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Thermoelastic Rayleigh waves in an isotropic half-space

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ABSTRACT

The process of the Rayleigh wave propagation in an elastic isotropic half-space with consideration of the interaction between the temperature and the mechanical fields is studied. The mathematical setting of the problem includes a related system of equations of thermoelasticity in displacements. The solution is sought in the form of a progressing thermoelastic wave. A dispersion equation is obtained that relates the frequency of the propagating wave to the components of the wave vector, the solution of which is found analytically. Taking into account the trivial boundary conditions expressing the absence of mechanical loads and heat fluxes at the boundary of the half-space, an equation is formulated to determine the frequency of a thermoelastic Rayleigh wave. Analytical expressions for displacement amplitudes and temperature are obtained. A series of numerical calculations was performed for various materials. The calculation results are presented in the form of tables and graphs. The effect of the mechanical characteristics of materials on the amplitudes and frequencies of elastic and thermoelastic Rayleigh waves is studied. It is demonstrated that the interaction of mechanical and temperature fields in real materials is very weak, and the amplitudes and frequencies of Rayleigh waves coincide in the elastic and thermoelastic cases. Thermoelastic effects depend essentially on the linear thermal expansion coefficient, and the parameters of elastic and thermoelastic Rayleigh waves start to differ significantly as the latter increases.

KEYWORDS

isotropic half-space • isotropic semi-plane • Rayleigh wave • surface acoustic wave • thermoelasticity progressive wave • propagation of waves • dispersion equation

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Introduction

Rayleigh waves are surface acoustic waves named after Lord Rayleigh (John Strutt), who theorized them in 1885. Waves of this type propagate near the solid surface and the research of their properties is of great importance for seismic exploration in oil production [1] and when studying the elastic properties of rocks and soils located behind the tunnel lining, reinforced concrete slabs, masonry constructions, etc. [2]. Surface wave tomography, used to study the crustal structure, is based on the analysis of the group-velocity dispersion of the Love and Rayleigh waves [3]. In seismology, the use of the surface waves is associated with the study of crustal disturbances caused by earthquakes [4].

Since the middle of the 20th century, Rayleigh waves of the ultrasonic range (~ 10^8 Hz) have been used as the non-destructive inspection aids for the surface layer of the manufactured metal parts. Ultrasonic inspection was used to determine the depth of thermal hardening, the residual mechanical stresses, the surface treatment quality,

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the defects, etc. High-frequency Rayleigh surface waves are used in acoustoelectronics, which studies the interaction processes of the ultra- and hypersonic waves ($10^8 - 10^{10}$ Hz) with conduction electrons in crystals [5].

In addition to technology, the Rayleigh waves are also used in purely scientific experiments to study the surface properties of materials. Thus, the problem of the surface acoustic wave propagation in solids is relevant and practically significant, which takes attention of many researchers. Works [6–12] are devoted to the simulation of the Rayleigh wave propagation in continuous media, with account of the temperature and diffusion fields. In articles [13–17], surface acoustic waves in electromagnetic elastic media were investigated. The influence of piezoelectric effects on the propagation of Rayleigh waves is considered in [18,19], and papers [20–23] take into account the rotation of the medium. The influence of anisotropy on wave processes in continuous media is studied in [24–29]. Of these, papers [24–26] consider the propagation of the surface waves in transversally isotropic media, while the articles [27–29] study the propagation of surface waves in orthotropic media.

Almost all of these papers consider heat transfer models with a finite velocity of heat flux propagation based on the Lord-Shulman theory. The scientific reviews [30,31] have numerous references to experimental studies supporting the existence of wave heat-transfer processes in metals, crystals, polymers, biological tissues and mixtures. However, these effects exhibit themselves over very short spaces of time, about $10^{-14} \div 10^{-6}$ sec, and, given the stationary nature of Rayleigh waves, they are unlikely to have any effect on their propagation mode in practice.

On the other hand, the solutions given in [6–29] are more descriptive than specific. The proposed algorithms are essentially focused on the use of symbolic mathematics packages, Maple type, which perform all the necessary transformations and numerical calculations instead of the researcher. Herewith, the solution itself, written in the form of a specific formula, the truth of which can be verified by substituting it into the original problem, is missing in these works.

The paper introduces an analytical approach to the Rayleigh wave propagation in a thermoelastic isotropic half-space, corresponding to the traditional ideas of the "solution" concept. The amplitudes of the thermoelastic waves are explicitly defined, and the solution of the dispersion equation connecting the Rayleigh wave frequency with the components of the wave-number vector is obtained. An equation is explicitly obtained for the Rayleigh frequency, and a scheme for its approximate solution formulation is described. The proposed form of the solution makes it possible to make an efficient parametric analysis of the surface wave propagation depending on the medium properties and the degree of coupling of the mechanical and temperature fields.

General statement of the problem

Let there be an isotropic two-dimensional half-space (half-plane), the boundary of which is free from mechanical loads and heat fluxes. For mathematical description of the physical-mechanical processes in the considered medium, we use the following system of equations [32–34]:

$$(\lambda+2\mu)\frac{\partial^2 u_1}{\partial x_1^2}+\mu\frac{\partial^2 u_1}{\partial x_2^2}+(\lambda+\mu)\frac{\partial^2 u_2}{\partial x_1\partial x_2}-\Lambda\frac{\partial\vartheta}{\partial x_1}=\rho\ddot{u}_1,$$
$$(\lambda+\mu)\frac{\partial^2 u_1}{\partial x_1\partial x_2}+\mu\frac{\partial^2 u_2}{\partial x_1^2}+(\lambda+2\mu)\frac{\partial^2 u_2}{\partial x_2^2}-\Lambda\frac{\partial\vartheta}{\partial x_2}=\rho\ddot{u}_1, \quad (1)$$
$$\kappa\frac{\partial^2\vartheta}{\partial x_1^2}+\kappa\frac{\partial^2\vartheta}{\partial x_2^2}-T_0\Lambda\frac{\partial^2 u_1}{\partial t\partial x_1}-T_0\Lambda\frac{\partial^2 u_2}{\partial t\partial x_2}=\rho c_0\dot{\vartheta},$$

where t is time, x_i are rectangular Cartesian coordinates, u_i are displacement vector components, λ and μ are Lamé elastic constants, ρ is the density, $\Lambda=\alpha(3\lambda+2\mu)$ is the temperature constant, α is a linear thermal expansion coefficient, κ is a thermal conduction coefficient, $\vartheta=T-T_0$, T is the current ambient temperature, T_0 is an initial ambient temperature, c_0 is a specific thermal capacity.

