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Diagnostics of coating and adhesive properties on load-bearing elements of complex shape

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ABSTRACT

Protective coatings are used to solve many technical and economic problems. Information on known methods and approaches to studying the mechanical and adhesive properties of coatings is provided. It is noted that modern thin-walled structures, as a rule, have a complex geometry. In this case, protective coatings are formed directly on the surfaces of load-bearing elements. Known adhesion meters have a number of limitations. There are practically no works on determining the mechanical properties of coatings and adhesive formed on surfaces of complex shapes. An effective two-dimensional experimental - theoretical approach to diagnosing the rigidity and adhesive properties of a thin-layer coating formed directly on the surface of a load-bearing element of complex shape is described. At the theoretical stage of the study, the spline version of the finite element method is effective, when varying the properties of the material, we approach the shape of the experimental dome of the considered loading stage.

KEYWORDS

load-bearing element • protective coating • mechanical and adhesive properties • deformed surface diagnostics • complex structure and geometry

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Introduction

On the role of protective coatings

Since the 20 th century, thin-walled structures have garnered significant attention. To ensure the reliable operation of such structures, which are often exposed to diverse environments, physical fields, and substantial loads, maximizing the protection of their components from external influences is crucial. Protective coatings are commonly employed for this purpose. Research in this direction remains highly relevant.

Protective coatings are widely employed to address a multitude of technical and economic challenges. The degradation or failure of these protective coatings leads to the exposure of the surface of load-bearing structural components. Consequently, the load-bearing element becomes directly subjected to the effects of the surrounding environment and physical fields. This, in turn, results in the development of various corrosion defects, scratches, localized depressions, and the like on the surface of these structural components [1,2]. These defects induce alterations in the stiffness properties of thin-walled structural elements and give rise to stress concentrations in the defect regions [3,4].

The requisite properties of coatings, including smart coatings, are achieved through the development of sophisticated thin-film composite structures and adhesives [5–8]. Various methods and approaches have been devised for applying coatings to the surface of load-bearing components.

However, during structural operation, various defects also emerge within the coatings themselves, arising from the environment, physical fields, and deformation of the load-bearing component. Local delaminations also develop in multilayer composites. All of these factors induce significant alterations in the structure and stiffness properties of both the coating and the adhesive [9].

Modern coatings are typically formed directly onto the surfaces of load-bearing thin-walled structures, which generally possess a non-planar geometry. During operation, the coating deforms in concert with the load-bearing component. Research into the mechanical properties of coatings and adhesives on complex-shaped surfaces, considering the deformations of the load-bearing components, is notably scarce. Consequently, when determining the mechanical properties of coatings and adhesives, it is essential to account for the shape and deformation of the load-bearing component [10].

On methods and approaches for investigating the mechanical properties of coatings

