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A model for the temperature dependence of the fracture strength of ceramic matrix composites

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ABSTRACT

High fracture strength at elevated temperatures is a crucial characteristic of ceramic matrix composites for applications in extreme heat environments. Here we suggest a model that describes the temperature dependence of the fracture strength of particulate-reinforced ceramic matrix composites. Within the model, the variation of the fracture strength with temperature is given by the competition of the temperature dependences of the thermal stresses and cohesive strength of the matrix. It is demonstrated that the temperature dependence of the fracture strength can have a maximum if the coefficient of thermal expansion of the filler is significantly smaller than that of the matrix. The results of the model agree with experimental data and confirm that a small coefficient of thermal expansion difference between the matrix and the filler is beneficial for a high fracture strength of ceramic matrix composites.

KEYWORDS

high-temperature ceramics • ceramic matrix composites • fracture • strength • thermal stresses

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Introduction

Due to their high strength, oxidation resistance, and high-temperature stability high-temperature ceramics and ceramic matrix composites are increasingly used in extreme environments, such as gas turbines, rocket nozzles, and hypersonic vehicle leading edges [1–3]. Among these, ultra-high-temperature ceramics (UHTCs) based on diborides (ZrB_2, HfB_2) and carbides (HfC, TaC) exhibit outstanding melting points (> 3000 °C), high stiffness, and resistance to ablation and thermal shock [4,5]. The incorporation of reinforcements such as SiC, ZrC, and carbon fibers produces UHTC-based composites that combine high temperature capability with improved fracture toughness and damage tolerance [6,7].

However, the fracture strength and toughness of high- and ultrahigh-temperature ceramics and ceramic composites strongly vary with temperature [8–20]. While some researchers report on a monotonous decrease in the fracture toughness [16] and flexural strength [8,10,17] of ceramics and ceramic matrix composites, others [9–16,18] demonstrate the presence of maxima in such dependences at certain temperatures. The enhanced strength or toughness of such materials was attributed to the formation of core-

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shell structures [11,17], the relaxation of thermal mismatch stresses [8,10], the brittle-to-ductile transition in one of the phases [11] or to the healing of microcracks [17].

The effect of the thermal mismatch stresses on the temperature dependence of the flexural strength of composite ceramics has also been addressed theoretically in [21–23], which considered the nucleation and growth of cracks at spherical particles but have not demonstrated the possibility for the growth of the fracture strength with temperature. In [24], the effect of the thermal mismatch stresses on the temperature dependence of fracture toughness has been considered for ceramic matrix composites containing long parallel whiskers. The solution used in[24] adopts the concept [25] of crack growth in a two-dimensional periodic stress field and the assumption that the coefficient of thermal expansion (CTE) creates compressive stresses in the matrix [26], which is valid if the CTE of the matrix is smaller than that of particles. In contrast to [24], in the present paper, we consider the situation of particulate reinforced ceramic matrix composites and focus on the case where the CTE of the matrix is larger than that of particles. The aim of the present paper is to suggest a model that describes the temperature dependence of the fracture strength of particulate reinforced ceramic matrix composites.

Model

Consider a composite ceramic solid that consists of an elastically isotropic ceramic matrix and elastically isotropic ceramic particles and is subject to a uniaxial tensile load σ_0 (Fig. 1). Within the model, we assume that all the particles are identical and represent spheres of radius R. Let the thermal stresses, associated with the cooling of the ceramic composite after sintering and the difference of the CTE of the particles and the matrix, act in the composite solid. We focus on the case where the thermal stresses in the particles are compressive. Let a mode I crack propagate across the ceramic solid. The compressive thermal stresses acting inside the particles hinder crack growth across them. In this case, when the crack front approaches a particle, the crack can grow in three different ways: it can cut the particle (Fig. 2(a)), change its direction and bend the particle (Fig. 2(b)) or envelope the particle, which will eventually produce a bridge in the crack wake (Fig. 2(c)). The mode of crack growth should depend on the stress level, as well as on how far the crack plane lies from the "equator" of the particle. (The closer the crack plane to a particle pole, the easier the crack to bend around the particle due to the smaller deflection angle.) Here we focus on the case where the compressive stresses acting in

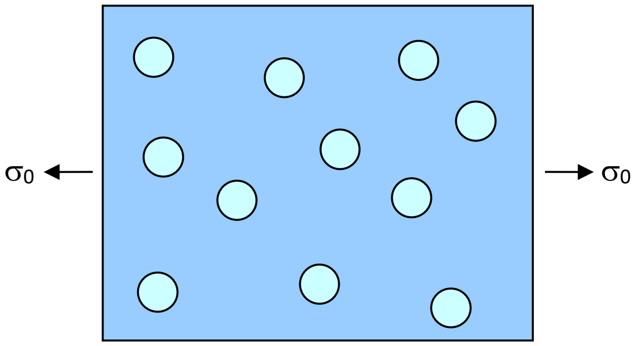

Fig. 1. A ceramic composite solid containing spherical particles under a uniaxial tensile load σ_0