To find Rayleigh waves, it is necessary to consider a boundary value problem involving equations (1) and the following boundary conditions, evaluating the absence of mechanical loads and heat fluxes at the boundary of the half-space:

$$\tau_{12}|_{x_2=0}=\mu\left(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}\right)\Big|_{x_2=0}=0, \quad q_2|_{x_2=0}=\kappa\frac{\partial\vartheta}{\partial x_2}\Big|_{x_2=0}=0,$$
$$\sigma_2|_{x_2=0}=\left(\lambda\frac{\partial u_1}{\partial x_1}+(\lambda+2\mu)\frac{\partial u_2}{\partial x_2}-\Lambda\vartheta\right)\Big|_{x_2=0}=0, \quad (2)$$
$$u_i,\vartheta\to O(1)\quad (x_2\to\infty).$$

Here σ_2 is the longitudinal stress, τ_{12} is the shearing stress, q_2 is the normal component of the heat flux vector, $x_2=0$ are the half-plane boundary.

Since the Rayleigh wave amplitude decays as it moves away from the surface at an exponential rate, and the wave energy is concentrated in the region at a distance of the order of the wavelength from the surface [6–8,35], we seek the solution of the problem (1) and (2) in the form of a progressive wave:

$$u_j=U_j e^{i(\omega t-k_1x_1)-k_2x_2}, \quad \vartheta=\Phi e^{i(\omega t-k_1x_1)-k_2x_2}, \quad (3)$$

where ω are the Rayleigh wave frequency to be determined, $k=\{k_1,k_2\}$ is a wave vector, in which k_1 is the given value, k_2 is to be determined.

For further description, we pass on to dimensionless correlations (if they have the same trace as their dimensional counterpart, they are indicated by the "*" symbol, which is omitted in further treatment):

$$u_i^*=\frac{u_i}{L}, \quad x_i^*=\frac{x_i}{L}, \quad \tau=\frac{Ct}{L}, \quad C=\sqrt{\frac{\lambda+2\mu}{\rho}}, \quad F_\vartheta^*=\frac{LF_\vartheta}{Cc_0T_0},$$
$$F_i^*=\frac{\rho LF_i}{\lambda+2\mu}, \quad \omega^*=\frac{L\omega}{C}, \lambda^*=\frac{\lambda}{\lambda+2\mu}, \quad \mu^*=\frac{\mu}{\lambda+2\mu}, \quad b=\frac{T_0\Lambda}{\lambda+2\mu}, \quad (4)$$
$$B=\frac{\Lambda}{\rho c_0}, \ \kappa^*=\frac{\kappa}{\rho c_0CL}, \quad \vartheta^*=\frac{\vartheta}{T_0}, \quad k_i^*=Lk_i.$$

Thereupon, the equations (1) and boundary conditions (2) are written as:

$$\frac{\partial^2 u_1}{\partial x_1^2}+\mu\frac{\partial^2 u_1}{\partial x_2^2}+(\lambda+\mu)\frac{\partial^2 u_2}{\partial x_1\partial x_2}-b\frac{\partial\vartheta}{\partial x_1}+F_1=\ddot{u}_1,$$
$$(\lambda+\mu)\frac{\partial^2 u_1}{\partial x_1\partial x_2}+\mu\frac{\partial^2 u_2}{\partial x_1^2}+\frac{\partial^2 u_2}{\partial x_2^2}-b\frac{\partial\vartheta}{\partial x_2}+F_2=\ddot{u}_2, \quad (5)$$
$$\kappa\frac{\partial^2\vartheta}{\partial x_1^2}+\kappa\frac{\partial^2\vartheta}{\partial x_2^2}-B\frac{\partial^2\dot{u}_1}{\partial x_1}-B\frac{\partial^2\dot{u}_2}{\partial x_2}+F_\vartheta=\dot{\vartheta}.$$

$$\tau_{12}|_{x_2=0}=\mu\left(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}\right)\Big|_{x_2=0}=0, \quad q_2|_{x_2=0}=\kappa\frac{\partial\vartheta}{\partial x_2}\Big|_{x_2=0}=0,$$
$$\sigma_2|_{x_2=0}=\left(\lambda\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}-b\vartheta\right)\Big|_{x_2=0}=0, \quad (6)$$
$$u_i,\vartheta\to O(1)\quad (x_2\to\infty).$$

Solution method

Substituting Eq. (3) in Eq. (5) we derive:

$$(-k_1^2+\mu k_2^2+\omega^2)U_1+i(\lambda+\mu)k_1k_2U_2+ibk_1\Phi=0,$$
$$i(\lambda+\mu)k_1k_2U_1+(-\mu k_1^2+k_2^2+\omega^2)U_2+bk_2\Phi=0, \quad (7)$$
$$-B\omega k_1U_1+iB\omega k_2U_2-(\kappa k_1^2-\kappa k_2^2+i\omega)\Phi=0.$$

Express Φ from the last equation and substitute it in the first two equalities. We get:

$$\Phi=B\omega\frac{ik_2U_2-k_1U_1}{i\omega-\kappa K^2}, \quad (8)$$
$$[\Pi(\omega,K)-k_1^2Q(\omega,K)]U_1+ik_1k_2Q(\omega,K)U_2=0,$$
$$ik_1k_2Q(\omega,K)U_1+[\Pi(\omega,K)+k_2^2Q(\omega,K)]U_2=0, \quad (9)$$

where

$$\Pi(\omega,K)=(i\omega-\kappa K^2)(\mu K^2+\omega^2),$$
$$Q(\omega,K)=(\lambda+\mu)(i\omega-\kappa K^2)+ibB\omega, \quad (10)$$
$$K=\sqrt{k_2^2-k_1^2}\in\mathbb{C}.$$

For a non-trivial system solution (9) to exist, it is necessary to request its determinant to be next to none. We arrive at what is known as the dispersion equation

$$\begin{vmatrix}\Pi(\omega,K)-k_1^2Q(\omega,K) & ik_1k_2Q(\omega,K)\\ ik_1k_2Q(\omega,K) & \Pi(\omega,K)+k_2^2Q(\omega,K)\end{vmatrix}=\Pi(\omega,K)[\Pi(\omega,K)+K^2Q(\omega,K)]=0, \quad (11)$$

which, taking into account the agreed notations (10), is equivalent to the following set of equalities:

$$(i\omega-\kappa K^2)(\mu K^2+\omega^2)=0, \quad -\kappa K^4+(i(1+bB)-\kappa\omega)\omega K^2+i\omega^3=0. \quad (12)$$

