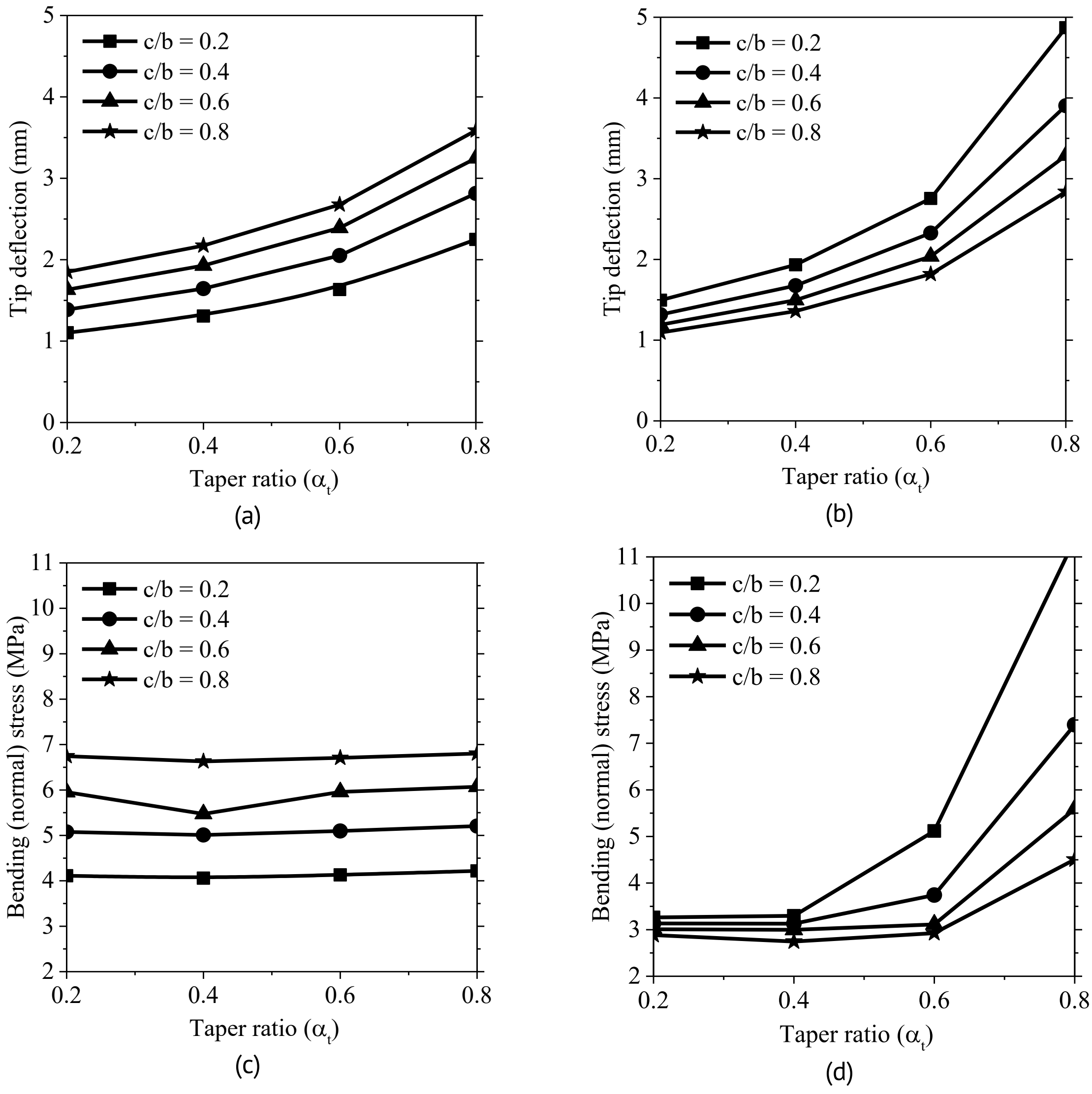

Fig. 5. Deflection and bending stress of trapezoidal cantilever plate with variable taper ratio and variable c/b ratio: (a) deflection under UDL; (b) deflection under edge load; (c) bending stress under UDL; (d) bending stress under edge load