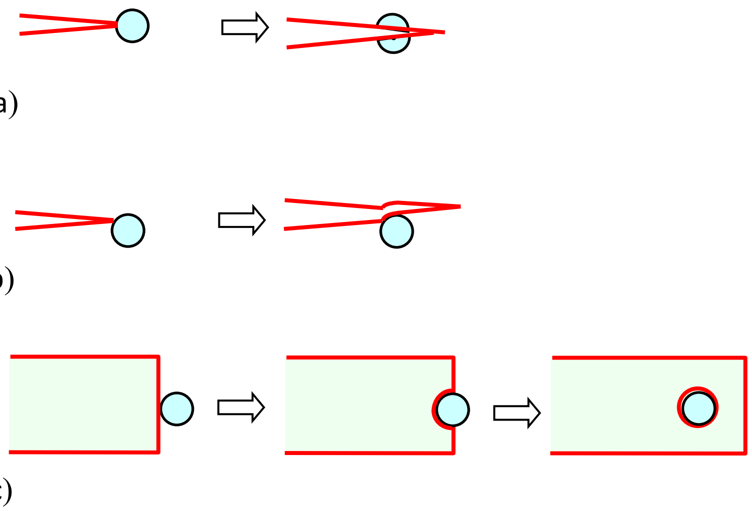

Fig. 2. Three modes of crack interaction with a spherical particle: (a) the crack cuts the particle; (b) the crack deflects and bends around the particle; (c) the crack envelops the particle, which eventually produces a bridge in the crack wake. Figures (a) and (b) display the plane normal to the crack plane, while figure (c) shows the crack plane

the particles are large enough to prevent cutting and assume that the crack does not cut the particles. Under this assumption, the crack propagates only inside the matrix, where tensile thermal stresses act in addition to the applied tensile stress σ_0. These stresses are temperature-dependent and promote crack growth inside the matrix. (It should be noted here that crack deflection and crack bridging hamper crack growth, and so particles can inhibit crack propagation, even though their thermal stresses promote crack growth.)

To calculate the effect of the thermal stresses on crack growth inside the matrix, as a first approximation, we employ the mean field theory, that is, treat the crack as advancing in a spatially uniform stress field equal to the average stress field acting in the matrix (and in those particles which are cut by the crack) in the crack plane. Within this theory, it is sufficient to consider the matrix as a homogeneously stressed medium, which is filled by the particles bypassed by the crack and characterized by the volume fraction f_V. The average stress field acting in the homogeneously stressed matrix incorporates the tensile stress σ_0 normal to the crack plane and the average thermal mismatch stress $\bar{\sigma}_{ij}^{m}$, where *i,j=x,y,z*, and *x*, *y* and *z* are the coordinate axes of a Cartesian coordinate system. In the following, we assume that the crack plane coincides with the (*xy*) plane of the coordinate system.

We assume that the fracture of the ceramic composite solid occurs when the total tensile stress acting in the direction normal to the macroscopic direction of crack propagation reaches a critical stress σ_c, which is temperature-dependent and incorporates the effects of crack bridging or crack deflection. This implies that the fracture criterion has the form $\sigma_0 + \bar{\sigma}_{zz}^{m} = \sigma_c$. We assume that the stress σ_c does not depend on the crack length. (For the case of crack bridging, this assumption is valid if the crack length exceeds the length of the bridging zone.) On the other hand, the condition of fracture can also be presented as $\sigma_0 = \sigma_f$, where σ_f is the fracture strength. From the two latter relations one obtains:

$$\sigma_f = \sigma_c - \bar{\sigma}_{zz}^m. \tag{1}$$

For elastically isotropic spherical particles characterized by the Young modulus E_p and Poisson's ratio ν_p in an elastically isotropic matrix with the Young modulus E_m and Poisson's ratio ν_m, the average stress $\bar{\sigma}_{zz}^m$ is given [26] by:

$$\bar{\sigma}_{zz}^m = \frac{2\delta f_V E_m \varepsilon_0}{(1-f_V)(\delta+2)(1+\nu_m)+3\delta f_V(1-\nu_m)}, \tag{2}$$

where $\delta = (1+\nu_m)E_p/[(1-2\nu_p)E_m]$, $\varepsilon_0 = \int_T^{T_s}[\alpha_m(T') - \alpha_p(T')]\,dT'$ and T_s is the sintering temperature.