Based on the equality (8) $\kappa K^2-i\omega\neq0$. In addition, taking account of (3) $\lambda+2\mu=1$. Denoting $s=K^2$, this system is written as follows:

$$(\mu s+\omega^2)=0,\ \kappa s^2-(i(1+bB)-\kappa\omega)\omega s-i\omega^3=0. \quad (13)$$

We get the root equation from the first equation:

$$s_1=-\frac{\omega^2}{\mu}. \quad (14)$$

From the second equation (13) we determine:

$$s_2=\omega\frac{i-\kappa\omega+ibB-\sqrt{(i-\kappa\omega+ibB)^2+4i\kappa\omega}}{2\kappa},\ s_3=\omega\frac{i-\kappa\omega+ibB+\sqrt{(i-\kappa\omega+ibB)^2+4i\kappa\omega}}{2\kappa}. \quad (15)$$

Using the solutions of the dispersion equation, we determine the values of k_{2j} of the unknown component k_2 of the wave vector k. These values are complex, as well as ω frequency-dependent. According to the last equality in Eq. (10) let us introduce the following notations:

$$K_j=\sqrt{s_j},\quad k_{2j}=\sqrt{s_j+k_1^2}\quad (j=\overline{1,3}). \quad (16)$$

Amplitude factors U_1 and U_2 are related using the equation system (9). Using the first equality and considering that $\Pi(\omega,K_1)=0$, we have:

$$U_{21}=i\frac{\Pi(\omega,K_1)-k_1^2Q(\omega,K_1)}{k_1k_{21}Q(\omega,K_1)}U_{11}=-i\frac{k_1}{k_{21}}U_{11}. \quad (17)$$

For the rest of the roots $\Pi(\omega,K_j)+K_j^2Q(\omega,K_j)=0$ $(j=2,3)$, then:

$$U_{2j}=i\frac{\Pi(\omega,K_j)-k_1^2Q(\omega,K_j)}{k_1k_{2j}Q(\omega,K_j)}U_{1j}=i\frac{\Pi(\omega,K_j)-\left(k_1^2+K_j^2-K_j^2\right)Q(\omega,K_j)}{k_1k_{2j}Q(\omega,K_j)}U_{1j}=$$
$$=i\frac{\Pi(\omega,K_j)-\left(k_1^2-K_j^2\right)Q(\omega,K_j)}{k_1k_{2j}Q(\omega,K_j)}U_{1j}=i\frac{-k_{2j}^2Q(\omega,K_j)}{k_1k_{2j}Q(\omega,K_j)}U_{1j}=-i\frac{k_{2j}}{k_1}U_{1j}\quad (j=2,3) \quad (18)$$

Omitting further the first index of the values U_{1j}, taking into account Eq. (8), we finally obtain:

$$\Phi_1 = B\omega\frac{ik_{21}U_2-k_1U_1}{i\omega-\kappa K_1^2}=0, \quad \Phi_j = B\omega\frac{ik_{2j}U_2-k_1U_1}{i\omega-\kappa K_j^2}=\frac{B\omega K_j^2U_j}{k_1\left(i\omega-\kappa K_j^2\right)} \quad (j=2,3). \tag{19}$$

Substituting the determined values Eqs. (17)–(19) to Eq. (3) we determine:

$$\begin{aligned} u_1 &= \sum_{j=1}^{3} U_j e^{i(\omega t-k_1x_1)-k_{2j}x_2}, \\ u_2 &= \sum_{j=1}^{3} A_jU_j e^{i(\omega t-k_1x_1)-k_{2j}x_2}, \\ \vartheta &= \sum_{j=1}^{3} B_jU_j e^{i(\omega t-k_1x_1)-k_{2j}x_2}, \end{aligned} \tag{20}$$

where

$$A_1 = -i\frac{k_1}{k_{21}}, \quad B_1=0, \quad A_j=-i\frac{k_{2j}}{k_1}, \quad B_j=\frac{B\omega K_j^2}{k_1\left(i\omega-\kappa K_j^2\right)} \quad (j=2,3). \tag{21}$$

To find ω frequency, we use the solution (20) in the boundary conditions (6). As a result, we arrive at the system of equations below with respect to U_j parameters:

$$\sum_{j=1}^{3}\left(k_{2j}+ik_1A_j\right)U_j=0, \quad \sum_{j=1}^{3}\left(i\lambda k_1+k_{2j}A_j+bB_j\right)U_j=0, \quad \sum_{j=1}^{3}B_jk_{2j}U_j=0. \tag{22}$$

The matrix of this system is written as:

$$A=\begin{pmatrix} k_{21}^2+k_1^2 & 2k_{22}k_{21} & 2k_{23}k_{21} \\ i(\lambda-1)k_1^2 & i(\lambda k_1^2-k_{22}^2)+bB_2k_1 & i(\lambda k_1^2-k_{23}^2)+bB_3k_1 \\ 0 & B_2k_{22} & B_3k_{23} \end{pmatrix}.$$

With the aid of elementary transformations, it is reduced to

$$S=\begin{pmatrix} 1 & \frac{2k_{22}k_{21}}{k_{21}^2+k_1^2} & \frac{2k_{23}k_{21}}{k_{21}^2+k_1^2} \\ 0 & 1 & \frac{B_3k_{23}}{B_2k_{22}} \\ 0 & 0 & \Delta(\omega) \end{pmatrix},$$

where $\Delta(\omega)=\frac{(k_{21}^2+k_1^2)[i(\lambda k_1^2-k_{23}^2)+bB_3k_1]-2ik_{23}k_{21}(\lambda-1)k_1^2}{(k_{21}^2+k_1^2)[i(\lambda k_1^2-k_{22}^2)+bB_2k_1]-2ik_{22}k_{21}(\lambda-1)k_1^2}-\frac{B_3k_{23}}{B_2k_{22}}$, is the system determinant with the reduced matrix S.