When selecting coatings, adhesives, and their application technologies, based on a specified service life and operational conditions, questions arise concerning the determination of their structure, geometric parameters, and physico-mechanical characteristics. To ensure the integrity of coatings on structural components, it is crucial to effectively design and reliably assess both the initial mechanical properties and those acquired during operation for protective coatings and adhesives. The challenges of assessing the mechanical properties of coatings and the adhesion of coatings to load-bearing components, as well as investigating the patterns of change in coating and adhesion characteristics under the influence of the environment, physical fields, and operational factors, are highly relevant. The instrumentation for evaluating the mechanical properties of coatings remains underdeveloped. The "indenter" method, while capable of determining material properties in the vicinity of a point of interest, exhibits limited effectiveness when investigating coatings with complex structures [11]. In [12], the indentation method is noted as an effective tool for studying the mechanical properties of polyethylene. This method can be used to determine such material properties as hardness, elastic modulus and rheological characteristics. However, the "indenter" method, which allows determining the properties of the material in the vicinity of the point under consideration, is ineffective when studying coatings of complex structure. Indentation methods, as well as the stretch test method, which consists of stretching a material sample until it breaks to assess its strength and plasticity, are noted in [13]. Adhesion assessments based on the destruction of the coating in the area of application of the indenter are considered in [14,15]. The advent of powerful computers has facilitated the increasing prevalence of computational modeling. Molecular approaches to investigating the mechanical properties of thin structures are currently in their nascent stages. Difficulties arise in describing the complex structure and defects of coatings at the nano- and macro-scales [16]. Uniaxial tensile testing of specimens is a standard method [17–19]. However, when investigating the stiffness properties of complex-structured coatings, uniaxial testing often reveals a significant scatter in test results [20]. On the topic under consideration, there exist inventor's certificates (A.c.) and patents for inventions, including: A.c. 1742671 USSR, publ. 23.06.92; A.c. 1458766 USSR, publ. 5.02.89; SU 601599 A, 05.04.1978; SU 1441243 A1, 30.11.1988; SU 765697 A, 23.09.1980; US Patents US 5764068A, 09.06.1998 and Japan JP 8313422 A, 29.11.1996, which provide solutions to certain questions related to this subject. When investigating the properties of complex-structured coatings, the experimental-theoretical method of investigation proves indispensable. In particular, for studying initially flat thin coatings with complex structures, a two-dimensional shell approach is recommended [21]. This method can also be employed to determine the mechanical characteristics of nano-coatings within a "coating-substrate" system without separating the coating from the substrate [22]. The experimental-theoretical method has been further developed for investigating initially spherical and cylindrical coatings with variable radii [23].

On approaches to determining the adhesion of coatings to a load-bearing element

Determining the adhesion of coatings to load-bearing structural components has received considerable attention. A review of standard methods for assessing the adhesive strength of special coatings is given in [24]. The interaction between the adhesion strength and the tensile properties of coated laminates is noted in [25]. It is obvious that the adhesion strength of the coating is affected by both the properties and the technology of coating application [26,27]. An analysis of some adhesion assessment methods for heat-protective coatings is given in [28]. A technique for assessing the adhesion of thin-film coatings is considered in [29]. Some aspects of increasing the adhesion of metal coatings are presented in [30]. The effect of temperature on the adhesion of films to substrates is noted in [31]. The choice of adhesion assessment method for antifriction coatings is considered in [32]. In [33], it is noted that grinding and sandblasting are effective in improving the adhesion strength.

The following methods are known for determining adhesion properties: the tear-off method, which allows determining, in particular, the adhesion of the paint and varnish coating to various substrates; the method of lattice cuts, including *X*-shaped cuts and parallel cuts for visual assessment of coating delamination.

Methods have been developed to determine the adhesion strength of coatings to substrates, as evidenced by inventor's certificates and patents, including: A.c. USSR No. 183459, publ. 17.06.1966; Patent RF No. 689411, publ. 10.05.1995; Patent RF No. 2207544, publ. 27.06.2003. However, these methods suffer from drawbacks such as low accuracy, technological complexity, and low throughput. A known method for assessing the adhesion of elastic films utilizes "bubble" parameters [34], but this approach, along with its inherent limitations, exhibits a scatter in results.

Specialized instruments known as adhesiometers (e.g., PSO-XMG series) are available for evaluating coating adhesion to substrates [35]. However, these devices exhibit limitations, notably the difficulty in ensuring measurement consistency when studying the influence of different factors, among others.

A method for evaluating coating adhesion to a flat substrate has been developed [36]. Further developments of this method, refining the results of adhesion studies on substrates, are detailed in [37].

Modern thin-walled structures are typically characterized by complex geometries, harmoniously blending functional purpose with architectural expressiveness. In these designs, protective coatings are formed directly on the surfaces of load-bearing elements. Notably, research is scarce regarding the determination of mechanical properties of coatings and adhesives on complexly shaped surfaces.

An algorithm for diagnosing the mechanical and adhesive properties of a coating system on a complex surface of a structural element is outlined below. This algorithm assumes the availability of experimental coating pull-off data, specifically the parameters defining the dome shape and base as a function of air pressure introduced through a central aperture in the structural element.