The response of tip deflection and bending stress under UDL and edge load is presented in Fig. 5. In Fig. 5(a), as taper ratio increases then the smallest free end width of the trapezoidal plate at *c/b*= 0.2 have the lowest deflection under uniformly distributed load (q_0 = 10 × 10^3 N/m^2). Whereas, when the load on the trapezoidal cantilever plate applied at the end edge (q_0 = 100 kN), then as taper ratio (α_t) increases from 0.2 to 0.8 and root to cord ratio (*c/b*) decreases from 0.8 to 0.2, deflection of the plate is maximum at the lowest tip width *c/b* = 0.2 and taper ratio α_t = 0.8 as shown in Fig. 5(b). In Fig. 5(c,d) bending stresses under UDL and edge load plotted respectively, which shows that under UDL stresses will be highest when *c/b* = 0.8 but there is no major variation on the stress due to change in the taper ratio along the *x*-direction as presented in Fig. 5(c); whereas under edge load plate will be highly stressed when taper ratio is max (α_t = 0.8) and root to cord ratio is minimum (*c/b* = 0.2). So, we can conclude that when load is not uniform and only at the end then trapezoidal plate will experience more deflection and stresses, as illustrated in Fig. 5(d).

Dynamic analysis

Next, free vibration analysis of the trapezoidal cantilever plate, firstly validation of the results obtained from ANSYS for different modes shape done with the available results in the literature for free vibration. Then results of trapezoidal cantilever plate for various parameters like thickness, aspect ratio, tip to root width ratio (*c/b*) and combination of these obtained here.

Validation of 3D geometry using ANSYS with available 2D literature results

For validation of free vibration results, a rectangular plate (*a* = 1 m, *b* = 0.5 m, h_0 = 0.01 m) is created using ANSYS 18.1 design modeller and meshing has been done for uniform quality mesh which resulted in 1272 nodes and 162 elements of uniform quads with four node each. DOF of the node was 6 (displacement *u*, *v*, *w* and rotation ϑ_x, ϑ_y, ϑ_z in *x*, *y* and *z* direction respectively).

Results have been validated from different authors [27–30] with non-dimensional frequency parameter where different taper ratio ($\alpha = 1-h_{min}/h_0$) is varied for thickness variation from base to tip of the plate which is given by $h = h_0(1-\alpha_t (x/a))$. In Table 2, cantilever rectangular plate with *a/b* = 2 where *a* is length of the plate and *b* is width and h_0 is thickness of the plate at the fixed point or root. Elastic constant and poison's ratio is 100 kPa, 0.3, respectively. Non-dimensional frequency parameter (ϖ) is calculated for different taper ratio (α_t), where thickness of the plate varies in x-direction. Eight modes of the plate have been evaluated and shown in Fig. 6 and corresponding non-dimensional frequency written in Table 2. From Table 2, we can observe that result obtained from ANSYS is well converged and showing very minimal % of error. Result obtained in this work can be considered more accurate as 3D analysis has been done in this work. Error % at α = 1 is highest as we can see in Table 2 this is because of the triangular tip at the end and triangular elements in 3D work which resulted in more error. As taper ratio increases thickness at the tip decreases which resulted in decrement of ϖ as taper ratio decreases except for mode 1. For a particular α, ϖ increases with modes which can be seen in Table 2.

Table 2. Fundamental non-dimensional frequency parameter ($\varpi = \omega \times a^2\sqrt{\rho h_0/D_0}$) of isotropic rectangular cantilever (*CFFF*) plate (a/b = 2, a/h_0 = 100, h_0 = 1.0, ν = 0.3) considering variable thickness function is linearly tapered along x-direction $h = h_0\big(1 - \alpha_t(x/a)\big)$