For the availability of a non-trivial system solution (22) it is necessary to request the S matrix determinant to be next to none. After transformation of the last equality by its reduction to a common denominator, we obtain the following equation:

$$\begin{aligned} f(\omega) &= B_2k_{22}\{(k_{21}^2+k_1^2)[i(\lambda k_1^2-k_{23}^2)+bB_3k_1]-2ik_{23}k_{21}(\lambda-1)k_1^2\}- \\ &-B_3k_{23}\{(k_{21}^2+k_1^2)[i(\lambda k_1^2-k_{22}^2)+bB_2k_1]-2ik_{22}k_{21}(\lambda-1)k_1^2\}=0. \end{aligned} \tag{23}$$

This equation contains numerous radicals and its solution, even by numerical methods, is very difficult. Let us proceed as follows. Taking advantage of the bB product smallness, we omit all summands proportional to this value in Eq. (23). Herewith, the values k_{2i} and s_i will be roughly determined as follows:

$$\begin{aligned} s_1 &= -\frac{\omega^2}{\mu}, \quad k_{21}=\sqrt{k_1^2-\frac{\omega^2}{\mu}}, \\ s_2 &= \omega\frac{i-\kappa\omega+ibB-\sqrt{(i-\kappa\omega+ibB)^2+4i\kappa\omega}}{2\kappa}\approx\omega\frac{i-\kappa\omega-i-\kappa\omega}{2\kappa}=-\omega^2, \quad k_{22}\approx\sqrt{k_1^2-\omega^2}, \\ s_3 &= \omega\frac{i-\kappa\omega+ibB+\sqrt{(i-\kappa\omega+ibB)^2+4i\kappa\omega}}{2\kappa}\approx\omega\frac{i-\kappa\omega+i+\kappa\omega}{2\kappa}=\frac{i\omega}{\kappa}, \quad k_{23}\approx\sqrt{k_1^2+\frac{i\omega}{\kappa}}. \end{aligned} \tag{24}$$

Considering these equalities $i\omega-\kappa s_3\approx i\omega-i\omega=0$, $i\omega-\kappa s_2=-\kappa(s_2-s_3)$ Eq. (23) is as follows:

$$(k_{21}^2+k_1^2)(\lambda k_1^2-k_{22}^2)+2k_{22}k_{21}k_1^2(1-\lambda)=0, \tag{25}$$

which, in general, corresponds to the purely elastic case.

From Eq. (24) we determine the approximate value of the Rayleigh wave frequency, which we denote as ω_{el}. Further refinement of the obtained value is carried out as follows. Equation (23) is written in an equivalent form:

$$\omega = \varphi(\omega), \tag{26}$$

where the function $\varphi(\omega)$ is defined as follows:

$$\varphi(\omega) = \left[\omega^{\beta} + \gamma f^{\delta}(\omega)\right]^{\frac{1}{\beta}}, \quad \delta > 0, \tag{27}$$

where β, γ and δ are numerical parameters.

Next, an iterative sequence is constructed:

$$\omega_n = \varphi(\omega_{n-1}) \quad (n \in \mathbb{N}), \tag{28}$$

where, as an initial approximation, we assume:

$$\omega_0 = \omega_{el}. \tag{29}$$

Numerical calculations show that with a certain choice of parameters β, γ and δ, in the vicinity of the value ω_{el} the condition is satisfied $\left|\varphi'(\omega)\right| < 1$, which ensures the convergence of the iterative sequence (28) to the solution of Eq. (26) for a given initial approximation (29).

Thus, the frequency of the Rayleigh thermoelastic wave ω_T is found using the iterative procedure Eqs. (28) and (29). Then we find the amplitude values U_j from the system (22):

$$U_2 = -\frac{B_3 k_{23}}{B_2 k_{22}} U_3, \quad U_1 = -\frac{2k_{22}k_{21}}{k_{21}^2+k_1^2} U_2 - \frac{2k_{23}k_{21}}{k_{21}^2+k_1^2} U_3, \quad U_3 \in \mathbb{R}. \tag{30}$$

Here, k_{2i} and s_i values are already found by Eqs. (14)–(16). Parameters k_1 and U_3 are determined by the external disturbing factors, generating a Rayleigh wave.

Numerical calculations

For calculation, let us consider the half-space, made of duralumin, with the following characteristics [36]:

$$\lambda = 6.93 \cdot 10^{10} \frac{N}{m^2}, \quad \mu = 2{,}56 \cdot 10^{10} \frac{N}{m^2}, \quad \rho = 2780 \ \frac{kg}{m^3}, \quad T_0 = 300 \ K, \tag{31}$$

$$c_0 = \ 920 \ \frac{J}{kg \cdot K}, \quad \kappa = 134 \ \frac{W}{m \cdot K}, \quad \alpha = 2{,}3 \cdot 10^{-5} \ \frac{1}{K}.$$

The initial parameters of the Rayleigh wave are:

$$k_1 = 1, \quad U_3 = -\frac{B_2 k_{22}}{B_3 k_{23}}. \tag{32}$$

In that event, from Eq. (30) we obtain: $U_2 = 1, \quad U_1 = \frac{2k_{22}k_{21}}{k_{21}^2+k_1^2}\left(\frac{B_2}{B_3} - 1\right)$.

From Eq. (25), we find the Rayleigh wave frequency rate corresponding to the specified designed parameters (31) and (32) $\omega_{el} = 4.6948 \cdot 10^{-3}$, which is 2909.7 Hz when converted to dimensional quantities. Figure 1 represents the graph of $|f(\omega)|$ function, entering into the left-hand side of Eq. (23), in the neighborhood of the ω_{el} value.

Figure 1 shows that the Rayleigh wave frequency rate $\omega_T$for a thermoelastic wave is slightly higher than the ω_{el} value. Refinement of this value using the iterative algorithm (28) and (29) gives the value $\omega_T \approx 4.6991 \cdot 10^{-3}$ (2912.4 Hz). Thus, the ratio error $\delta\omega_T$ in Rayleigh frequency rate calculation using Eq. (25) is $\delta\omega_T = \frac{|\omega_T - \omega_{el}|}{\omega_{el}} \cdot 100\% =$ $= \frac{0.0046991 - 0.0046948}{0.0046948} \cdot 100\% \approx 0.09\%$, thus, it is small to negligible.