Algorithm for determining mechanical and adhesion properties of a coating on a load-bearing surface of a structural element of complex shape

Numerical investigation tool

Established software packages can be utilized during the theoretical phase of the investigation. However, when examining the mechanical and adhesive properties of a coating applied to the surface of a load-bearing element with complex geometry, the spline-based finite element method proves most effective [38].

A load-bearing structural element of complex geometry 1 with a protective coating 2 is under consideration (Fig. 1(a)). The structural element 1 is parameterized by parameters t^1, t^2, t^3 of a parallelepiped 3 [39], where:

, (1)

and the coating 2 is parameterized by parameters t^1, t^2 of a rectangle 4 (Fig. 1(b)) [37–39]:

$$r = \bar{r}(t^1, t^2). \quad (2)$$

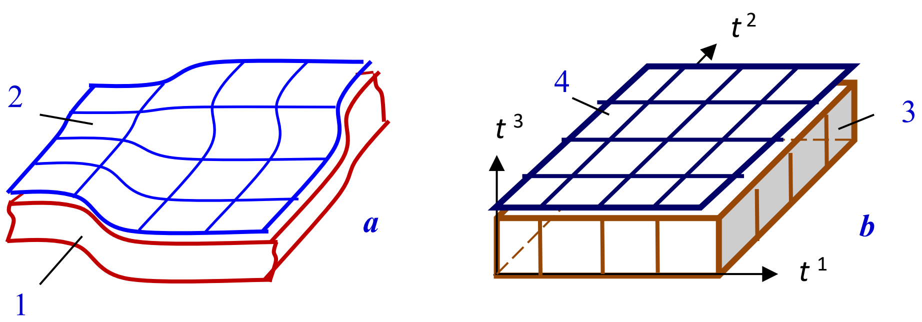

Fig. 1. A scheme of parametrization

A distinction is made between thin-walled structural elements of complex canonical geometry, where the mid-surface is described by analytical formulas, and elements of complex non-canonical geometry, where the mid-surface is not described analytically, but rather specified point-by-point. When considering thin-walled elements with complex non-canonical geometry, challenges arise in the parameterization stage [40]. For cases where the structural element geometry is not analytically described, an experimental parameterization approach can be employed, as exemplified by Patents of the Russian Federation Nos. 2374697 and 2665499.

By differentiating expression (1) with respect to t^1 и t^2 for all nodal and integration points, the coordinate vectors $\bar{r}_1$ и $\bar{r}_2$, the first fundamental metric tensor a_{ij} and the fundamental determinant a are calculated for the coating:

$$\bar{r}_1 = \frac{\partial \bar{r}}{\partial t^1},\ \bar{r}_2 = \frac{\partial \bar{r}}{\partial t^2},\ \bar{m} = \frac{[\bar{r}_1, \bar{r}_2]}{\sqrt{a}},\ a_{11} = \bar{r}_1\bar{r}_1,\ a_{12} = \bar{r}_1\bar{r}_2,\ a_{22} = \bar{r}_2\bar{r}_2,\ a = a_{11}a_{22} - a_{12}^2. \quad (3)$$

Subsequently, Christoffel symbols of the second kind are calculated:

$$a\Gamma_{11}^1 = \frac{a_{22}}{2}\frac{\partial a_{11}}{\partial t^1} - a_{12}\left(\frac{\partial a_{12}}{\partial t^1} - \frac{1}{2}\frac{\partial a_{11}}{\partial t^2}\right), a\Gamma_{12}^1 = \frac{1}{2}\left(a_{22}\frac{\partial a_{11}}{\partial t^2} - a_{12}\frac{\partial a_{22}}{\partial t^1}\right),$$
$$a\Gamma_{22}^1 = a_{22}\left(\frac{\partial a_{12}}{\partial t^2} - \frac{1}{2}\frac{\partial a_{22}}{\partial t^1}\right) - \frac{a_{12}}{2}\frac{\partial a_{22}}{\partial t^2}, a\Gamma_{11}^2 = a_{11}\left(\frac{\partial a_{12}}{\partial t^1} - \frac{1}{2}\frac{\partial a_{11}}{\partial t^2}\right) - \frac{a_{12}}{2}\frac{\partial a_{11}}{\partial t^1}, \quad (4)$$
$$a\Gamma_{12}^1 = \frac{1}{2}\left(a_{11}\frac{\partial a_{22}}{\partial t^1} - a_{12}\frac{\partial a_{11}}{\partial t^2}\right), a\Gamma_{22}^2 = \frac{a_{11}}{2}\frac{\partial a_{22}}{\partial t^2} - a_{12}\left(\frac{\partial a_{12}}{\partial t^2} - \frac{1}{2}\frac{\partial a_{22}}{\partial t^1}\right),$$

along with the components of the second fundamental metric tensor b_{ij}:

$$b_{11} = \bar{m}\frac{\partial^2 \bar{r}}{(\partial t^1)^2},\ b_{12} = \bar{m}\frac{\partial^2 \bar{r}}{\partial t^1 \partial t^2},\ b_{22} = \bar{m}\frac{\partial^2 \bar{r}}{(\partial t^2)^2}. \quad (5)$$

Tangential forces T^{ik} and bending moments M^{ik} for the coating in a physically linear formulation can be expressed as [41]:

$$T^{ik} = \frac{Eh}{2(1-\nu^2)}(a^{ij}a^{ks} + \nu c^{ij}c^{ks})\left[\nabla_j u_s - b_{js}w + \nabla_s u_j - b_{sj}w + (\nabla_j w + b_j^l u_l)(\nabla_s w + b_j^m u_m)\right],$$
$$M^{ik} = \frac{Eh^3}{12(1-\nu^2)}\left(a^{ij}a^{ks} + \nu c^{ij}c^{ks}\right)\left[-\nabla_j(\nabla_s w + b_s^l u_l) - b_j^m(\nabla_s u_m - b_{sm}w)\right], \quad (6)$$
$$c^{ii} = 0,\ c^{12} = -c^{21} = 1/\sqrt{a};\ i, j, k, s, l, m = 1,2,$$

where T^{ik}, M^{ik} are tangential forces and bending moments, E is the elastic modulus of of the coating; ν is Poisson's ratio, h is the coating thickness, $u = u_1$, $v = u_2$, w are the components of displacements of the middle surface of the coating, ∇_i is the symbol of covariant differentiation with respect to a_{ij}; a^{ij} are the contravariant components of the first fundamental metric tensor.

The governing relations are derived from Lagrange's variational equation:

$$\delta W - \delta A = 0, \quad (7)$$

where δW is the variation of the shell's strain potential energy, δA is the variation of the work done by external forces acting on the structural element.

The solution within each mesh cell is represented using an interpolating Hermite bicubic spline, and the problem is then reduced to solving a system of algebraic equations:

$$[B]\{U\} = \{R\}, \quad (8)$$

where $[B]$ is the stiffness matrix of a system with a banded structure, $\{U\}$ is the vector of unknowns, $\{R\}$ is the load vector.

Diagnostic algorithm

Initially, the stiffness characteristics of the coating are investigated. The components of parameterization are calculated using Eqs. (1)–(5) for the initial state of the load-bearing element with a coating (Fig. 1). Further, for each loading stage, based on the experimentally determined coordinates of the dome base (Fig. 2), a numerical model of coating deformation under pressure p is constructed – all components of parameterization are calculated using Eqs. (1)–(5); in nodes coinciding with the dome base, displacements are zeroed, the load is applied to elements located under the dome, and the stress-strain state calculation is performed according to Eq. (8).