Taper ratio α_t	Reference	Modes							
		1	2	3	4	5	6	7	8
0.0	**Present**	**3.458**	**14.769**	**21.544**	**48.080**	**60.414**	**92.373**	**92.929**	**118.712**
	Kumari [27]	3.439	14.779	21.424	48.089	60.108	92.327	93.130	118.556
	Liew et al. [28]	3.4394	14.803	21.435	48.183	60.154	92.530	93.104	118.450
	Liu and Chang [29]	3.429	14.53	21.34	47.50	60.26	92.110	92.930	119.200
	Huang et al. [30]	3.436	14.703	21.414	47.856	59.940	92.907	-	-
0.2	**Present**	**3.543**	**14.192**	**20.159**	**43.176**	**54.994**	**80.175**	**82.747**	**107.373**
	Kumari [27]	3.5250	14.206	20.054	43.079	54.743	80.382	82.702	107.297
	Liew et al. [28]	3.5257	14.225	20.063	43.243	54.766	80.320	82.826	107.130
	Liu and Chang [29]	3.526	14.090	20.070	43.100	55.160	81.500	83.320	108.600
0.4	**Present**	**3.662**	**13.545**	**18.684**	**38.007**	**49.237**	**66.807**	**72.592**	**95.222**
	Kumari [27]	3.646	13.560	18.595	38.006	49.036	67.013	72.552	95.233
	Liew et al. [28]	3.6470	13.574	18.602	38.046	49.041	66.926	72.610	95.025
	Liu and Chang [29]	3.649	13.600	18.690	38.490	49.660	69.660	74.000	96.900
	Huang et al. [30]	3.646	13.500	18.585	37.848	48.902	66.763	-	-
0.6	**Present (9 × 18)**	**3.848**	**12.790**	**17.096**	**32.478**	**42.954**	**52.823**	**61.682**	**81.663**
	Present (31 × 61)	**3.837**	**12.772**	**17.039**	**32.409**	**42.808**	**52.806**	**61.516**	**81.476**
	Kumari [27]	3.834	12.805	17.023	32.471	42.807	53.010	61.645	81.823
	Liew et al. [28]	3.834	12.813	17.027	32.490	42.800	52.909	61.654	81.600
	Liu and Chang [29]	3.813	13.020	17.110	33.570	43.440	57.010	63.940	84.120
0.8	**Present**	**4.191**	**11.872**	**15.397**	**26.451**	**35.481**	**38.464**	**49.546**	**64.487**
	Kumari [27]	4.178	11.881	15.341	26.434	35.439	38.544	49.523	64.776
	Liew et al. [28]	4.179	11.886	15.345	26.442	35.417	38.461	49.490	64.583
	Liu and Chang [29]	4.047	12.280	15.120	27.980	35.880	43.610	52.520	70.600
	Huang et al. [30]	4.179	11.846	15.331	26.352	35.495	38.209	-	-
1.0	**Present (11772 triangular)**	**5.182**	**12.22**	**14.979**	**25.771**	**30.449**	**34.440**	**48.755**	**53.063**
	Present (202744 triangular)	**5.176**	**11.497**	**14.867**	**21.644**	**24.706**	**29.625**	**37.404**	**41.286**
	Kumari [27]	5.176	11.483	14.866	21.507	24.123	29.619	36.591	39.372
	Liew et al. [28]	5.177	11.484	14.870	21.500	24.021	29.600	36.159	37.978
	Liu and Chang [29]	4.388	10.840	11.570	20.550	26.730	27.880	40.580	52.930