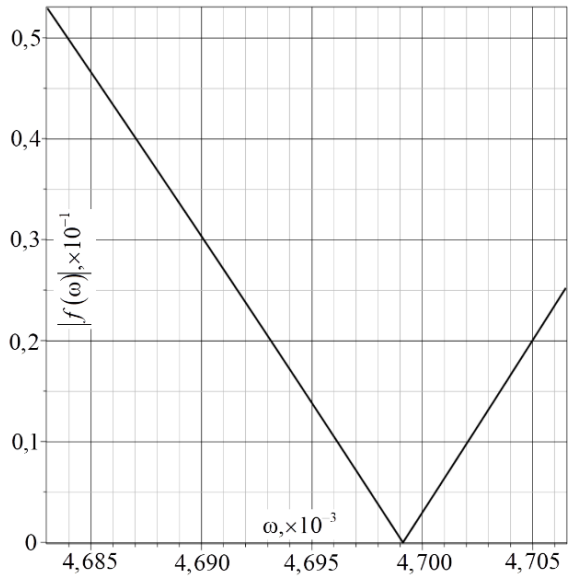


Fig. 1. Graph of $|f(\omega)|$ function for duralumin

The output computation is presented in Figs. 2–7, which demonstrate the spatiotemporal distribution of the mechanical displacements and the temperature field of the Rayleigh wave.

Comparison of amplitudes and frequencies of the elastic and thermoelastic Rayleigh waves allows to conclude that the effect of the temperature factors on the mode of the Rayleigh wave propagation in duralumin is next to none (Fig. 8). Sampling control makes it possible to state that material properties do not markedly affect coupling of temperature and mechanical fields in the surface Rayleigh waves. Table 1 lists the frequency values of the elastic and thermoelastic Rayleigh waves for various materials, whose properties are presented in Table 2 [36]. As can be seen from Table 1, frequency values of the Rayleigh waves in elastic and thermoelastic cases for the controlled materials are little different. Relative difference hardly exceeds 0.1 %. Herewith, such materials as duralumin, copper, and lead have significant expansion when heated. The same goes for other constructional materials used in engineering equipment, as well.