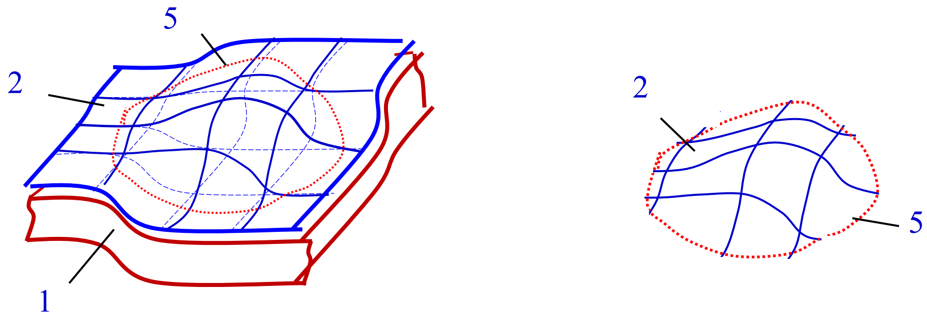

Fig. 2. Formation of a dome

The source material employed in this study was Class F fly ash sourced from Raichur Thermal Power Corporation Limited (RTPCL), Karnataka, India. The fly ash confirmed to IS 3812 2003 (part 1&2) [26].

At each loading stage p, using the iterative adjustment method– by varying material properties (Young's modulus E and Poisson's ratio ν), the shape of the experimental dome at the considered loading stage is approached. This determines process the distribution of the stress-strain state of the coating. When required, curves "strain ε – stress σ", "Young's modulus E – strain ε" are constructed. Thus, the mechanical properties of the coating are determined.

Next, the adhesion properties of the coating to the surface of the load-bearing element are investigated. In general, the coating detachment forces T_{otr} vary significantly along the contour. Knowing the tangential forces T^{ik} at the calculation nodes along line 5 of the dome base (Fig. 2) from the current loading stage solution, we can determine the local value of the normal adhesion stress η_{otr} at the surface of the load-bearing element 1 (Fig. 1) using the following equation:

$$\eta_{otr} = T_{otr}(\bar{m}_0 \times \bar{m}_i)/\,[h_0(1-\varepsilon_1-\varepsilon_2)], \tag{9}$$

where $T_{otr} = f\,(T^{ik})$ is the detachment force, which depends on the values of tangential forces T^{ik} at each point along line 5 of the dome base; h_0 is the coating thickness before deformation, ε_1, ε_2 is the coating deformation in the normal and tangential directions along line 5 of the dome base.

Conclusions

Employing protective coatings is an effective strategy for safeguarding structural load-bearing elements from environmental and physical field exposure. The requisite properties of coatings, including smart coatings, are achieved through the development of sophisticated composite structures and adhesives.

Selecting coatings, adhesives, and their application technologies raises questions regarding the determination of their parameters. The available tools for assessing the mechanical properties of coatings remain underdeveloped.

To determine the mechanical and adhesive properties of a coating system with a complex structure applied to a load-bearing element with complex geometry, a two-dimensional approach proves effective. This approach involves an experimental-theoretical method rooted in a spline-based finite element method (FEM).

An algorithm for diagnosing the mechanical and adhesive properties of coatings on the surface of complexly shaped load-bearing structural elements is described. This algorithm serves as a reliable tool for researchers, designers, and practicing engineers alike.

CRediT authorship contribution statement

Samat N. Yakupov Sc: writing – review & editing, writing – original draft.

Conflict of interest

The author declares that he has no conflict of interest.