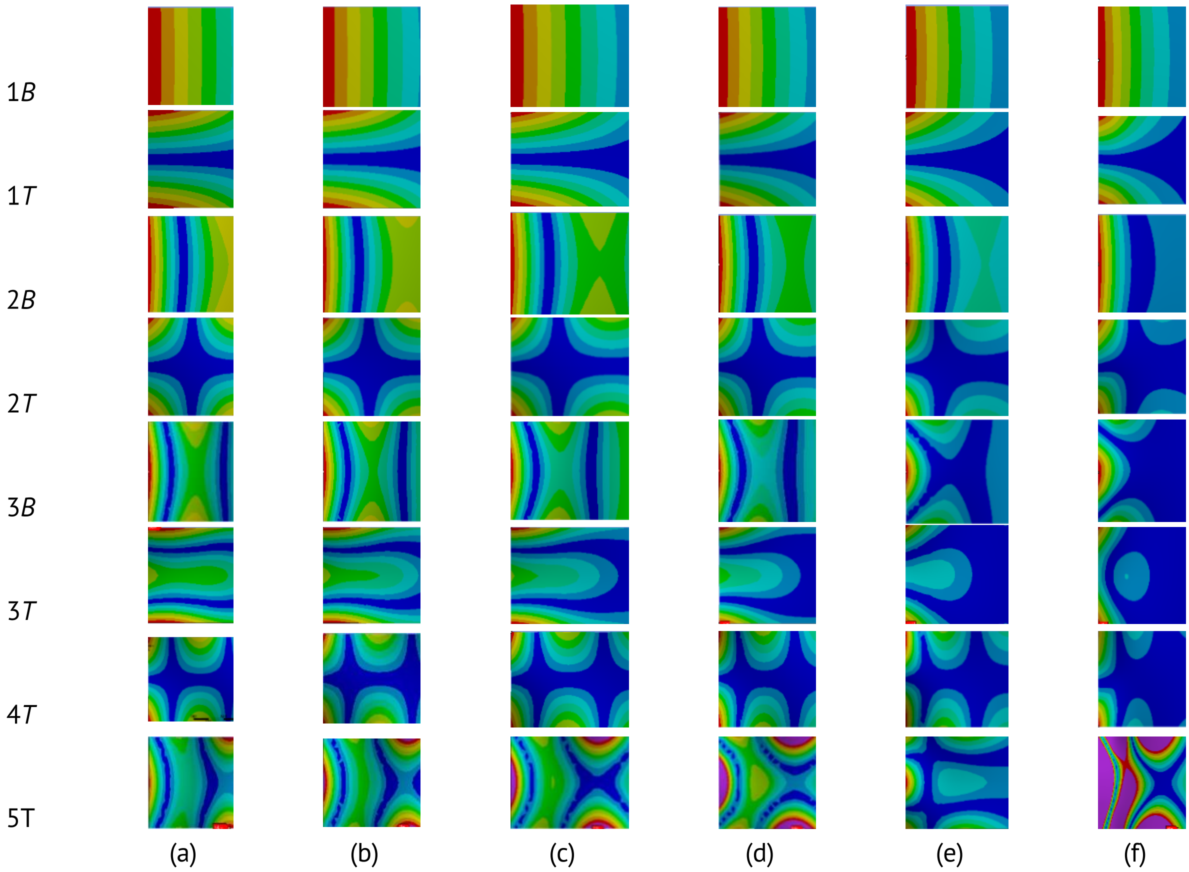

Fig. 6. Mode shapes of first eight modes of cantilever plate having variable thickness along span: (a) α = 0; (b) α = 0.2; (c) α = 0.4; (d) α = 0.6; (e) α = 0.8; (f) α = 1

Non dimensional frequency of cantilever trapezoidal plate with different taper ratios

Next, performed the modal analysis results of the isotropic (v = 0.3) trapezoidal plate having aspect ratio a/b = 3, a/h_0 = 15 and c/b = 0.6, and validated the present results with available published results. The aspect ratio and geometry considered here is shown in Fig. 7, that will be equivalent to an aeroplane wing size. Dimensions of the symmetric trapezoidal plate is as follows: a = 7.5 m, b = 2.5 m, c = 1.5 m, h_0 = 0.5 m. Non-dimensional frequency parameter: $\varpi = \omega \times a^2\sqrt{\rho h_0/D_0}$, where ω is circular frequency in rad/s, density of the material ρ = 1800 kg/m^3, flexural rigidity of plate $D_0 = \frac{E{h_0}^3}{12(1-\nu^2)}$, Young's modulus or elastic constant E = 45×10^3 MPa.

Table 3. Non-dimensional frequency parameter ($\varpi = \omega \times a^2\sqrt{\rho h_0/D_0}$) of isotropic trapezoidal cantilever (*CFFF*) plate (a/b = 3, a/c = 5, a/h_0 = 15, h_0 = 0.5, v = 0.3) considering variable thickness function is linearly tapered along x-direction $h = h_0\big(1-\alpha_t(x/a)\big)$

Taper ratio α_t	Present	Modes							
		1	2	3	4	5	6	7	8
0.0	10 × 38 × 3	3.943	16.86	21.792	28.198	57.466	68.781	74.088	86.304
0.2	10 × 38 × 2	4.030	17.922	20.48	27.766	52.902	68.729	70.266	90.014
0.4	10 × 38 × 2	4.151	19.049	19.348	27.165	47.884	62.577	72.172	89.135
0.6	10 × 38 × 2	4.337	17.482	21.415	26.346	42.281	55.378	74.868	78.054
0.8	10 × 38 × 2	4.681	15.775	24.806	25.144	35.777	46.357	64.702	74.345
1.0	4636 elements	5.687	15.225	23.479	29.482	31.889	34.651	48.426	51.832

In Table 3, non-dimensional frequency (ϖ) is calculated for trapezoidal cantilever plate having variable taper ratio ranging from 0.2 to 1. Modes for all taper ratio up to 8 modes drawn. As previously seen in validation results here also first Mode increases with increase in taper ratio and all other modes decreases with increase in taper ratio. These signifies that as thickness at tip of the cantilever plate decreases vibration frequency also decreases as shown in Table 3 and modes of the results in different taper ratio shown in Fig. 7.