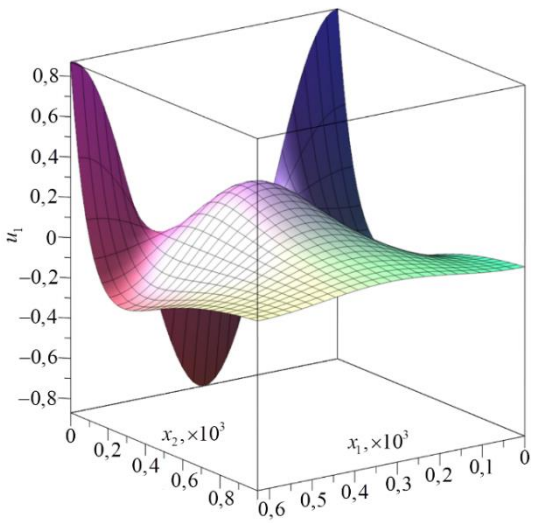


Fig. 2. Displacement $u_1(x_1, x_2, 0)$ for duralumin

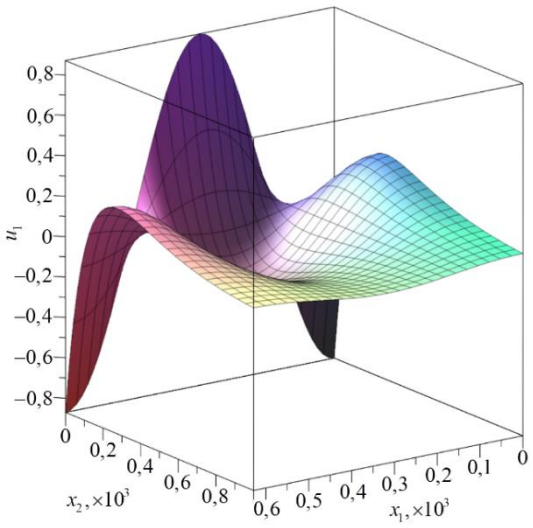


Fig. 3. Displacement $u_1(x_1, x_2, \pi/\omega_T)$ for duralumin

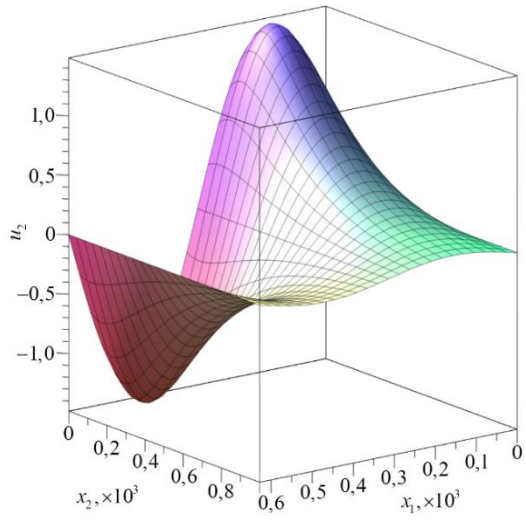


Fig. 4. Displacement $u_2(x_1, x_2, 0)$ for duralumin

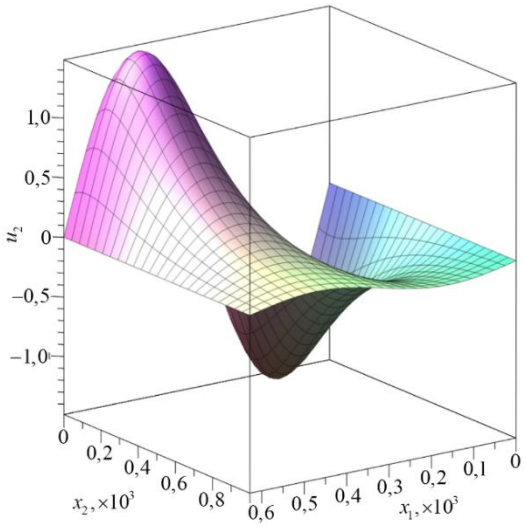


Fig. 5. Displacement $u_2(x_1, x_2, \pi/\omega_T)$ for duralumin

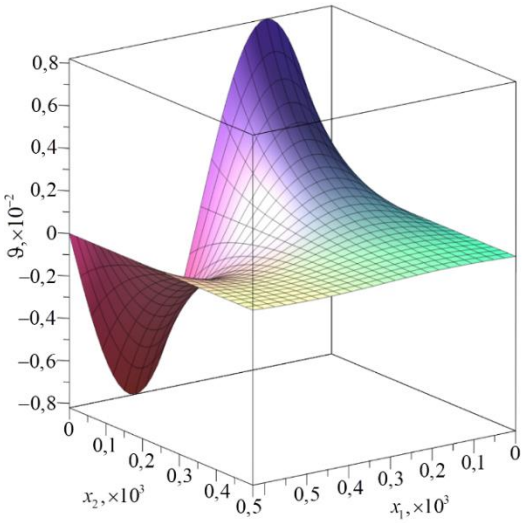


Fig. 6. Temperature increment $\vartheta(x_1, x_2, 0)$ for duralumin

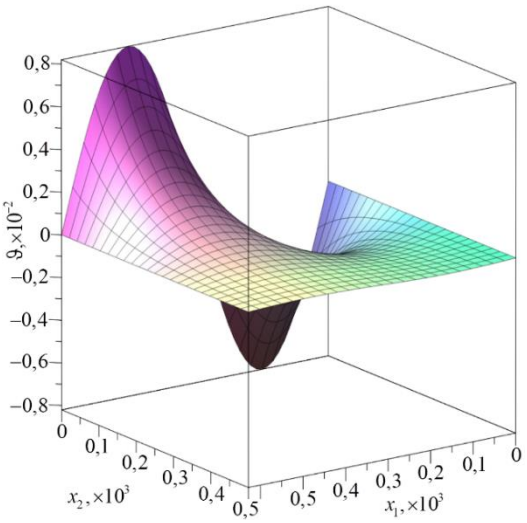


Fig. 7. Temperature increment $\vartheta(x_1, x_2, \pi/\omega_T)$ for duralumin

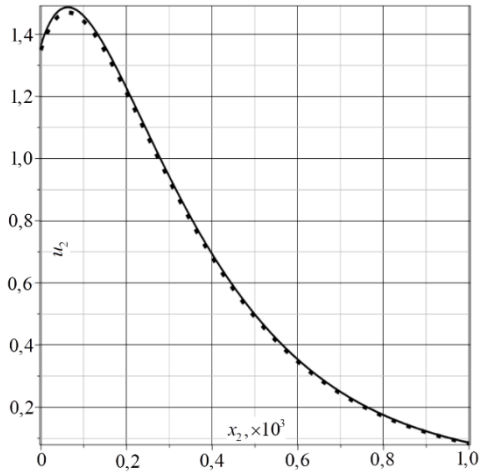


Fig. 8. Displacement $u_2(0, x_2, 0)$ for duralumin: solid line – thermoelastic wave, dotted line – elastic wave

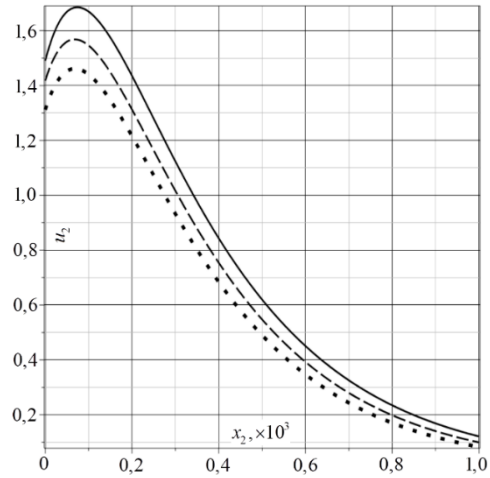


Fig. 9. Displacement $u_2(0, x_2, 0)$ for the hypothetical material: solid line – thermoelastic wave (TCLE is increased 5 times), dashed line – thermoelastic wave (TCLE is increased 3 times), dotted line – elastic wave

Table 1. Rayleigh wave frequency

	ω_{el}, Hz	ω_T, Hz	$\delta\omega_T$, %
Duralumin	2909.7	2912.7	0.092
Copper	2090.4	2091.5	0.053
Iron	3490.5	3492.8	0.066
Silicon	4344.7	4349.6	0.11
Window glass	2700.5	2700.8	0.011
Plumbage	1268.89	1268.9	0.0006
Diamond	9939.8	9940.2	0.004
Granite	2858.9	2859.2	0.0099
Floor ceramic	6880.2	6882.9	0.04
Epoxide resin (E-181)	1215.1	1215.9	0.07
Lead	705.0	705.4	0.060

Table 2. Physical properties of materials

	λ, $\frac{N}{m^2}$	μ, $\frac{N}{m^2}$	ρ, $\frac{kg}{m^3}$	c_0, $\frac{J}{kg\cdot K}$	κ, $\frac{W}{m\cdot K}$	α, $\frac{1}{K}$
Duralumin	$6.93\cdot10^{10}$	$2.56\cdot10^{10}$	2780	920	134	$2.3\cdot10^{-5}$
Copper	$9.51\cdot10^{10}$	$2.48\cdot10^{10}$	8930	400	401	$1.6\cdot10^{-5}$
Iron	$16.7\cdot10^{10}$	$11.2\cdot10^{10}$	7874	460	80.4	$1.13\cdot10^{-5}$
Silicon	$5.88\cdot10^{10}$	$5.17\cdot10^{10}$	2330	20.1	149	$2.5\cdot10^{-5}$
Window glass	$2.2\cdot10^{10}$	$2.2\cdot10^{10}$	2550	1000	0.96	$5.0\cdot10^{-6}$
Plumbage	$4.0\cdot10^{9}$	$4.0\cdot10^{9}$	2100	750	1750	$2.5\cdot10^{-6}$
Diamond	$2.78\cdot10^{11}$	$4.17\cdot10^{11}$	3500	502	2000	$1.0\cdot10^{-6}$
Granite	$1.81\cdot10^{10}$	$2.71\cdot10^{10}$	2750	800	3.5	$7.0\cdot10^{-6}$
Floor ceramic	$1.4\cdot10^{11}$	$1.4\cdot10^{11}$	2500	840	25	$6.0\cdot10^{-6}$
Epoxide resin (E-181)	$9.98\cdot10^{9}$	$1.90\cdot10^{9}$	1150	1000	1.2	$5.0\cdot10^{-5}$
Lead	$3.87\cdot10^{10}$	$6.29\cdot10^{9}$	11340	130	35.1	$2.9\cdot10^{-5}$

Yet, if we take a hypothetical material, for example, duralumin, whose linear thermal expansion coefficient (TCLE) is increased 3 times, the effect of the temperature field on the Rayleigh wave characteristics is already appreciable. The relative difference between the frequency of the elastic wave and that of the thermoelastic one is 0.6 %. TCLE increase by 5 times results in 1.2 % relative difference of frequencies. Displacement graphs for the analyzed cases are represented in Figs. 8 and 9.