References

1. Collins JA. *Failure of Materials in Mechanical Design. Analysis, Prediction, Prevention.* NewYork: John Wiley & Sons; 1981.
2. Yakupov SN. Influence of Scratches on the Stiffness Properties of Thin-walled Elements. *Lobachevskii Journal of Mathematics.* 2019;40(6): 834–839.
3. Neuber H. *Theory of Notch Stress.* Michigan: J.W. Edwards; 1946.
4. Peterson RE. *Stress Concentration Factors.* New York: John Wiley & Sons; 1974.
5. Montemor MF. Functional and smart coatings for corrosion protection: A review of recent advances. *Surface and Coatings Technology.* 2014;258: 17–37.
6. Taylor SR, Shiflet GJ, Scully JR, Buchheit RG, W, Van Ooij WG, Sieradzki K, Diaz R, Brinker CJ, Moran AL. Increasing coating functionality using nanodimensioned materials. In: *Nanotechnology Applications in Coatings. (ACS Symposium Series).* American Chemical Society; 2009. p. 126–155.
7. Yakupov SN, Yakupov NM. Thin-layer films and coatings. *Journal of Physics: Conference Series.* 2017;857: 012056.
8. Belyaev AK, Galyautdinova AR, Smirnov SA, Bessonov NM. Analytical and numerical solution to the problem of hydrogen diffusion in rotating cylindrical elastic bodies. *Materials Physics and Mechanics.* 2022;50(1): 56–65.
9. Yakupov NM, Kharislamova LU. Stiffness of Compositions with Delaminations and the Influence of Ultraviolet on Adhesion. *Lobachevskii Journal of Mathematics.* 2019;40(6): 840–845.
10. Yakupov SN, Gubaidullin RI, Yakupov NM. Investigation of the influence of the nature of surface deformation on coating adhesion. *Journal of Physics: Conference.* 2021;1954: 012053.
11. Tsui TY, Oliver W, Pharr GM. Nanoindentation Hardness of Soft Films on Hard Substrates: Effects of the Substrate. *MRS Online Proceedings Library.* 1997;473: 57–62.
12. Vinogradova AA, Gogolinskiy KV, Shchiptsova EK. A study of applicability of instrumented indentation method to determine the mechanical properties of thermoplastic. *Industrial laboratory. Diagnostics of materials.* 2025;91(5): 67–76. (In Russian)
13. Wang YM, Weng WX, Chi MH, Lui BL, Li Q. Investigation into the evolution of interface fracture toughness of thermal barrier coatings with thermal exposure treatment by wedge indentation. *Journal of Materials Research.* 2020;35: 1715–1725.
14. Vidakis N, Antoniadis A, Bilalis N. The VDI 3198 indentation test evaluation of a reliable qualitative for layered compounds. *Journal of Materials Processing Technology.* 2003;143–144: 481–485.
15. Hatic D, Cheng X, Stephani T, Rauhut M, Gäbler J, Bethke R, King H, Hagen H. Use of machine learning for automatic Rockwell adhesion test classification based on descriptive and quantitative features. *Surface and Coatings Technology.* 2021;427: 127762.
16. Yanovsky YuG, Nikitina EA, Nikitin SM, Karnet YuN. Quantum mechanical studies of the mechanism of deformation of carbon nanotubes. *Nanoscience and Technology: An International Journal.* 2010;1(1): 1–29.