To calculate the temperature dependence of the fracture strength σ_f, we employ the following equation for the critical stress σ_c [27]:

$$\sigma_c(T) = \sigma_c(T_0)\left\{\frac{E(T)}{E(T_0)}\left[1 - \frac{\int_{T_0}^{T} c_p(T')dT'}{\int_{T_0}^{T_m} c_p(T')dT' + \Delta H_m}\right]\right\}^{1/2}, \tag{3}$$

where T_0 is the arbitrary temperature, whose choice does not affect the critical stress $\sigma_c(T)$, T_m is the melting temperature, $E(T)$ is the Young modulus of the composite, $c_p(T)$ is the specific heat capacity, ΔH_m is the specific heat of melting. The Young modulus of the composite can be roughly estimated using the mixture rule as $E = E_p f_V + E_m(1-f_V)$.

Using Eqs. (1)–(3), let us calculate the temperature dependence of the fracture strength σ_f for the case of α-SiC particles in a ZrB_2 matrix. To do so, we exploit the following temperature dependence of the model parameters [23]:

$$E_m(T) = 507 - [2.54T + 1.9(T - 0.363T_m + |T - 0.363T_m|)]\exp[-T_m/T], \tag{4}$$

$$E_p(T) = 460 - 0.04T\exp[-962/T], \tag{5}$$

$$c_p(T) = 64.18 + 9.41\times10^{-3}T - 16.57\times10^{5}T^{-2}, \tag{6}$$

where E_m and E_p are given in GPa, T is given in K, c_p is given in $\mathrm{J/(mol\cdot K)}$.

The temperature dependence of the CTE can be written based on the experimental data [28,29] as:

$$\alpha_p(T) = (0.882 + 8.974\times10^{-3}T - 5.680\times10^{-6}T^2 + 1.264\times10^{-9}T^3)\times10^{-6}, \tag{7}$$

$$\alpha_m(T) = (6.40 + 7.65\times10^{-4}T - 3.62\times10^{-7}T^2 + 9.67\times10^{-11}T^3)\times10^{-6}, \tag{8}$$

where α_p and α_m are given in K^{-1}. We also put $\nu_m = 0.14$ [30], $\nu_p = 0.19$ [31], $T_m = 3323$ K [23], T_s=2223 K [23], T_0=300 K. Since the values of $\sigma_c(T_0)$ and ΔH_m for these composites are not known, they have been chosen to fit the calculated curve and the experimental data as follows: $\sigma_c(T_0) = 1.05$ GPa, $\Delta H_m = 50$ KJ/mol.

Results

The temperature dependence of the fracture strength σ_f is presented in Fig. 3(a), for ZrB_2–30 % SiC composite (with $f_V = 0.3$). The squares in this figure depict the experimental data from [10]. It is seen in Fig. 3(a) that the fracture strength first increases and then decreases with temperature and that the calculated curve agrees with the experimental data. An increase in σ_f with temperature up to 500 °C is related to a decrease in the thermal stresses. Since these stresses promote crack propagation, their decrease hinders crack growth and thereby increases the fracture strength σ_f. At the same time, an increase in σ_f with temperature due to a reduction of thermal stresses competes with a decrease in the fracture strength due to the reduction of the cohesive strength (which manifests itself by a decrease in the critical stress σ_c with temperature). Above a