Finally, graphs in Figs. 10 and 11 illustrate the dependence of the Rayleigh wave frequency on the wave number k_1 for the materials listed in Table 2 and for the hypothetical material with the increased TCLE. As it is evident, linear dependence, typical of the elastic waves [21], remains with reasonable accuracy for the thermoelastic waves as well and even for the hypothetical materials (Fig. 11).

The fact that the Rayleigh wave frequency in the case of the elastic wave is in linear dependence with k_1 is immediate from Eq. (25). Just assume that $\omega = ck_1$, where c is a phase velocity. This setting taking account of Eq. (24) reduces Eq. (25) to the below form:

$$\left(2-\frac{c^2}{\mu}\right)(\lambda-1+c^2)+2(1-\lambda)\sqrt{1-c^2}\sqrt{1-\frac{c^2}{\mu}}=0.$$

As this equation indicates, the phase velocity is not k_1 dependent, thus, the Rayleigh wave frequency in the elastic case is in linear dependence with k_1. Herewith, the similar setting in Eq. (23) does not allow to conclude the same for the thermoelastic wave. Moreover, as the roots s_i in Eq. (15) are not homogeneous functions of k_1 parameter, it

may be deduced that the function$f(\omega)$ in Eq. (23) is not a homogenous function of this parameter as well. In this way, the phase velocity of the thermoelastic wave is dependent on the wave number k_1, and the graphs in Figs. 10 and 11 are solely roughly linear.

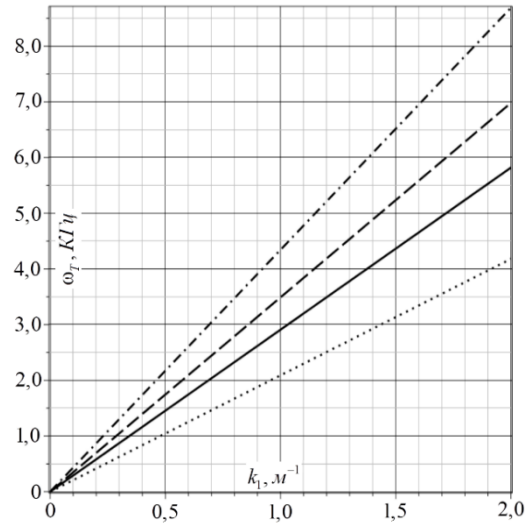


Fig. 10. Dependence of the Rayleigh wave frequency on the wave number for duralumin (solid line), copper (dotted line), iron (dashed line) and silicon (dotted-dashed line)

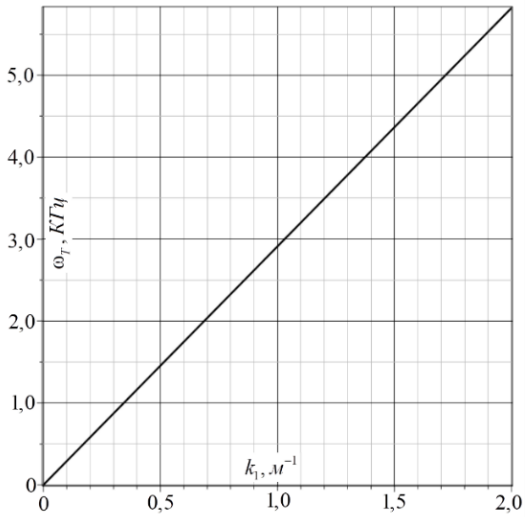


Fig. 11. Dependence of the Rayleigh wave frequency on the wave number for hypothetical material (TCLE is increased 3 times)

Conclusions

Hence, the paper developed a mathematical model of the thermoelastic Rayleigh wave propagation in an isotropic half-space. The equation of the thermoelastic Rayleigh wave was obtained using analytical procedures.

A parametric analysis of the obtained results has been performed, which makes it possible to estimate that the effect of temperature factors on the propagation of Rayleigh waves in real materials is practically not noticeable. Frequency and amplitude of the thermoelastic and elastic Rayleigh waves almost coincide for various real materials. The relative difference between the elastic and thermoelastic frequencies has an order of about $0{,}10\%$ even for materials with a high thermal coefficient of linear expansion.

However, numerical simulation based on the hypothetical material allows us to establish that the coefficient of linear expansion has a major effect on the degree of coupling of the temperature and mechanical fields. As it increases, there is a noticeable difference in the amplitudes and frequencies of the Rayleigh wave in the elastic and thermoelastic cases. Therein, in the case of the thermoelastic wave, unlike the elastic one, the linear dependence of the Rayleigh frequency on the wave numberk_1 is not maintained, but remains sufficiently close to it. This proximity to the linear relationship is also preserved for hypothetical materials sensitive to thermoelastic phenomena.

CRediT authorship contribution statement

Andrei V.Zemskov: writing – original draft, conceptualization, investigation.

Conflict of interest

The author declares that he has no conflict of interest.