17. State Standard. GOST 14236-81. *Polymer films. Tensile testing method.* Moscow: Publishing house of standards; 1989. (In Russian)
18. State Standard. GOST 29088-91. *Polymer cellular elastic materials. Determination of conditional strength and elongation at break.* Moscow: Publishing house of standards; 1991. (In Russian)
19. American Society for Testing and Materials. ASTM D 412-41. *Standard Test Methods for Vulcanized Rubber and Thermoplastic Elastomers-Tension.* West Conshohocken; 2016.
20. Kupriyanov VN. *Film – fabric materials for building structures.* Kazan: KISI; 1989.
21. Jakupov NM, Nurullin RG, Nurgaliev AR, Jakupov SN. *Method of determining strength of thin-layer materials.* Russian Patent RU 2310184 C2, 10 November 2007. (In Russian)
22. Yakupov SN. Mechanical characteristics of thin titanium oxide coatings in the “coating – polymer film” system. *Mechanics of Composite Materials and Structures*. 2010;16: 436–444. (In Russian)
23. Yakupov NM, Galimov NK, Yakupov SN. Methodology of studying non-planar films and membranes of complex structure. *Zavodskaya Laboratoriya. Diagnostika Materialov.* 2019;85(2): 55–59. (In Russian)
24. Zaytsev AN, Aleksandrova YP, Yagopolskiy AG. Comparative Analysis of Methods for Assessing Adhesion Strength of Thermal Spray Coatings. *BMSTU Journal of Mechanical Engineering.* 2021;5: 48–59. (In Russian)
25. Luo X, Smith GM, Sampath S. On the Interplay between Adhesion Strength and Tensile Properties of Thermal Spray Coated Laminates– Part I: High Velocity Thermal Spray Coatings. *Journal of Thermal Spray Technology.* 2018;27: 296–307.
26. Mittal KL. *Adhesion Measurement of Films and Coatings.* London: CRC Press; 1995.
27. Mittal KL. *Adhesion Measurement of Films and Coatings, Volume 2.* London: CRC Press;2001.
28. Fedorova EN, Sukhodoeva NV, Moskvichev VV, Ogoreltceva NV, Klimkin YuO. Methods for characterizing the interfacial adhesion in thermal barrier coating systems. *Industrial laboratory. Diagnostics of materials.* 2022;88(12): 51–63. (In Russian)
29. Topolyanskiy P, Ermakov S, Topolyansky A. Adhesion characteristics of thin-film coating deposited by finishing plasma hardening. *Voronezh Scientific-Technical Bulletin.* 2022;3(3): 11–27. (In Russian)
30. Kolesnik LL, Zhuleva TS, Predtechenskiy PO, Hlaing MK, Aung ZP. Processing of metallization technology aluminum oxide ceramics for electro-vacuum devices elements and power electronics devices. *Journal of Physics: Conference Series.* 2017;872: 012018.
31. Bovaldinova KA, Feldstein MM, Sherstneva NE, Moscalets AP, Khokhlov AR. Thermo-switchable pressure-sensitive adhesives with strong tunable adhesion towards substrate surfaces of different hydrophilicity. *Polymer.* 2017;125: 10–20.
32 Lunin EA, Zakharova NV, Trifanov IV. Selecting a method for determining the adhesion of an antifriction coating. *Current issues in aviation and astronautics.* 2019;2: 613–615. (In Russian)
33. Ito K, Shima T, Fujioka M. Improvement of Oxidation Resistance and Adhesion Strength of Thermal Barrier Coating by Grinding and Grit-Blasting Treatments. *Journal of Thermal Spray Technology.* 2020;29: 1728–1740.
34. Bilbi B. Fracture. In: Taplin J. (eds.). *Fracture Mechanics. The destruction of the materials.* Moscow: Mir;1979. p.222–224. (In Russian)
35. Bousfield DW, Coleman PS, Hassler JC, Osgood AK. *Devices and methods for testing tack uniformity of a coating on a substrate.* US6050139A (Patent), 2000.
36. Gol'dshtejn RV, Jakupov NM, Nurullin RG, Jakupov SN, Jakupova RN. *Method of determining adhesion of film substrate.* RU 2421707 C1 (Patent), 2011. (In Russian)
37. Yakupov SN. Experimental-Theoretical Approach to Determining the Film–Substrate Adhesion. *Mechanic of Solids.* 2017;52: 587–593.
38. Yakupov NM. Method of calculating shells of complex geometry. *Research in Shell Theory. Seminar Proceedings.* 1984;17(2):4–17. (In Russian)
39. Yakupov SN, Kiyamov HG, Yakupov NM, Mukhamedova IZ. A new variant of the fem for evaluation the strenght of structures of complex geometry with heterogeneous material structure. *Case Studies in Construction Materials.* 2023;19: e02360.
40. Badriev IB, Paimushin VN. Refined models of contact interaction of a thin plate with positioned on both sides deformable foundations. *Lobachevskii Journal of Mathematics.* 2017;38: 779–793.
41. Galimov KZ. *Foundations of Nonlinear Theory of Thin Shells.* Kazan: Kazan University Press; 1975. (In Russian)