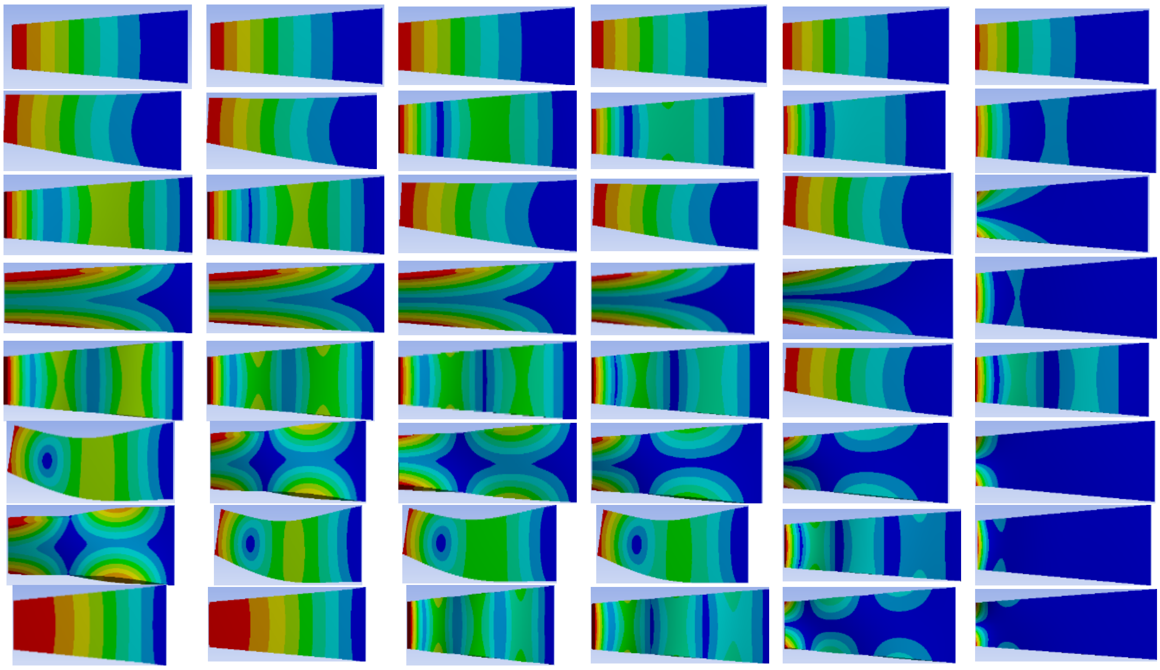

Fig. 7. Mode shapes of different taper ratio of isotropic trapezoidal cantilever (*CFFF*)

Non dimensional frequency of cantilever trapezoidal plate with uniform thickness and different tip to root width ratios (*c*/*b*)

Now keeping the thickness of the trapezoidal plate uniform and varying the tip to root ratio (c/b = 0.9 to 0.1) of the plate non-dimensional frequency (ϖ) calculated for eights modes. In Table 4, keeping the whole dimension and material properties same as previous

Table 4. Non-dimensional frequency parameter ($\varpi = \omega \times a^2\sqrt{\rho h_0/D_0}$) of isotropic trapezoidal cantilever (*CFFF*) plate (a/b = 3, a/h_0 = 15, h_0 = 0.5, v = 0.3) considering uniform thickness along x-direction

Tip to root width ratio (*c/b*)	Present	Modes					
		1	2	3	4	5	6
0.9	12 × 38 × 3	3.505	15.857	20.908	21.024	56.831	62.367
0.8	12 × 38 × 3	3.629	16.15	21.246	22.957	57.013	65.763
0.7	11 × 38 × 3	3.773	16.479	21.498	25.362	57.218	69.626
0.6	10 × 38 × 3	3.943	16.86	21.792	28.198	57.466	68.781
0.5	9 × 38 × 3	4.147	17.31	22.146	31.548	57.774	67.711
0.4	8 × 38 × 3	4.401	17.864	22.589	35.49	58.19	66.426
0.3	7 × 38 × 3	4.727	18.582	23.18	40.075	58.788	64.917
0.2	5 × 38 × 3	5.165	19.578	24.043	45.288	59.764	63.226
0.2	7 × 65 × 4	5.164	19.575	24.035	45.27	59.741	63.217
0.1	4 × 66 × 4	5.798	21.131	25.509	51.01	61.694	61.703
0.1	10 × 152 × 10	5.796	21.129	25.501	50.986	61.684	61.684

analysis except the thickness parameter. Here first five modes of the plate corresponding to the different *c/b* ratio increases with decrease in *c/b* ratio whereas 6th mode of the cantilever plate increase up to 0.7 *c/b* ratio then decreases. So, the more taper towards the tip then vibration frequency of the trapezoidal plate increases.