References

1. Li X, Li Q, Shen H, Yang F, Zhang X. Research on oil exploration seismic Rayleigh wave imaging based on multi-channel analysis of surface waves and genetic-damped least squares joint inversion. *Journal of Applied Geophysics.* 2022;202: 104670.
2. Zhao W, Chen W, Yang D, Gao H, Xie P. Analytical solution for seismic response of tunnels with composite linings in elastic ground subjected to Rayleigh waves. *Soil Dynamics and Earthquake Engineering.* 2022;153: 107113.
3. Filippova AI, Solovey OA. Surface Wave Tomography of the Kola Peninsula and Adjacent Territories: Data on Dispersion of Group Velocities of Rayleigh and Love Waves. *Dokl. Earth Sc.* 2022;504: 380–384.
4. Yanovskaya TB. *Fundamentals of Seismology.* St. Petersburg: S.-Peterb. gos. un-ta; 2008. (In Russian)
5. Gulyaev YuV, Plesskij VP. Rasprostranenie poverhnostnyh akusticheskih voln v periodicheskih strukturah. *Uspekhi fizicheskih nauk.* 1989;157(1): 85-127. (In Russian)
6. Kumar R, Ahuja S, Garg SK. Rayleigh waves in isotropic microstretch thermoelastic diffusion solid half space. *Latin American Journal of Solids and Structures.* 2014;11: 299–319.
7. Sharma JN., Sharma YD., Sharma PK. On the propagation of elasto-thermodiffusive surface waves in heat conducting materials. *J. Sound Vib.* 2008;315(4–5): 927–938.
8. Sharma JN, Thakur N., Singh S. Propagation characteristics of elasto-thermodiffusive surface waves in semiconductor material half-space. *Therm Stresses.* 2007;30(4): 357–380.
9. Gupta V, Kumar M, Goel Sh. Study of different theories of thermoelasticity under the Rayleigh wave propagation along an isothermal boundary. *Materials Physics and Mechanics.* 2025;53(1): 22–37.
10. Sharma S, Batra D, Kumar R. Fractional strain analysis on reflection of plane waves at an impedance boundary of non-local swelling porous thermoelastic medium. *Materials Physics and Mechanics.* 2025;53(1): 1–21.
11. Bucur AV, Passarella F, Tibullo V. Rayleigh surface waves in the theory of thermoelastic materials with voids. *Meccanica.* 2014;49: 2069–2078.
12. Kumar R, Gupta V. Rayleigh waves in generalized thermoelastic medium with mass diffusion. *Canadian Journal of Physics.* 2015;93(10): 1039–1049.
13. Anya AI. Rayleigh waves in a rotating inhomogeneous half-space with magnetic effect under impedance and variable amplitudes of corrugation. *Materials Physics and Mechanics.* 2025;53(6): 164–181.
14. Roy I, Acharya DP, Acharya S. Propagation and reflection of plane waves in a rotating magneto-elastic fiber-reinforced semi-space with surface stress. *Mechanics and Mechanical Engineering.* 2018;22(4): 939–957.
15. Anya AI, Khan A. Reflection and propagation of magneto-thermoelastic plane waves at free surfaces of a rotating micropolar fiber-reinforced medium under GL theory. *International Journal of Acoustics and Vibration.* 2020;25(2): 190–199.
16. Anya AI, Khan A. Plane waves in a micropolar fiber-reinforced solid and liquid interface for non-insulated boundary under magneto-thermo-elasticity. *Journal of Computational Applied Mechanics.* 2022;53(2): 204–218.
17. Mahmoud SR. Influence of rotation and generalized magneto-thermoelastic on Rayleigh waves in a granular medium under effect of initial stress and gravity field. *Meccanica.* 2012;47: 1561–1579.
18. Rakshit S, Mistri KC, Das A, Lakshman A. Effect of interfacial imperfections on SH-wave propagation in a porous piezoelectric composite. *Mechanics of Advanced Materials and Structures.* 2022;29(25): 4008–4018.
19. Sharma JN, Walia V, Gupta SK. Effect of rotation and thermal relaxation on Rayleigh waves in piezothermoelastic half space. *International Journal of Mechanical Science.* 2008;50(3): 433–444.
20. Sharma JN., Othman MIA. Effect of rotation on generalized thermoviscoelastic Rayleigh-Lamb waves. Intern. *J. Solids Structures.* 2007;44: 4243–4255.
21. Sharma JN., Thakur MD. Effect of rotation on Rayleigh-Lamb waves in magneto-thermoelastic media. *J. Sound Vibration.* 2006;296(4–5): 871–887.
22. Kumar R., Kansal T. Effect of rotation on Rayleigh waves in an isotropic generalized thermoelastic diffusive half-space. *Arch. Mech.* 2008;60(5): 421–443.
23. Kumar R, Kansal T. Effect of Rotation on Rayleigh—Lamb Waves in an Isotropic Generalized Thermoelastic Diffusive Plate. *J Appl Mech Tech Phys.* 2010;51: 751–761.

24. Kumar R, Kansal T. Propagation of Rayleigh waves on free surface of transversely isotropic generalized thermoelastic diffusion. *Appl. Math. Mech. – Engl. Ed.* 2008;29(11): 1451–1462.
25. Kumar R, Kansal T. Rayleigh-Lamb Waves in Transversely Isotropic Thermoelastic Diffusive Layer. *Int J Thermophys.* 2009; 30: 710–733.
26. Kumar R, Kansal T. Propagation of cylindrical Rayleigh waves in a transversly isotropic thermoelastic diffusive solid half-space. *Journal of Theoretical and Applied Mechanics.* 2013;43(3): 3–20.
27. Singh B, Kaur B. Rayleigh-type surface wave on a rotating orthotropic elastic half-space with impedance boundary conditions. *Journal of Vibration and Control.* 2020;26(21–22): 1980–1987.
28. Sahu SA, Mondal S, Nirwal S. Mathematical analysis of Rayleigh waves at the nonplanner boundary between orthotropic and micropolar media. *International Journal of Geomechanics.* 2023;23(3): 04022313.
29. Shaw S, Othman MIA. Characteristics of Rayleigh wave propagating in an orthotropic magneto-thermoelastic half-space: An eigen function expansion method. *Applied Mathematical Modelling.* 2019;67: 605–620.
30. Kovács R. Heat equations beyond Fourier: From heat waves to thermal metamaterials. *Physics Reports.* 2024;1048: 1-75.
31. Astapov AN, ZHavoronok SI, Kurbatov AS, Rabinskij LN, Tushavina OV. Main Problems in the Creation of Thermal-Protection Systems Based on Structurally Heterogeneous Materials and the Methods of Their Solution. *High Temp.* 2021;59: 346–372.
32. Zemskov AV, Tarlakovskij DV. *Modeling of mechanical diffusion processes in multicomponent bodies with flat boundaries*. Moscow: FIZMATLIT; 2021 (In Russian)
33. Vestyak VA, Zemskov AV, Tarlakovskij DV, Fedotenkov GV. *Mathematical Foundations of Thermoelasticity: A Tutorial*. Moscow: MAI; 2021. (In Russian)
34. Novackij V. *Dynamic problems of thermoelasticity*. Moscow: MIR; 1970. (In Russian)
35. Amenzade YuA. *Theory of Elasticity: A Textbook for Universities*. Baku: Azerbajdzhanskoe gosudarstvennoe izdatel'stvo uchebno-pedagogicheskoj literatury; 1968. (In Russian)
36. Babichev AP, Babushkina NA, Bratkovsky AM. Physical quantities: reference manual. Moscow: Energoatomizdat; 1991. (In Russian)