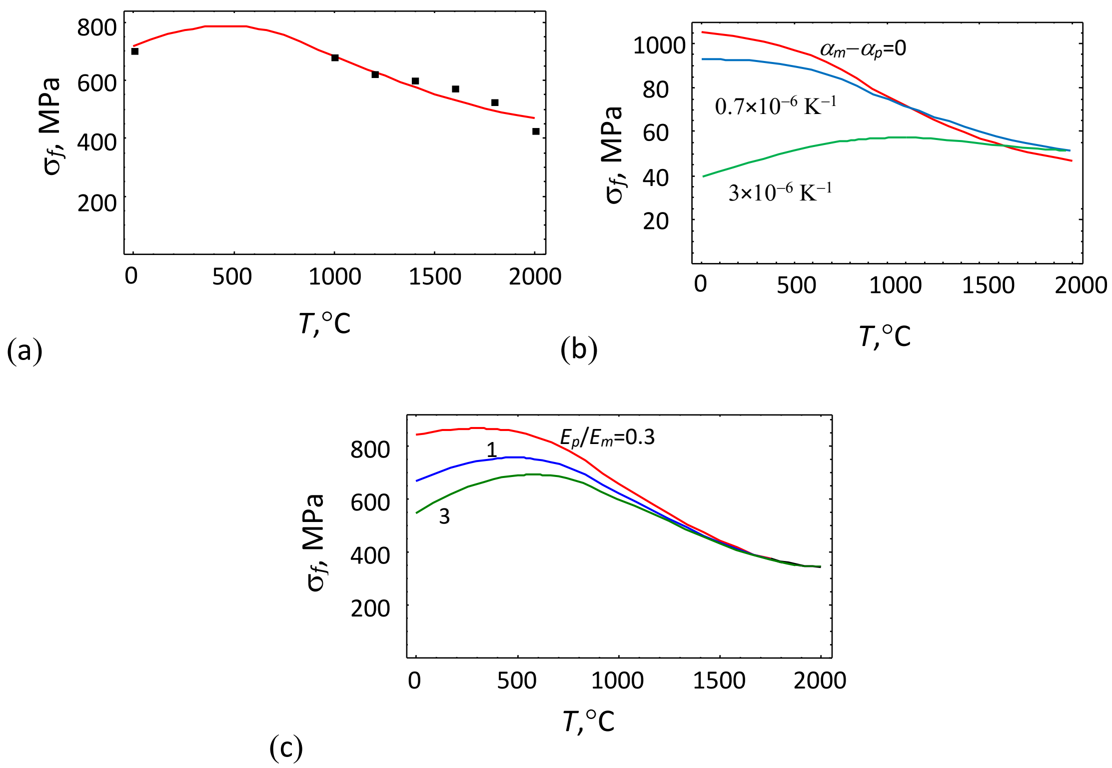

Fig. 3. Dependence of the fracture strength σ_f on temperature *T*, for the case of the ZrB_2–30 % SiC composite (a), for various values of $\alpha_m - \alpha_p$ (b), and for various values of E_p/E_m (c). The squares in figure (a) show the experimental data [10] for ZrB_2–30 % SiC–2 % B_4C

certain temperature (500 °C for the case shown in Fig. 3(a)) the latter factor (a reduction of the cohesive strength with temperature) becomes dominant, which leads to a decrease in the fracture strength σ_f with increasing temperature.

To demonstrate further that the presence of a maximum on the temperature dependence of the fracture strength σ_f is associated with the thermal stresses, Fig. 3(b) illustrates the model situation where the CTE α_m and α_p do not depend on temperature. Figure 3 displays the fracture strength σ_f vs temperature *T* for various values of the difference $\alpha_m - \alpha_p$ and the values of the other parameters specified above. It is seen in Fig. 3(b) that in the situation where the thermal stresses are absent ($\alpha_m - \alpha_p = 0$) or relatively small ($\alpha_m - \alpha_p = 0.7 \times 10^{-6}\ K^{-1}$), the fracture strength σ_f monotonously decreases with an increase in temperature. In contrast, if the thermal stresses are large enough ($\alpha_m - \alpha_p = 3 \times 10^{-6}\ K^{-1}$), the curve $\sigma_f(T)$ has a maximum.

The thermal stresses also depend on the ratio E_p/E_m of the Young moduli of the particles and the matrix. To elucidate the effect of the elastic moduli on fracture strength σ_f, Fig. 3(c) shows the model situation where the ratio E_p/E_m is temperature independent. The dependences of the fracture strength σ_f on temperature *T* for various values of E_p/E_m and the values of the other parameters specified above are presented in Fig. 3(c). This figure shows that the fracture strength σ_f increases with a decrease in E_p/E_m. This points to the effectiveness of a decrease in the Young modulus of the particles compared to that of the matrix for an increase in the fracture strength of ceramic matrix composites. At the same time, if the Young modulus of the particles becomes too small, their cohesive strength significantly decreases, and cracks stop to bypass particles. This, in turn, eliminates strengthening via crack deflection and bridging. Therefore, the maximum fracture strength should be achieved at intermediate values of the particle Young modulus, which are sufficiently small to reduce the thermal mismatch stresses but also sufficiently high to provide strengthening via crack deflection and bridging.

Conclusions

Thus, in the present paper, we have suggested a model that explains the experimentally observed non-monotonous temperature dependences of the fracture strength of ceramic matrix composites. Within the model, such dependences can be observed if the CTE of the matrix is significantly larger than that of the particles and if the crack cannot cut all the particles. The model also demonstrates that although moderate compressive thermal mismatch stresses in particles can be beneficial for toughening via crack deflection and bridging, high compressive stresses inside particles (that originate if the CTE of the matrix significantly exceeds that of the particles) can reduce the fracture strength of ceramic matrix composites. The high compressive stresses can be reduced by choosing the filler with a smaller CTE mismatch or smaller Young modulus.

CRediT authorship contribution statement

Alexander G. Sheinerman Sc: writing – review & editing, writing – original draft; conceptualization; investigation.

Conflict of interest

The author declares that he has no conflict of interest.

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