Non dimensional frequency of cantilever trapezoidal plate with variable thickness along x-direction and variable tip to root width ratios (*c/b*)

In Table 5, thickness of the plate along the *x*-direction (α_t = 0.2, 0.4, 0.6 and 0.8) and tip to root ratio (*c/b* = 0.8 to 0.2) both varied. The non- dimensional frequency parameter obtained for different cases up to 6 modes as presented in Table 5.

Table 5. Non-dimensional frequency parameter ($\varpi = \omega \times a^2\sqrt{\rho h_0/D_0}$) of isotropic trapezoidal cantilever (*CFFF*) plate (*a/b* = 3, a/h_0= 15, h_0 = 0.5, *v* = 0.3) considering variable thickness function is linearly tapered along *x*-direction $h = h_0\big(1 - \alpha_t(x/a)\big)$ and different ratio of *c/b*

Taper ratio α_t	Tip to root width ratio (*c/b*)	Present study (elements)	Modes					
			1	2	3	4	5	6
0.2	0.8	15 × 50 × 3	3.715	17.194	19.960	22.514	52.430	60.460
	0.6	13 × 50 × 3	4.028	17.919	20.470	27.750	52.869	68.683
	0.4	10 × 50 × 3	4.487	18.941	21.218	35.116	53.565	67.818
	0.2	6 × 50 × 3	5.253	20.679	22.592	45.086	55.055	64.525
0.4	0.8	15 × 50 × 2	3.837	18.574	18.600	21.952	47.454	54.546
	0.6	13 × 50 × 2	4.15	19.045	19.346	27.163	47.875	62.572
	0.4	10 × 50 × 2	4.609	19.741	20.391	34.589	48.529	69.631
	0.2	6 × 50 × 2	5.377	21.031	22.158	44.735	49.926	66.235
0.6	0.8	15 × 50 × 2	4.023	17.050	20.635	21.191	41.882	47.707
	0.6	13 × 50 × 2	4.337	17.480	21.413	26.345	42.274	55.368
	0.4	10 × 50 × 2	4.796	18.119	22.490	33.814	42.887	66.823
	0.2	6 × 51 × 2	5.566	19.319	24.293	44.145	44.177	68.715
0.8	0.8	15 × 50 × 2	4.366	15.393	20.088	23.966	35.421	39.361
	0.6	13 × 50 × 2	4.681	15.773	24.803	25.143	35.772	46.352
	0.4	10 × 50 × 2	5.141	16.351	25.938	32.675	36.326	57.307
	0.2	6 × 51 × 2	5.910	17.457	27.781	37.494	43.280	66.244

Conclusion

Static and dynamic analysis of trapezoidal cantilever plate has been carried out in this study using ANSYS 18.1. To analyse trapezoidal plate in vibration and bending, two parameters (taper ratio α_t and tip to root width ratio *c/b*) of the trapezoidal plate and combination of these parameters varied.

It can be concluded from this study that at lower aspect ratio, stresses and deflection of a trapezoidal cantilever plate is lower than higher aspect ratio. In modal analysis at α_t = 0.6 and *c/b* = 0.2 maximum non-dimensional frequency for the plate observed when both parameters varied. In bending analysis under UDL when α_t = 0.8 and *c/b* = 0.8 then deflection and stresses will be maximum, whereas under Edge load when α_t = 0.8 and *c/b* = 0.2 (smallest tip width) will have maximum deflection and stresses.

Trapezoidal cantilever plate is used in various field of engineering and this geometry widely acceptable for its high strength to weight ratio. So, this analysis can be helpful for selecting thickness, aspect ratio and free end width of the trapezoidal plate for various applications based on different industry uses. Also, future scope of the analysis available for different coating of the plate for surface strength for in-plane stresses and composite materials with different boundary conditions.

CRediT authorship contribution statement

Emarti Kumari: writing – review & editing, writing – original draft, conceptualization, supervision; **Brajesh Choudhary**: data curation, investigation.

Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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